A. Zee

Quantum Field Theory in a Nutshell

简明量子场论

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To my parents,

who valued education above all else

Preface

As a student, I was rearing at the bit, after a course on quantum mechanics, to learn quantum field theory, but the books on the subject all seemed so formidable. Fortunately, I came across a little book by Mandl on field theory, which gave me a taste of the subject enabling me to go on and tackle the more substantive texts. I have since learned that other physicists of my generation had similar good experiences with Mandl.

In the last three decades or so, quantum field theory has veritably exploded and Mandl would be hopelessly out of date to recommend to a student now. Thus I thought of writing a book on the essentials of modern quantum field theory addressed to the bright and eager student who has just completed a course on quantum mechanics and who is impatient to start tackling quantum field theory.

I envisaged a relatively thin book, thin at least in comparison with the many weighty tomes on the subject. I envisaged the style to be breezy and colloquial, and the choice of topics to be idiosyncratic, certainly not encyclopedic. I envisaged having many short chapters, keeping each chapter "bite-sized."

The challenge in writing this book is to keep it thin and accessible while at the same time introducing as many modern topics as possible. A tough balancing act! In the end, I had to be unrepentantly idiosyncratic in what I chose to cover. Note to the prospective book reviewer: You can always criticize the book for leaving out your favorite topics. I do not apologize in any way, shape, or form. My motto in this regard (and in life as well), taken from the Ricky Nelson song "Garden Party," is "You can't please everyone so you gotta please yourself."

This book differs from other quantum field theory books that have come out in recent years in several respects.

I want to get across the important point that the usefulness of quantum field theory is far from limited to high energy physics, a misleading impression my generation of theoretical physicists were inculcated with and which amazingly enough some recent textbooks on quantum field theory (all written by high energy physicists) continue to foster. For instance, the study of driven surface growth provides a particularly clear, transparent, and physical example of the importance of the renormalization group in quantum field theory. Instead of being entangled in all sorts of conceptual irrelevancies such as divergences, we have the obviously physical notion of changing the ruler used to measure the fluctuating surface. Other examples include random matrix theory and Chern-Simons gauge theory in quantum Hall fluids. I hope that condensed matter theory students will find this book helpful in getting a first taste of quantum field theory. The book is divided into eight parts,[1] with two devoted more or less exclusively to condensed matter physics.

I try to give the reader at least a brief glimpse into contemporary developments, for example, just enough of a taste of string theory to whet the appetite. This book is perhaps also exceptional in incorporating gravity from the beginning. Some topics are treated quite differently than in traditional texts. I introduce the Faddeev-Popov method to quantize electromagnetism and the language of differential forms in developing Yang-Mills theory, for example.

The emphasis is resoundingly on the conceptual rather than the computational. The only calculation I carry out in all its gory details is that of the magnetic moment of the electron. Fortunately, many excellent texts focus on calculational techniques. Throughout, specific examples rather than heavy abstract formalism will be favored. Instead of dealing with the most general case, I always opt for the simplest.

I had to struggle constantly between clarity and wordiness. In trying to anticipate and to minimize what would confuse the reader, I often find that I have to belabor certain points more than what I would like.

I tried to avoid the dreaded phrase "It can be shown that . . . " as much as possible. Otherwise, I could have written a much thinner book than this! There are indeed thinner books on quantum field theory: I looked at a couple and discovered that they hardly explain anything. I must confess that I have an almost insatiable desire to explain.

As the manuscript grew, the list of topics that I reluctantly had to drop also kept growing. So many beautiful results, but so little space! It almost makes me ill to think about all the stuff (bosonization, instanton, conformal field theory, etc., etc.) I had to leave out. As one colleague remarked, the nutshell is turning into a coconut shell!

Shelley Glashow once described the genesis of physical theories: "Tapestries are made by many artisans working together. The contributions of separate workers cannot be discerned in the completed work, and the loose and false threads have been covered over." I regret that other than giving a few tidbits here and there I could not go into the fascinating history of quantum field theory, with all its defeats and triumphs. On those occasions when I refer to original papers I suffer from that disconcerting quirk of human psychology of tending to favor my own more than decorum might have allowed. I certainly did not attempt a true bibliography.

The genesis of this book goes back to the quantum field theory course I taught as a beginning assistant professor at Princeton University. I had the enormous good fortune of having Ed Witten as my teaching assistant and grader. Ed produced lucidly written solutions to the homework problems I assigned, to the extent that the next year I went to the chairman to ask "What is wrong with the TA I have this year? He is not half as good as the guy last year!" Some colleagues asked me

[1] Murray Gell-Mann used to talk about the eightfold way to wisdom and salvation in Buddhism (M. Gell-Mann and Y. Ne'eman, *The Eightfold Way*). Readers familiar with contemporary Chinese literature would know that the celestial dragon has eight parts.

to write up my notes for a much needed text (those were the exciting times when gauge theories, asymptotic freedom, and scores of topics not to be found in any texts all had to be learned somehow) but a wiser senior colleague convinced me that it might spell disaster for my research career. Decades later, the time has come. I particularly thank Murph Goldberger for urging me to turn what expository talents I have from writing popular books to writing textbooks. It is also a pleasure to say a word in memory of the late Sam Treiman, teacher, colleague, and collaborator, who as a member of the editorial board of Princeton University Press persuaded me to commit to this project. I regret that my slow pace in finishing the book deprived him of seeing the finished product.

Over the years I have refined my knowledge of quantum field theory in discussions with numerous colleagues and collaborators. As a student, I attended courses on quantum field theory offered by Arthur Wightman, Julian Schwinger, and Sidney Coleman. I was fortunate that these three eminent physicists each has his own distinctive style and approach.

The book has been tested "in the field" in courses I taught. I used it in my field theory course at the University of California at Santa Barbara, and I am grateful to some of the students, in particular Ted Erler, Andrew Frey, Sean Roy, and Dean Townsley, for comments. I benefitted from the comments of various distinguished physicists who read all or parts of the manuscript, including Steve Barr, Doug Eardley, Matt Fisher, Murph Goldberger, Victor Gurarie, Steve Hsu, Bei-lok Hu, Clifford Johnson, Mehran Kardar, Ian Low, Joe Polchinski, Arkady Vainshtein, Frank Wilczek, Ed Witten, and especially Joshua Feinberg. Joshua also did many of the exercises.

Talking about exercises: You didn't get this far in physics without realizing the absolute importance of doing exercises in learning a subject. It is especially important that you do most of the exercises in this book, because to compensate for its relative slimness I have to develop in the exercises a number of important points some of which I need for later chapters. Solutions to some selected problems are given.

I will maintain a web page http://theory.kitp.ucsb.edu/~zee/nuts.html listing all the errors, typographical and otherwise, and points of confusion that will undoubtedly come to my attention.

I thank my editors, Trevor Lipscombe, Joe Wisnovsky, and Sarah Green and the staff of Princeton Editorial Associates (particularly Cyd Westmoreland and Evelyn Grossberg) for their advice and for seeing this project through. Finally, I thank Peter Zee for suggesting the cover painting and Catherine Zee for playing Mozart sonatas while I proofread.

Convention, Notation, and Units

For the same reason that we no longer use a certain king's feet to measure distance, we use natural units in which the speed of light c and the Dirac symbol $\hbar$ are both set equal to 1. Planck made the profound observation that in natural units all physical quantities can be expressed in terms of the Planck mass $M_{\text{Planck}} \equiv 1/\sqrt{G_{\text{Newton}}} \simeq 10^{19}$Gev. The quantities c and $\hbar$ are not so much fundamental constants as conversion factors. In this light, I am genuinely puzzled by condensed matter physicists carrying around Boltzmann's constant k, which is no different from the conversion factor between feet and meters.

Spacetime coordinates x^μ are labeled by Greek indices ($\mu = 0, 1, 2, 3$) with the time coordinate x^0 sometimes denoted by t. Space coordinates x^i are labeled by Latin indices ($i = 1, 2, 3$) and $\partial_\mu \equiv \partial/\partial x^\mu$. We use a Minkowski metric $\eta^{\mu\nu}$ with signature ($+, -, -, -$) so that $\eta^{00} = +1$. We write $\eta^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi = \partial_\mu\varphi\partial^\mu\varphi = (\partial\varphi)^2 = (\partial\varphi/\partial t)^2 - \sum_i(\partial\varphi/\partial x^i)^2$. The metric in curved spacetime is always denoted by $g^{\mu\nu}$, but often I will also use $g^{\mu\nu}$ for the Minkowski metric when the context indicates clearly that we are in flat spacetime.

Since I will be talking mostly about relativistic quantum field theory in this book I will without further clarification use a relativistic language. Thus, when I speak of momentum, unless otherwise specified, I mean energy and momentum. Also since $\hbar = 1$, I will not distinguish between wave vector k and momentum, and between frequency ω and energy.

In local field theory I deal primarily with the Lagrangian density $\mathcal{L}$ and not the Lagrangian $L = \int d^3x\, \mathcal{L}$. As is common practice in the literature and in oral discussion, I will often abuse terminology and simply refer to $\mathcal{L}$ as the Lagrangian. I will commit other minor abuses such as writing 1 instead of I for the unit matrix. We use the same symbol φ for the Fourier transform $\varphi(k)$ of a function $\varphi(x)$ whenever there is no risk of confusion, as is almost always the case. I prefer an abused terminology to cluttered notation and unbearable pedantry.

The symbol $*$ denotes complex conjugation, and $\dagger$ hermitean conjugation: The former applies to a number and the latter to an operator. I also use the notation c.c. and h.c. Often when there is no risk of confusion I abuse the notation, using $\dagger$ when I should use $*$. For instance, in a path integral, bosonic fields are just number-valued fields, but nevertheless I write $\varphi^\dagger$ rather than φ^*. For a matrix M, then of course $M^\dagger$ and M^* should be carefully distinguished from each other.

I made an effort to get factors of 2 and π right, but some errors will be inevitable.

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PART I

MOTIVATION AND FOUNDATION

Chapter I.1

Who Needs It?

Who needs quantum field theory?

Quantum field theory arose out of our need to describe the ephemeral nature of life.

No, seriously, quantum field theory is needed when we confront simultaneously the two great physics innovations of the last century of the previous millennium: special relativity and quantum mechanics. Consider a fast moving rocket ship close to light speed. You need special relativity but not quantum mechanics to study its motion. On the other hand, to study a slow moving electron scattering on a proton, you must invoke quantum mechanics, but you don't have to know a thing about special relativity.

It is in the peculiar confluence of special relativity and quantum mechanics that a new set of phenomena arises: Particles can be born and particles can die. It is this matter of birth, life, and death that requires the development of a new subject in physics, that of quantum field theory.

Let me give a heuristic discussion. In quantum mechanics the uncertainty principle tells us that the energy can fluctuate wildly over a small interval of time. According to special relativity, energy can be converted into mass and vice versa. With quantum mechanics and special relativity, the wildly fluctuating energy can metamorphose into mass, that is, into new particles not previously present.

Write down the Schrödinger equation for an electron scattering off a proton. The equation describes the wave function of one electron, and no matter how you shake and bake the mathematics of the partial differential equation, the electron you follow will remain one electron. But special relativity tells us that energy can be converted to matter: If the electron is energetic enough, an electron and a positron ("the antielectron") can be produced. The Schrödinger equation is simply incapable of describing such a phenomenon. Nonrelativistic quantum mechanics must break down.

You saw the need for quantum field theory at another point in your education. Toward the end of a good course on nonrelativistic quantum mechanics the interaction between radiation and atoms is often discussed. You would recall that the electromagnetic field is treated as a field; well, it is a field. Its Fourier components are quantized as a collection of harmonic oscillators, leading to creation and annihilation operators for photons. So there, the electromagnetic field is a quantum

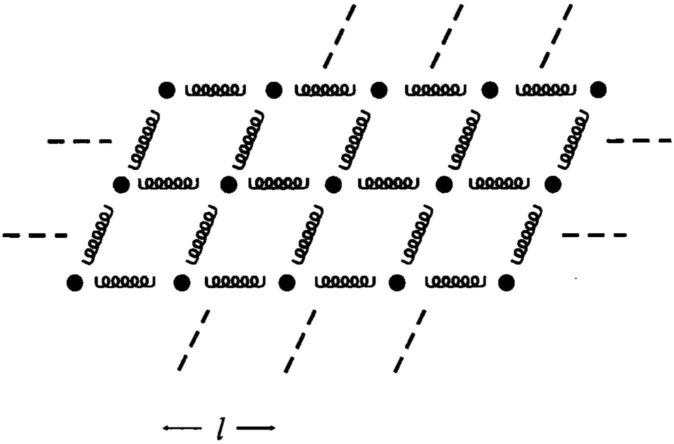

Figure I.1.1

field. Meanwhile, the electron is treated as a poor cousin, with a wave function $\Psi(x)$ governed by the good old Schrödinger equation. Photons can be created or annihilated, but not electrons. Quite aside from the experimental fact that electrons and positrons could be created in pairs, it would be intellectually more satisfying to treat electrons and photons, as they are both elementary particles, on the same footing.

So, I was more or less right: Quantum field theory is a response to the ephemeral nature of life.

All of this is rather vague, and one of the purposes of this book is to make these remarks more precise. For the moment, to make these thoughts somewhat more concrete, let us ask where in classical physics we might have encountered something vaguely resembling the birth and death of particles. Think of a mattress, which we idealize as a 2-dimensional lattice of point masses connected to each other by springs (Fig. I.1.1) For simplicity, let us focus on the vertical displacement [which we denote by $q_a(t)$] of the point masses and neglect the small horizontal movement. The index a simply tells us which mass we are talking about. The Lagrangian is then

$$L = \tfrac{1}{2}\Big(\sum_a m\dot{q}_a^2 - \sum_{a,b} k_{ab}q_aq_b - \sum_{a,b,c} g_{abc}q_aq_bq_c - \cdots\Big) \tag{1}$$

Keeping only the terms quadratic in q (the "harmonic approximation") we have the equations of motion $m\ddot{q}_a = -\sum_b k_{ab}q_b$. Taking the q's as oscillating with frequency ω, we have $\sum_b k_{ab}q_b = m\omega^2 q_a$. The eigenfrequencies and eigenmodes are determined, respectively, by the eigenvalues and eigenvectors of the matrix k. As usual, we can form wave packets by superposing eigenmodes. When we quantize the theory, these wave packets behave like particles, in the same way that electromagnetic wave packets when quantized behave like particles called photons.