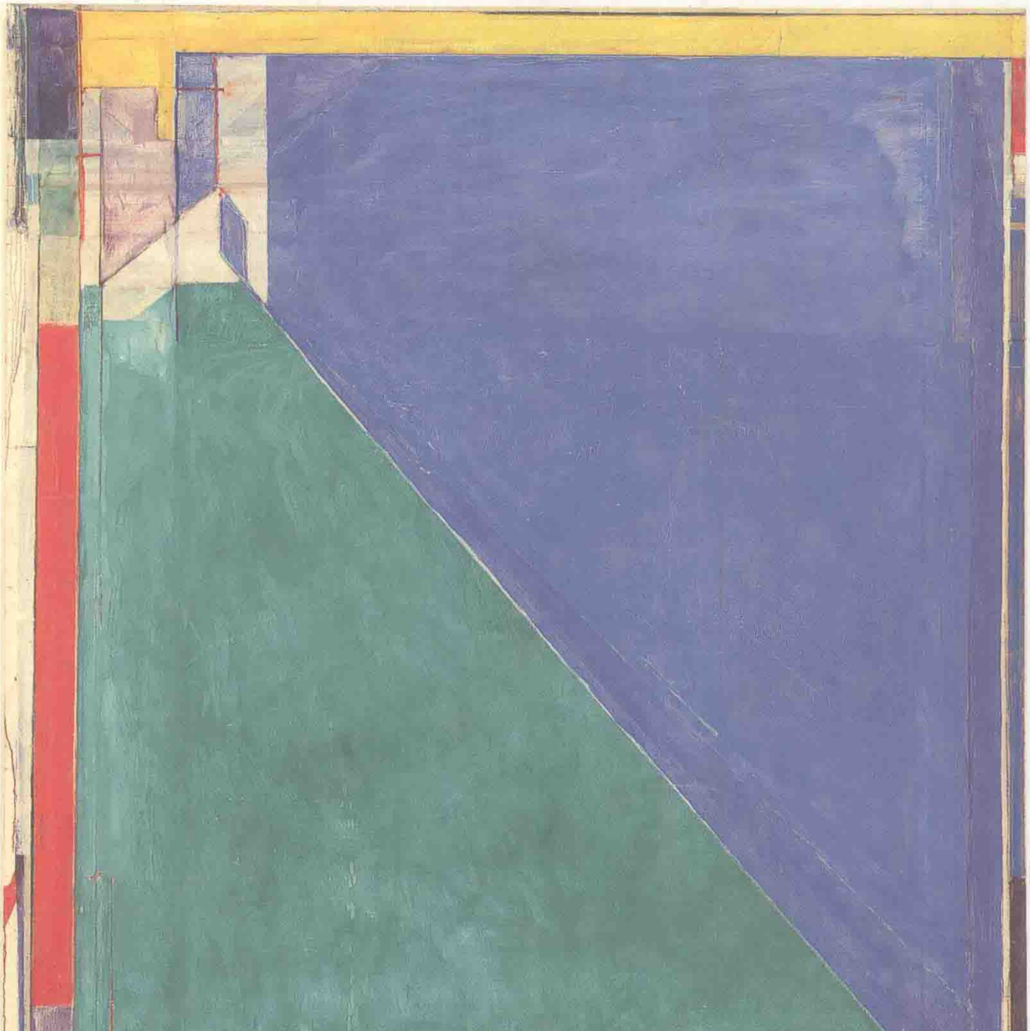
The Art and Science of Logic



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The Art And Science Of Logic

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Preface

This book is a comprehensive introduction to logic. Logic continues to occupy an important position in contemporary university curriculums for much the same reason it occupied such a position in ancient academies, medieval centers of learning, and Enlightenment universities: Its object of study, reasoning, is fundamental to all intellectual activity and to most other human endeavors. My aim in this book is to introduce readers to the traditional areas of logic: (1) "informal logic," concerned with language, communication, and fallacies; (2) "formal logic," including the theory of the syllogism and modern symbolic techniques; and (3) "inductive logic," the study of empirical reasoning.

In its structure, therefore, this book follows the now common pattern for introductory logic texts established by John Stuart Mill, Morris Cohen, Ernest Nagel, and others. Part I introduces two of the basic concepts of logic, truth and validity, in the context of a general theory of argument. It proceeds to discuss other criteria of good reasoning—evidence, relevance, and grounding—and ways of recognizing violations of them. Part I concludes with a discussion of meaning and definition. Part II covers sentential logic, developing three methods for evaluating sentential arguments: truth tables, semantic tableaux, and natural deduction. Part III covers predicate logic. It presents syllogistic logic and the modern theory of quantification, extending semantic tableau and natural deduction methods to the latter. Part IV covers inductive logic, discussing generalizations, analogies, causal inferences, and explanations. Finally, two appendixes present alternative deduction systems for sentential and quantificational logic.

As Richard Whately observed in his 1826 book, *Elements of Logic*, logic is both an art and a science. Logic is concerned with constructing a theory of correct reasoning, making it a science. Indeed, modern symbolic developments have led to sophisticated mathematical theories of reasoning. But logic is also concerned with applying theory to practice, making it an art. Most people study logic to improve their ability to reason: to argue, to analyze, and to think critically about issues that concern them. This book, therefore, concentrates on the art of logic. It focuses on applying logical concepts and theories to reasoning in everyday language in a wide variety of contexts. But it's important to remember that without theories of reasoning, there would be nothing to apply. The art of logic depends crucially on the science of logic.

Organization and New Features

Despite its traditional structure, this book contains a number of new features. The first is a novel treatment of fallacies as aspects of a general theory of argumentation and communication. Part I of the book concerns the art of logic. Much of it is devoted to a discussion of fallacies. Too often, such discussions read like mere lists of mistakes. As a chorus of authors has pointed out, there has been no theory of fallacy to give these discussions structure. Moreover, if fallacies are invalid forms of argument, as many authors have taken them to be, then there can be no theory of fallacy. Part I locates the discussion of fallacies in the context of a theory of conditions for good argument. The theory derives from a general theory of communication that Robert Stalnaker, Hans Kamp, and Irene Heim, among others, have developed over the past ten to fifteen years. From this approach, fallacies emerge not as kinds of invalidity but as violations of other conditions of good argumentation.

Part I begins with a chapter developing this general theory. It presents fundamental logical concepts—argument, validity, truth, implication, equivalence, logical truth, and contradictoriness—defining these concepts rigorously while concentrating on their usefulness in understanding natural-language inference. Truth and validity emerge as two of the conditions good arguments ought to meet. Chapter 2 discusses two other conditions for good arguments: evidence and relevance. Violating these conditions leads to fallacies such as begging the question and *ignoratio elenchi*. Chapter 3 discusses the grounding condition for good arguments: An argument should take for granted only the set of beliefs and assumptions that occupies the common ground of the conversation in which it occurs. Violations of this condition lead to fallacies such as appeals to authority, emotion, force, and pity, as well as accident and misapplication. Chapter 4 concerns problems arising from miscommunication: equivocation, amphiboly, accent, composition, and division. The chapter also presents the traditional theory of definition as a way of clarifying meaning and avoiding such problems.

A second novelty of this book is its treatment of semantic tableaux. Based on E. W. Beth's tableaux, the method bears much similarity to Richard Jeffrey's truth trees. It is very easy to teach and to learn; it directly mirrors the semantics of the logical operators. Tableaux provide a ready test for validity and other logical properties in both sentential and predicate logic. (Of course, in full predicate logic, the method is not a decision procedure.) Semantic tableaux are even simpler than truth trees, for tableaux have the subformula property: On a tableau, formulas are decomposed into their subformulas.

A third novelty of this book is its presentation of three deduction systems, each of which has some special characteristics. The system in the text itself descends from the systems presented in Irving Copi's *Introduction to Logic*. It uses only simple rules, that is, rules stating that formulas of certain kinds can be deduced from formulas of other kinds. There are, consequently, no subordinate proofs. The system of this book goes beyond Copi's system, however, in three respects. First, it doesn't require premises; categorical as well as hypothetical proofs are possible. Second, the rules divide into basic and derived rules. The derived rules make proofs much easier, but they aren't required. The distinction allows instructors to omit some rules if they choose while maintaining the completeness of the deduction system. Also, a limited, indirect proof method can be added to simplify proofs further. Third, the system extends to encompass all of predicate logic. Many instructors, of course, will not want to cover polyadic predicates and multiply quantified formulas. They can avoid these topics without encountering unnecessary complications simply by omitting parts of Chapter 13. For those who want to cover full predicate logic, however, the extension of the system is presented there.

Appendix I presents a second deduction system, in the style of E. J. Lemmon and of Copi's *Symbolic Logic*. Appendix II presents a third deduction system, based on one developed by Donald Kalish and Richard Montague and presented in my earlier book, *Deduction*. Both systems use conditional and indirect proof, thus making use of subordinate proofs, and mark a distinction between basic and derived rules. The pattern of rules is easy to understand: Most connectives come with introduction and exploitation rules. The former introduce formulas with a given connective as main connective into proofs, while the latter exploit the presence of such formulas to deduce others. (In Appendix I, the rules retain their traditional names; in Appendix II, they have explicit introduction/exploitation labels.) The system used in Appendix II has an unusual form that greatly simplifies deductive rules and strategies.

The treatment of quantification in this book is slightly unorthodox. Neither the tableau system nor the deduction systems use free variables; instantiation always involves a constant. Indeed, in this book, formulas never contain free variables; quantifiers on the same variable never overlap in scope. There are no vacuous quantifiers. The need for distinguishing free from bound variables arises only in the final section of Chapter 13, in order to achieve a complete system of rules for full predicate logic without resorting to subordinate proofs. These unusual features of the language of predicate logic reflect the semantics and intended use of the language and simplify quantificational tableau and deduction rules.

Part IV discusses induction. Chapter 14 concerns generalization, both enumerative and statistical, and analogy. It analyzes the forms such arguments take and the means of evaluating them. Chapter 15 discusses the structure and evaluation of causal reasoning by considering Mill's methods.

Unlike most textbooks, this book does not maintain that the word 'cause' is ambiguous. The context-sensitive approach to language developed in Part I allows for a uniform analysis of the concept of causation. Chapter 16 discusses explanation. Here, too, taking context into account allows for a treatment simpler but richer than the usual.

Throughout, I have tried to present logical theories and their applications as simply and elegantly as possible. I have kept the focus on natural language: on constructing English arguments, analyzing their structure, and evaluating them. Natural language, after all, makes use of far richer resources than any introductory logic text can explicate. I hope that by equipping readers with a refined set of logical tools and making them aware of the issues that arise in using them, this book will enable its readers to go beyond its own limited realm.

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PART

I

Reasoning and Language

CHAPTER

1

TRUTH AND VALIDITY

Logic is the study of correct reasoning. Aristotle (384–322 B.C.) founded the discipline of logic as a system of principles on which all other knowledge rests. Indeed, logic pertains to all subjects, since people can reason about anything they can think about. Politics, the arts, literature, business, the sciences, and everyday problems are all subjects open to reasoning. Sometimes the reasoning is good; sometimes, not so good. People use logic to tell the difference.

Reasoning well is more than an academic exercise. Gathering information, making decisions, and carrying out plans all require reasoning. Good reasoning tends to lead to accurate information, good decisions, and appropriate plans, whereas bad reasoning tends to lead to inaccuracies, bad decisions, and misguided plans. The consequences of reasoning well or poorly also reach beyond the life of any one person. The twentieth century has witnessed remarkable advances in science and technology that have improved the lives of vast numbers of people. These applications of scientific method required a great deal of good and highly sophisticated reasoning. But the twentieth century has also suffered the results of reasoning gone astray. There were arguments—seriously flawed arguments, to be sure—that led to Gallipoli and Stalingrad, Dresden and Hiroshima, Auschwitz and Kolyma.

Part I of this book discusses both the fundamental features of good reasoning and common sources of bad reasoning. Parts II and III present the core of modern symbolic logic: a rigorous theory of inference. Part IV presents the fundamentals of the theory of induction, the kind of reasoning used frequently in the sciences.

Using logic, we can evaluate bits of reasoning as proper or improper, good or bad. Logic is not the study of how people do reason, but how they should reason. We might put this point differently by saying that logic does not describe the psychology of reasoning, with its flashes of insight and oversight; it prescribes methods for justifying reasoning, that is, for showing that a given bit of reasoning is proper. Just as arithmetic describes the rules for addition rather than the psychological process of addition, logic describes the rules for correct reasoning, not the process of reasoning. Logic thus describes an ideal that actual reasoning strives for but often fails to reach.

Logic, however, involves not only constructing abstract theories of reasoning but also applying them to arguments in natural languages such as English, French, Arabic, or Japanese. Logic can help in devising and evaluating arguments. In this sense, logic is an art as well as a science; it comprises both developing principles of correct reasoning and putting them into practice.

1.1 ARGUMENTS

Arguments are bits of reasoning in language. Frequently, we think of arguments as altercations or conflicts. Sometimes, however, we speak of a politician arguing for the passage of a bill, a lawyer arguing a case, or a reader of spy novels arguing that *Tinker, Tailor, Soldier, Spy* is the pinnacle of that genre. In this latter sense, an argument starts with some assertions and tries to justify a particular thesis. Arguments are attempts to establish a conclusion.

Arguments in natural language can be complicated. A lawyer arguing for the innocence of a client, for instance, offers many specific arguments in presenting the case. The lawyer may argue that a piece of evidence is inadmissible, that results from a lab test are ambiguous, that the client could not have reached the scene of the crime by the time it was committed, and so on. All these smaller arguments form part of the larger argument for the client's innocence.

We can divide arguments, then, into two groups: *extended* arguments, which contain other arguments, and *simple* arguments, which don't. A simple argument, like an extended argument, starts with some assertions trying to justify a thesis. The initial assertions of the argument are its *premises;* the thesis the argument tries to justify is its *conclusion.* Because extended arguments are good only if the simple arguments within them are good, we'll usually analyze simple arguments. In fact, we'll be so often concerned with simple arguments that we'll drop the adjective *simple* and speak simply of *arguments.*

> DEFINITION An *argument* consists of a finite sequence of sentences, called *premises,* together with another sentence, the *conclusion,* which the premises are taken to support.

An argument is a string or sequence of sentences.[1] The sentences that make up the argument are in a particular order, whether the argument is spoken, written, or encoded in a computer language. For our purposes in this text, the order in which an argument presents its premises rarely makes a difference. So, we generally won't worry about order of presentation. But it's important that the string of premises be finite. If the premises never end, the conclusion is never established.

The word 'sentence' has two uses relevant to logic. According to the first, a sentence is any grammatical string of words that ends with a period (or exclamation point, or question mark). Winston Churchill once said that no one who could write an English sentence would long be out of a job; he probably had this sense of the word in mind. Computer programs for counting the number of sentences in a text count sentences in this sense; they count the strings between the above punctuation marks (generally, by counting those marks themselves).

Logicians and linguists use 'sentence' in a second, slightly different way. Consider this variant of the traditional aphorism, "When the going gets tough, the tough get going," called Lynch's Law.

(1) When the going gets tough, everyone leaves.

How many sentences are there in (1)? In the first sense of 'sentence', the answer is one. But, in another sense, it would be reasonable to answer three. Lynch's law itself is a sentence. Within it, furthermore, are two other sentences: 'The going gets tough' and 'Everyone leaves'. The aphorism (1) results from combining these two shorter sentences. Logicians use 'sentence' in this second sense, roughly equivalent to 'independent clause', in which one sentence may be a part or component of another. The number of premises in an argument, therefore, is not always one less than the number of periods, exclamation points, and so on. Indeed, an entire argument, with premises and conclusion, can appear within a single sentence (in the first sense of 'sentence').

This example of a simple argument is Abraham Lincoln's explanation of why he did not expect to marry:

(2) I have come to the conclusion never again to think of marrying, and for this reason: I can never be satisfied with anyone who would be blockhead enough to marry me.

We can represent Lincoln's argument as

(3) I can never be satisfied with anyone who would be blockhead enough to marry me.
∴ I shall never again think of marrying.

When we write an argument "officially," in what we'll call *standard form,* we'll list the premises in the order in which they are given and then list the conclusion. In addition, we'll preface the conclusion with the symbol ∴, which means "therefore." So, in our official representations, conclusions will always come last. This isn't true in natural language, as Lincoln's argument shows. Conclusions may appear at the beginning, in the middle, or at the end of arguments, if they are stated at all.

Another simple argument is from French essayist Joseph Joubert (1754–1824):

(4) Nothing that is proved is obvious; for what is obvious shows itself and cannot be proved.

This argument, too, starts with its conclusion and then introduces premises to justify it. In standard form, the argument becomes:

(5) What is obvious shows itself and cannot be proved.
∴ Nothing that is proved is obvious.

Unlike simple arguments, extended arguments contain not only premises and conclusion but also other arguments. Extended arguments may consist of several simple arguments in sequence. They may contain other extended arguments. And they may consist of a list of premises, followed by several conclusions stated at once. Typically, the conclusion of one part of an extended argument serves as a premise for another part.

Consider an extended argument that the ancient skeptic Sextus Empiricus (*circa* A.D. 150–225) offered to challenge our usual notion of change:

(6) If Socrates died, he died either when he was living or when he was dead. But he did not die while living; for assuredly he was living, and as living he had not died. Nor when he was dead; for then he would be twice dead. Therefore Socrates did not die.

The argument as a whole tries to establish the startling conclusion that Socrates did not die. Moreover, it includes two subarguments. Signaling them is the word 'for', which, as in the argument from Joubert above, tends to follow a conclusion and introduce a premise supporting it. In this argument, the first 'for' introduces premises supporting the intermediate conclusion that Socrates did not die while living:

(7) Assuredly, Socrates was living.
As living, he had not yet died.
∴ Socrates did not die while living.

The second 'for' introduces premises to support the conclusion that Socrates didn't die while he was dead:

(8) If Socrates died while he was dead, he would be twice dead.
∴ Socrates did not die while he was dead.