

# Control Systems

Theory and Implementations

Sisil Kumarawadu

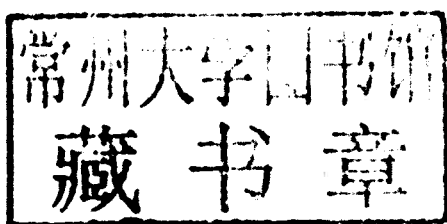


Alpha  
Science

# **Control Systems**

## **Theory and Implementation**

**Sisil Kumarawadu**



**Alpha Science International Ltd.**

Oxford, U.K.

## **Control Systems**

*Theory and Implementation*

212 pgs. | 78 figs. | 10 tbls.

**Sisil Kumarawadu**

Department of Electrical Engineering

University of Moratuwa

Moratuwa, Sri Lanka

Copyright © 2010

**ALPHA SCIENCE INTERNATIONAL LTD.**

7200 The Quorum, Oxford Business Park North

Garsington Road, Oxford OX4 2JZ, U.K.

**[www.alphasci.com](http://www.alphasci.com)**

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without prior written permission of the publisher.

Printed from the camera-ready copy provided by the Author.

ISBN 978-1-84265-605-1

Printed in India

# **Control Systems**

## **Theory and Implementation**

To my  
beloved father late Vincent Kumarawadu  
and mother Yasawathi Kumarawadu

# Preface

---

This book contains a comprehensive coverage of mathematical modeling of dynamical systems, analog and digital control principles, controller design and analysis, commercial microcontrollers/DSPs for control applications, and implementation of digital control systems using commercial processors. Theoretical contents of the book are presented as much practically oriented as possible. Heavy emphasis has been paid on the practical aspects and implementation of control systems. Digital signal processing is discussed with an explicit emphasis on realtime control applications. Control engineering is one of the broadest sub-disciplines of Engineering that can not be covered in a single book. Too much of content in the book often makes it difficult for undergraduate students and beginners to figure out which of the contents the most relevant. This book starts with the basic fundamentals, modeling of dynamical systems, discusses analog and digital control theories, and practical implementation using microprocessor-based systems. The contents cover typical syllabi of a control systems undergraduate course and postgraduate level taught courses and hence a compact yet comprehensive textbook on control systems for the budding practicing engineers.

Chapter 1 lays foundation for the reader to better understand the following chapters of the book. This chapter starts with an introduction to closed-loop control systems in terms of both hardware and system representation view points. There is a brief comparison between analog control and digital control from the points of view of theoretical as well as implementation aspects. Mathematical modeling of dynamical systems has been included. Essence of transforms in the analysis and design of dynamical systems is highlighted. Mapping between transforms used in analog and digital domains are also discussed. Concepts of system stability are discussed in the time domain, analog, and digital domains. Step response of a closed-loop system is discussed via formative mathematical analyzes. Time domain design specifications are introduced.

Chapter 2 covers the important classical graphical methods used in control theory. Root locus method is first discussed with the details of construction, reshaping, and overall design procedure outline. PID controller is detailed as one of most popular cascade compensators. The importance of frequency domain analysis and its various aspects have been included. Relation between time-domain and frequency domain specifications is discussed. Frequency domain analysis and design is addressed via reshaping the frequency response curves with special emphasis on Bode plots.

The important topics of state-space methods are covered in Chapter 3. Comparison between state-variable methods and classical methods is drawn throughout the chapter. The important topic of stability is looked at through a comprehensive mathematical analysis of the state equations.

The concepts are further elaborated by the use of some interesting practical examples. Chapter 3 also covers the concepts of controllability and observability. State-feedback control and state-feedback with integral control are discussed together with design examples.

Chapter 4 covers the important issues related to digital control theory. It covers the basic mathematics of discrete systems such as unit pulse response, difference equations,  $z$ -transform, discrete transfer functions, graphical methods and frequency response, and the methods of mapping from  $s$  to  $z$  domains. It compares and contrasts the important topic of design of digital control systems. This chapter also presents the state-space methods in the discrete-time domain.

Chapter 5 presents the fundamentals of DSPs with special emphasis on the commercial processors available for control applications and discusses general guidelines for selecting DSPs for specific control applications. Issues of sampling rate, range and round off errors associated with digital computing are discussed. DSP architectures are compared with general purpose processors. Different DSPs options are discussed in terms of arithmetic and hardware architectures, on-chip hardware and software resources. Their relative importance is discussed giving an in-depth coverage on the specific roles that each of the architectural features play in the implementation of realtime digital control systems. Software and hardware support tools for commercial DSPs are discussed with examples. Examples for practical digital implementation of variety of control algorithms and systems have also been included.

The topics nonlinear systems and intelligent control are discussed in Chapter 6. This chapter starts with an introduction to nonlinear systems. The topic of linearization of variety of nonlinear systems is then discussed giving examples. Lyapunov-based stability analysis methods are discussed in detail as the most general method for the determination of stability of nonlinear and/or time-varying systems. Rigid robot systems such as industrial robot manipulators are taken as a case study as a good practical example for complex nonlinear systems. The topic of robot control is discussed starting from fully model-based control to neural network-based online adaptive control. The concept of combined controller-observer design is discussed in detail. Design so that the nonlinearities are completely estimated online by an adaptive neural module is presented. Stability issues are addressed using formative mathematical analysis based on a Lyapunov approach. This chapter also presents fuzzy logic control (FLC) and describes two interesting design examples of FLCs.

**Sisil Kumarawadu**

# List of Figures

1.1	Block diagram of the standard error feedback control system. . .	2
1.2	Block diagram of the standard open-loop (feed-forward) control system. . . . .	3
1.3	Block diagram of the standard digital feedback control system. .	6
1.4	An operational amplifier circuit realization of the PD controller.	8
1.5	OOPic-R servo controller board that controls RC servos and small DC motors. Courtesy of Microchip Technology Inc. . . . .	9
1.6	Block diagram of an analog control system (Laplace domain representation). . . . .	10
1.7	Block diagram of a digital control system (z-domain representation). . . . .	11
1.8	Series RLC circuit. . . . .	12
1.9	The mass-spring-damper system. . . . .	14
1.10	DC motor electrical equivalent circuit. . . . .	15
1.11	Block diagram representation of the DC motor system. . . . .	16
1.12	Block diagram of an error feedback control system. . . . .	17
1.13	Purely exponential component. . . . .	20
1.14	Oscillatory components: (a) $\alpha_k = 0$ , (b) $\alpha_k < 0$ , (c) $\alpha_k > 0$ . . . .	21
1.15	Stable and unstable regions in the $s$ -plane. . . . .	22
1.16	System for example 1.3. . . . .	25
1.17	Transients of the unit step response of a second-order system. .	28
2.1	An analog position control system. . . . .	32
2.2	Variation of the roots with $K$ in the $s$ -plane. . . . .	34
2.3	The asymptotes. . . . .	38
2.4	The root locus. . . . .	39
2.5	Root locus produced by Matlab function, rlocus(sys) where sys = tf([1], [1 2 2 0]). . . . .	40
2.6	Root locus for $G(s)H(s) = K/s(s + a)$ where $a > 0$ . . . . .	41
2.7	Root locus for $G(s)H(s) = K/s(s + a)(s + b)$ where $a, b > 0$ . .	42
2.8	Root locus for $G(s)H(s) = K(s + b)/s(s + a)$ where $a, b > 0$ . .	43
2.9	An error feedback control system. . . . .	44
2.10	An Operational amplifier circuit realization of the PI controller.	45
2.11	Trapezoidal integration. . . . .	46
2.12	Typical gain-phase curves of a feedback control system. . . . .	50
2.13	Effect of adding a pole at $s = -1/b$ : (1) $b = 5$ , (2) $b = 1$ , (3) $b = 0.5$ , (4) $b = 0$ . . . . .	52
2.14	Bode plot of a stable system: $GM, PM > 0$ , $\omega_c > \omega_\phi$ . . . . .	53
2.15	Bode plot of an unstable system: $GM, PM < 0$ , $\omega_c < \omega_\phi$ . . . . .	54



2.16	Code plot produced by the Matlab command, <code>bode(50, [1 9 30 40])</code> .	56
2.17	Code plot produced by the Matlab command, <code>bode(40, [1 7 10 0 0])</code> .	58
3.1	Dynamical system with its internal states, inputs and outputs .	61
3.2	Equivalent circuit of a separately excited dc motor . . . . .	63
3.3	DC motor speed control system . . . . .	65
3.4	State space trajectory . . . . .	75
3.5	Wheatstone bridge circuit . . . . .	77
3.6	State feedback control. . . . .	83
3.7	State feedback control with an observer. . . . .	84
3.8	State feedback with integral control. . . . .	85
3.9	Inverted pendulum. . . . .	87
4.1	(a) Unit pulse function, (b) a continuous signal, (c) a sampler, and (d) the sampled signal. . . . .	93
4.2	zero-order hold . . . . .	94
4.3	Step response of the system: Example 4.3. . . . .	103
4.4	Mapping from $s$ -plane to $z$ -plane. . . . .	104
4.5	System for Example 4.5. . . . .	106
4.6	Sampled data system for Example 4.6. . . . .	109
4.7	Matlab plot for Example 4.6. . . . .	111
4.8	$z$ -grid plot for Example 4.6. . . . .	113
4.9	State feedback control. . . . .	121
5.1	ADSP 2102 based digital controller. Courtesy of Analog Devices, Inc. . . . .	124
5.2	DSP56000 family DSP core diagram. Courtesy of Motorola, Inc. . . . .	129
5.3	Data registers and MAC/ALU of DSP56000 core. X0, X1, Y0, Y1 are data input registers and A, B are accumulator registers. . . . .	130
5.4	Development tools for DSP. . . . .	135
5.5	DSP-based 16-axis motion control board. Courtesy of ADLINK Technology Inc. . . . .	136
5.6	2 <sup>nd</sup> order biquad filter . . . . .	138
5.7	AC servomotor control system. . . . .	140
5.8	DSP-based AC servomotor control system. . . . .	141
5.9	F.A.A.K mobile robot system. Courtesy of Control Systems Engineering Group, Fern Universität in Hagen . . . . .	144
5.10	Implementation of PFC. Courtesy of Microchip Technologies Inc. . . . .	147
6.1	Kinematic model of the robotic head. . . . .	152
6.2	Binocular head configuration parameters $(q_p, q_t, q_v)$ with respect to the world coordinate frame $\{\hat{X}_W, \hat{Y}_W, \hat{Z}_W\}$ . $B$ is the base-line distance. . . . .	153
6.3	The Katana robotic arm. Courtesy of Neuronics AG . . . . .	162
6.4	Block diagram of the autonomous vehicular system. . . . .	170
6.5	Block diagram of manual driving set up during data collection. . . . .	171
6.6	Joint angle trajectories-rigid robot arm. . . . .	177
6.7	Neural network functional estimates. . . . .	177
6.8	The relative membership in a fuzzy set. . . . .	179
6.9	Centroidal defuzzification. . . . .	181
6.10	The control system block diagram for autonomous boat. . . . .	182

6.11	Membership functions of the position variables. . . . .	182
6.12	Membership functions for the heading variable. Desired value is 180°. . . . .	183
6.13	Membership functions for rudder angle. . . . .	183
6.14	An example on how maneuvers are defined differently depending on the distance to the target point. . . . .	184
6.15	The control system block diagram for cyclical leg motion. . . . .	185
6.16	ANFIS architecture. . . . .	187

# List of Tables

---

1.1	Routh's array . . . . .	23
1.2	Routh's array for $1 + G(s)H(s) = s^3 + 4s^2 + 8s + 12$ . . . . .	23
1.3	Routh's array for $1 + G(s)H(s) = s^4 + s^3 + 2s^2 + 2s + 5$ . . . . .	24
1.4	Routh's array for $1 + G(s)H(s) = s^5 + 2s^4 + 3s^3 + 4s^2 + 7s + 5$ . . . . .	24
1.5	Routh's array for Example 1.2 . . . . .	25
2.1	Roots of $s(s + 2) + K = 0$ . . . . .	33
4.1	$z$ -transform of some common functions . . . . .	96
5.1	Some popular single-chip DSPs . . . . .	133
5.2	Some popular single-chip DSPs, contd. . . . .	134
6.1	Fuzzy PD control rule base for cyclic leg motion . . . . .	186

# Contents

Preface . . . . .	vii
List of Figures . . . . .	xiii
List of Tables . . . . .	xvii

## Chapter

<b>1</b>	<b>Introduction to Control Systems</b>	<b>1</b>
1.1	Background . . . . .	1
1.2	Open-loop Versus Closed-loop Control . . . . .	2
1.3	Essence of Transforms in Control Theory . . . . .	3
1.3.1	An $n$ th-order System . . . . .	3
1.3.2	Laplace Transform . . . . .	4
1.3.3	$z$ -Transform . . . . .	5
1.4	Digital Control Versus Analog Control . . . . .	5
1.4.1	Analog Versus Digital Implementation . . . . .	7
1.4.2	Analog Versus Digital System Representation . . . . .	9
1.5	Continuous Control Versus Discrete Control and PLCs . . . . .	11
1.6	Mathematical Modeling of Systems . . . . .	12
1.6.1	Modeling of Electrical Systems . . . . .	12
1.6.2	Modeling of Mechanical Systems . . . . .	13
1.6.3	Modeling of Electrical-mechanical Systems . . . . .	15
1.7	Concepts of Stability . . . . .	16
1.8	Stability and the Roots of the Characteristic Equation . . . . .	17
1.8.1	Pole-zero Cancellation . . . . .	21
1.8.2	Routh's Stability Criterion . . . . .	22
1.9	Step-response and Time-domain Specifications . . . . .	26
1.9.1	Undamped Natural Frequency and the Damping Ratio . . . . .	27
1.9.2	Transient Response Specifications . . . . .	27
1.9.3	Steady State Error . . . . .	29
<b>2</b>	<b>Graphical Methods in Control Theory</b>	<b>30</b>
2.1	Introduction . . . . .	30
2.1.1	Open-loop Transfer Function . . . . .	31
2.2	Root Locus Method . . . . .	31
2.2.1	Construction of the Root Locus . . . . .	32
2.2.2	Reshaping the Root Locus—effects of Addition of Poles . . . . .	39
2.2.3	Reshaping the Root Locus—effects of Addition of Zeros . . . . .	40
2.2.4	Design Procedure Outline . . . . .	41
2.3	PID Controller (PID Compensator) . . . . .	42
2.4	Implementation of the PID Controller . . . . .	45
2.4.1	Ziegler-Nichols Method of Tuning PID Controller . . . . .	48
2.5	Frequency-Domain Analysis . . . . .	48

2.5.1	Closed-loop Frequency Transfer Function . . . . .	49
2.5.2	Frequency-domain Specifications . . . . .	50
2.5.3	Reshaping the Frequency Response Curves . . . . .	51
2.5.4	Nyquist Stability Criterion . . . . .	52
2.5.5	Bode Plots . . . . .	53
2.5.6	Nyquist Plots . . . . .	57
<b>3</b>	<b>State-Space Methods</b>	<b>60</b>
3.1	Introduction . . . . .	60
3.2	State Variable Description . . . . .	61
3.3	Solution of the State Equation . . . . .	67
3.3.1	State-transition Matrix . . . . .	69
3.3.2	Characteristic Equation and the Eigenvalues . . . . .	71
3.3.3	Stability and the Eigenvalues . . . . .	71
3.4	Controllability and Observability . . . . .	74
3.4.1	Controllability . . . . .	75
3.4.2	Testing for Controllability . . . . .	76
3.4.3	Observability . . . . .	79
3.4.4	Testing for Observability . . . . .	81
3.5	State Feedback Control . . . . .	83
3.5.1	State Feedback with Integral Control . . . . .	84
3.6	Canonical Forms . . . . .	89
<b>4</b>	<b>Digital Control Theory</b>	<b>92</b>
4.1	Background . . . . .	92
4.2	Mathematical Methods of Discrete Systems . . . . .	92
4.2.1	A Sampled Signal . . . . .	92
4.2.2	Sampling and Data Hold . . . . .	93
4.2.3	The z-transform . . . . .	95
4.2.4	Properties of the z-transform . . . . .	96
4.2.5	Inverse z-transform . . . . .	97
4.2.6	Obtaining the z-transform when the Function in $s$ is Known . . . . .	99
4.3	Discrete Time Transfer Function . . . . .	102
4.3.1	Stability and Pole Locations . . . . .	103
4.3.2	Modified Routh's Criterion . . . . .	104
4.4	Design of Digital Control Systems . . . . .	108
4.4.1	Root Locus Method . . . . .	109
4.4.2	Bode Plots . . . . .	112
4.4.3	Digital PID Control . . . . .	114
4.4.4	State Variable Methods . . . . .	115
4.4.5	Controllability and Observability . . . . .	119
4.4.6	State-feedback Control . . . . .	120
<b>5</b>	<b>DSPs in Control Systems</b>	<b>122</b>
5.1	Background . . . . .	122
5.1.1	Selection of the Sampling Rate . . . . .	124
5.1.2	Range and Round-off Error . . . . .	125

5.1.3	Fixed-point Versus Floating-point Processors . . . . .	126
5.2	Single-chip DSPs . . . . .	128
5.2.1	Programming, Device Evaluation, and Debugging . . . . .	132
5.2.2	DSP-based Chip Sets and Control Boards . . . . .	135
5.3	Digital Implementation of Control Systems . . . . .	137
5.3.1	PID Controllers . . . . .	137
5.3.2	$n^{\text{th}}$ Order Digital Controllers . . . . .	137
5.3.3	Motor Control . . . . .	138
5.3.4	Robot Control . . . . .	142
5.3.5	Active Power Factor Correction . . . . .	145
6	<b>Intelligent Control</b> . . . . .	<b>148</b>
6.1	Linear Versus Nonlinear Systems . . . . .	148
6.2	Linearized Systems . . . . .	149
6.3	Lyapunov-based Stability Analysis of Systems . . . . .	156
6.3.1	Mathematical Background . . . . .	156
6.3.2	Stability in the Sense of Lyapunov . . . . .	159
6.3.3	Stability Analysis of Linear Systems . . . . .	160
6.4	Robot Control . . . . .	162
6.4.1	Feedback Linearizing Control . . . . .	163
6.4.2	Nonlinear Controller-observer Schemes . . . . .	164
6.5	Neurocontrol . . . . .	167
6.5.1	Radial Basis Function (RBF) NNs . . . . .	168
6.5.2	Multi-layer Perceptron (MLP) NNs . . . . .	169
6.5.3	Identification-based Indirect Control . . . . .	169
6.5.4	Direct Closed-loop Neurocontrol . . . . .	171
6.5.5	Implementation . . . . .	176
6.6	Fuzzy Logic Control (FLC) . . . . .	178
6.6.1	The Three-step Process of Generating FLCs . . . . .	179
6.6.2	Design Examples . . . . .	181
6.7	Neuro-fuzzy Systems . . . . .	186
	<b>References</b> . . . . .	<b>191</b>

# Chapter 1

## Introduction to Control Systems

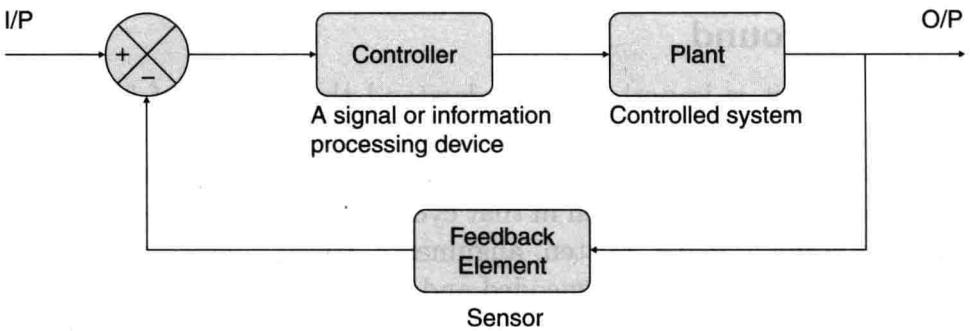
### 1.1 Background

To start with, it is important to understand the concept of feedback or closed-loop control. Consider writing *University of Moratuwa* on the white board with a marker pen attached to someone's fingers. Here, there is a perfect feedback control system in that eyes are used to capture the images of what is actually being written, alignments, spacing between the words, clarity etc. These images are encoded and sent to the brain for processing. Brain, knowing what is expected to be written and other specifications such as alignments and clarity, can now compare what is expected (reference) and what is actually happening (measured output based on visual sensor information) and control the movements of the muscles of the arm and hand in a way the difference between the reference and the output is always kept minimum. Difference between the reference and the output is usually referred to as the *error* in control systems terminology.

Above is an example for a feedback control system, which is completely biological. Automatic control is referred to as control of dynamical systems without a direct human involvement during the operation. For instance, by the use of today's technology, one can automate the aforementioned totally biological control system by replacing the eyes by video cameras, brain by a digital computer, hand by a robot arm, muscles by electric motors, and hand by a suitable robot end-effector. Marker pen is just a tool and may remain the same in both the systems. One may define the desired trajectory of the pen tip in terms of  $(x, y)$  coordinates in a Cartesian frame that is fixed on the white board. Machine vision or computer vision is a rapidly growing technology in which digital computer is used to process and analyze the images captured by cameras. Image processing followed by some coordinate transformations can help recover the actual trajectory of the pen tip. The control algorithm, which is a software program running on another digital computer should be capable

of producing control commands to the electric motors in a way the error is always regulated at zero.

Fig. 1.1 indicates a standard error feedback control system configuration. The plant or the controlled system is the system that is controlled. The feedback element is typically a sensor that feeds the plant output back to be used by the controller. Often, the most challenging task of the designer is to determine the structure of the controller, which is driven by the difference between the reference input and the fed-back output signals. Understandably, the reference input is the desired output.



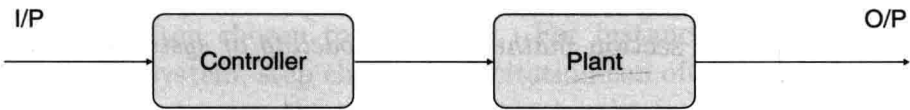
**Figure 1.1:** Block diagram of the standard error feedback control system.

## 1.2 Open-loop Versus Closed-loop Control

Consider writing *University of Moratuwa* on the white board with a marker pen, but this time with the eyes closed. Even though the past experience may enable doing a reasonable job, if tried to write it a bit faster may result in alignment, spacing issues etcetera producing a piece of writing that is unacceptable. The reason for this is the absence of feedback to be compared with the reference input or the desired way of writing. There is no way to correct the movement of the pen as the brain can not know what is exactly happening.

Another example is the fact that a deaf can not speak properly even though he may have gone deaf after he learned how to speak. In the error feedback control point of view, what he actually wants to speak represents the reference input and what is actually being spoken is the sensed output. Output quantity has no influence in the input quantity (and hence in the controller) and hence there is no feedback in the control system. Fig. 1.2 indicates a standard open-loop control system configuration.





**Figure 1.2:** Block diagram of the standard open-loop (feed-forward) control system.

A common automatic control example for open-loop control is domestic bread toaster. The setting of the darkness knob or timer represents the reference input, and the degree of darkness or crispness of the toasted bread is the output. If the degree of darkness is not satisfactory, may be because the type of bread is different, there is no way to automatically alter the length of time the heat is supplied. The sensors that can be considered here may be touch sensors or miniature color camera or both. But incorporating such sensors and accessories in high temperature environment is not an practical option due to added complexity and the cost. Furthermore, high precision control is not an issue of concern in this application. In general, the reasons for opting for the option of open-loop control may be either high-precision control is not required or sensing is practically impossible or financially unjustifiable or both.

### 1.3 Essence of Transforms in Control Theory

#### 1.3.1 An *n*th-order System

Applied mathematics is an essential tool in the studies and theoretical analyzes of control systems. If the mathematical models of the systems are available, the designer, through analyzing them, can arrive at reasonably predictable and reliable designs without depending heavily on thorny repetitive real-world experimentations and extensive computer simulations. Dynamical systems can be mathematically modeled by differential equations. These equations generally involve derivatives and integrals of the independent variables with respect to the dependent variable. Consider a moving ground, aerial, or underwater vehicle. Equations of its translational motion can be modeled by the use of Newton’s 2nd law,  $F = m\ddot{x}$ , where  $F$  is the total external force in the  $x$ -direction,  $m$  is the mass, and  $\ddot{x}$  is the acceleration in the  $x$ -direction. Once substituted for  $F$ , one ends up with a differential equation. Consider an electric circuit that in general consists of resistors, capacitors, and inductors. Writing Kirchoff’s laws give the governing differential equations of the circuit. Same is valid for other