ADVANCED TOPICS IN SCIENCE AND TECHNOLOGY IN CHINA

Kai Li

Electromagnetic Fields in Stratified Media

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Zhejiang University is one of the leading universities in China. In Advanced Topics in Science and Technology in China, Zhejiang University Press and Springer jointly publish monographs by Chinese scholars and professors, as well as invited authors and editors from abroad who are outstanding experts and scholars in their fields. This series will be of interest to researchers, lecturers, and graduate students alike.

Advanced Topics in Science and Technology in China aims to present the latest and most cutting-edge theories, techniques, and methodologies in various research areas in China. It covers all disciplines in the fields of natural science and technology, including but not limited to, computer science, materials science, life sciences, engineering, environmental sciences, mathematics, and physics.

Preface

The properties of the electromagnetic field over the boundary between two different media like earth and air are much more complicated when the source is on or near the boundary than when it is far away and the incident waves are plane waves. This subject has been intensively investigated for over a century. In the pioneering works of Zenneck and Sommerfeld, analytical solutions are carried out based on a flat-Earth model. With myriad practical applications in subsurface and close-to-the-surface communications, radar, and geophysical prospecting and diagnostics, the subject has been investigated widely and the findings have been summarized in the classic book *Dipole radiation in the presence of a conducting half-space* by Baños (Pergamon Press, Oxford, 1966). Lately, investigations are conducted to extend the two-layered case to three-layered and multi-layered cases, with a long series of papers published. The details of the research findings are well summarized in the book *Electromagnetic waves in stratified media* by Wait (Pergamon Press, New York, 1970) and the book *Lateral Electromagnetic Waves* by King, Owens, and Wu (Springer-Verlag, 1992).

In the past decade, the existence or nonexistence of trapped-surface-wave terms in the solution for a dipole radiating over a dielectric-coated lossy underling medium continued to be a controversial subject. Many investigators have revisited the old problem and some new progress has been made. In this book it is investigated with an emphasis placed on the solution for a dipole radiating over the planar or spherical boundary of a stratified medium. It is concerned with an approximate or rigorous analytical solution of the electromagnetic field radiated by a vertical or horizontal dipole. Usually, a simplified or idealized physical model is founded for a practical problem. Subject to the boundary conditions, the formulas of the electromagnetic field are always represented in the exact form of general integrals. These integrals are then evaluated by using mathematical techniques. The corresponding numerical results are carried out, and the conclusions are drawn.

Chapter 1 presents the historical and technical overview of the electromagnetic fields in stratified media in the past century. In Chapters 2-5, it is concerned with the approximate solutions for electromagnetic fields radiated by vertical and horizontal electric dipoles in the presence of three- and four-layered regions. Both the trapped surface wave and the lateral wave are addressed specifically. Chapters 6 and 7 deal with the propagation of the electromagnetic waves generated by a dipole on or near the spherical surface of a layered Earth while the transient field excited by delta function and Gaussian currents in a horizontal electric dipole on the boundary between two different media is dealt with in Chapters 8 and 9. In the entire book an $e^{-i\omega t}$ time dependence is assumed and suppressed.

The author would like to express his gratitude to his advisor, Professor Wei-Yan Pan of the China Research Institute of Radiowave Propagation, who introduced him to the fascinating world of electromagnetics. Special thanks are owed to Professor Hong-Qi Zhang of the China Research Institute of Radio Propagation. The author also wishes to thank Professor Seong-Ook Park of the Information and Communications University, Korea and Professor Yi-Long Lu of the Nanyang Technological University, Singapore for their insight suggestions and comments. The author is also grateful to his graduate students at Zhejiang University, especially to Yin-Lin Wang and Yi-Hui Xu. Many thanks are also owed to Professor Kang-Sheng Chen, Professor Xian-Min Zhang, Professor Sai-Ling He, Professor Dong-Xiao Yang, Professor Xi-Dong Wu, and Professor Wen-Yan Yin of Zhejiang University for their helpful support and encouragement. Finally, the author would like to acknowledge the support and guidance of the editorial staff of Zhejiang University Press and Springer.

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The author hopes that the book would help stimulate new ideas and innovative approaches to electromagnetic fields and waves in stratified media in the years to come.

Kai Li
Zhejiang University
August 2008

Contents

1

Historical and Technical Overview of Electromagnetic Fields in Stratified Media

The electromagnetic field in stratified media has been intensively investigated for over a century. In this chapter, we conduct a formal review on the electromagnetic field in stratified media. Attention should be paid to early analytical contributions on the electromagnetic field in two half-spaces by Zenneck and Sommerfeld. The subsequent contributions by other pioneers, such as Van der Pol, Baños, Fock, Wait, and King, are outlined. From the new developments on the electromagnetic field of a dipole in the presence of a three-layered region, it is seen that a trapped surface wave can be excited efficiently when both the dipole and the observation point are on or close to the boundary.

1.1 Electromagnetic Wave Along Air-Earth Boundary

The first analytical solution for the propagation of electromagnetic waves along the planar boundary between the air and the Earth was carried out by Zenneck (1907). With the existence and significance of the ionosphere still unknown, a possible explanation was offered that a surface wave could propagate over great distance with low attenuation. The properties of the Zenneck surface wave, a radial cylindrical surface wave, are well described by Barlow and Cullen (1953). The model considered is a conducting or dielectric half-space, for $z \geqslant 0$, with the conductivity σ_1 and permittivity ε_1. The air, for $z \leqslant 0$, has a uniform permittivity ε_0. It is assumed that both Region 1 and Region 2 are nonmagnetic so that $\mu_1 = \mu_2 = \mu_0$. Thus, the wave numbers of the two regions are

$$k_1 = \beta_1 + i\alpha_1 = \omega \left[\mu_0 \left(\varepsilon_1 + \frac{i\sigma_1}{\omega}\right)\right]^{1/2}, \tag{1.1}$$

$$k_2 = k_0 = \omega\sqrt{\mu_0\varepsilon_0}. \tag{1.2}$$

When the conditions

$$|k_1|^2 \gg k_2^2 \quad \text{or} \quad |k_1|^2 \geqslant 9|k_2|^2 \tag{1.3}$$

are satisfied, the simplified formulas are obtained in cylindrical coordinates ρ, ϕ, z readily.

For $z \geqslant 0$, the field components are expressed by

$$B_{1\phi} = \mu_0 A e^{ik_1 z} H_1^{(1)}(k_2\rho), \tag{1.4}$$

$$E_{1\rho} = -\frac{\omega}{k_1} B_{1\phi}, \tag{1.5}$$

$$E_{1z} = i\frac{\omega\mu_0 k_2}{k_1^2} A e^{ik_1 z} H_0^{(1)}(k_2\rho), \tag{1.6}$$

and the corresponding forms, for $z \leqslant 0$, by

$$B_{2\phi} = \mu_0 A e^{-i(k_2^2/k_1)z} H_1^{(1)}(k_2\rho), \tag{1.7}$$

$$E_{2\rho} = -\frac{\omega}{k_1} B_{2\phi}, \tag{1.8}$$

$$E_{2z} = i\frac{\omega\mu_0}{k_2} A e^{-i(k_2^2/k_1)z} H_0^{(1)}(k_2\rho). \tag{1.9}$$

Here H_0 and H_1 are Hankel functions and A is an amplitude that depends on the nature of the unspecified source at the origin. For $k_2\rho \geqslant 10$, the Hankel functions can be approximated by

$$H_0^{(1)}(k_2\rho) \sim \sqrt{\frac{2}{\pi k_2\rho}} e^{i(k_2\rho-\pi/4)}, \tag{1.10}$$

$$H_1^{(1)}(k_2\rho) \sim \sqrt{\frac{2}{\pi k_2\rho}} e^{i(k_2\rho-3\pi/4)}. \tag{1.11}$$

Evidently, it is seen that the far field decreases as $\rho^{-1/2}$ with radial distance ρ and decreases exponentially in a vertical direction. This is the essential characteristic of the Zenneck surface wave.

In pioneering works by Sommerfeld, detailed analysis was carried out on the electromagnetic field radiated by an infinitesimal vertical Hertzian dipole on the surface of the Earth, a lossy homogeneous isotropic half-space (Sommerfeld, 1909). In his classic solution, a radial Zenneck surface wave was contained. This seemed to explain the guiding-wave mechanism for the long-distance transmission of radio signals along the air-earth boundary. In 1919, Weyl published a paper on the same subject and obtained a solution similar to that found by Sommerfeld, but without the term of the Zenneck surface wave (Weyl, 1919). In 1926, Sommerfeld returned to the same problem by using a different approach, and concluded that the term of the Zenneck surface wave was not included in the complete solution for the electromagnetic field (Sommerfeld, 1926). This work confirmed the correctness of Weyl's solution.

Nevertheless, the controversies concerning the existence or nonexistence of the Zenneck surface wave continued for a long time and rekindled many investigators to revisit the subject (Van der Pol, 1930; 1935; Sommerfeld, 1935; North, 1937; Wait, 1953; 1961; 1962; Hill and Wait, 1978; Wait and Hill, 1979). The details are well summarized by Baños (1966), and they are not repeated here. Useful discussions are carried out by Wait (1998a), and Collin (2004b).

1.2 Surface Waves Along Surfaces of Stratified Media

The mechanisms of the propagation of electromagnetic waves over the surfaces of stratified media are depended on the nature of the boundary and the properties of the media. The typical surface waveguides that allow the surface waves to be guided along the boundary surfaces include dielectric-coated plane and cylinder as shown in Fig. 1.1, corrugated metal plane and cylinder in Fig. 1.2, and dielectric rod or optical fiber and dielectric image line in Fig. 1.3.

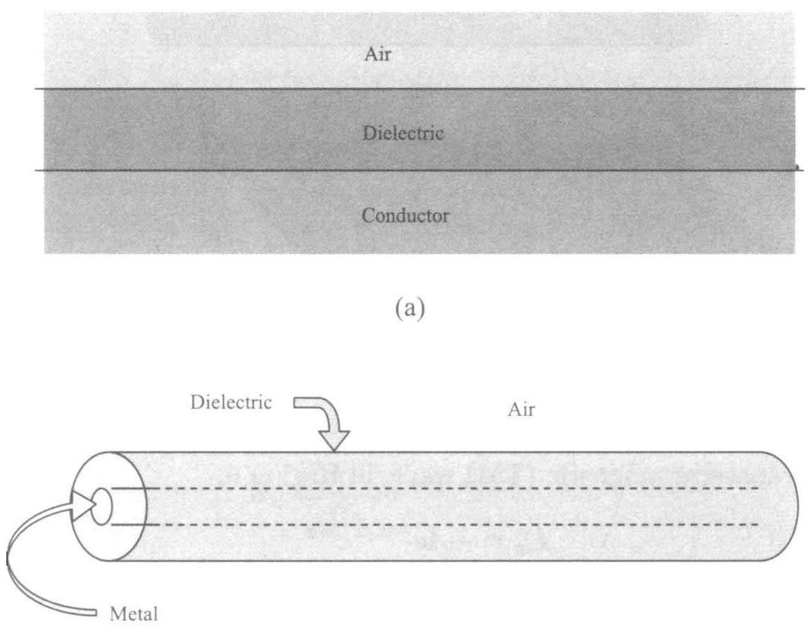

Fig. 1.1. Dielectric-coated metal plane and cylinder

Next, we attempt to describe the properties of the surface waves of the electromagnetic field in the presence of a three-layered region, which are addressed in the dissertation of Zhang (2001). The region of interest consists of the air (Region 0, $z > a$) over a dielectric layer (Region 1, $0 < z < a$) that coats a perfect conductor (Region 2, $z < 0$), as illustrated in Fig. 1.4. The simplified formulas for the electromagnetic field are obtained readily.

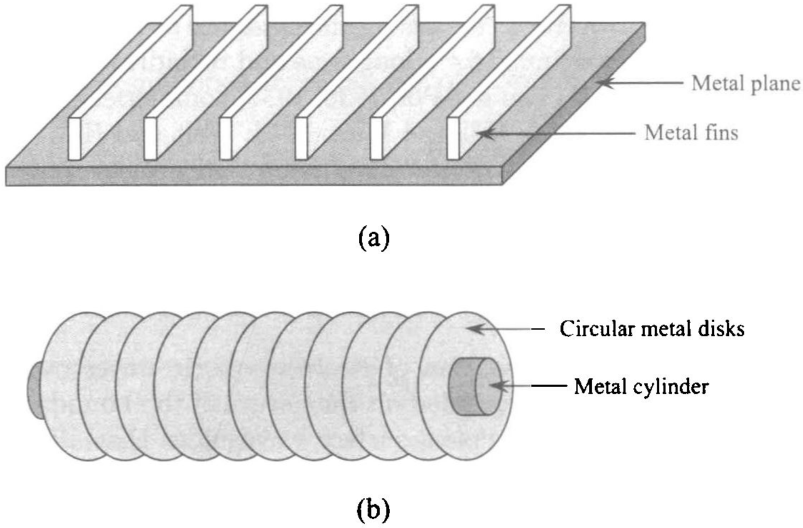

Fig. 1.2. Corrugated metal plane and cylinder

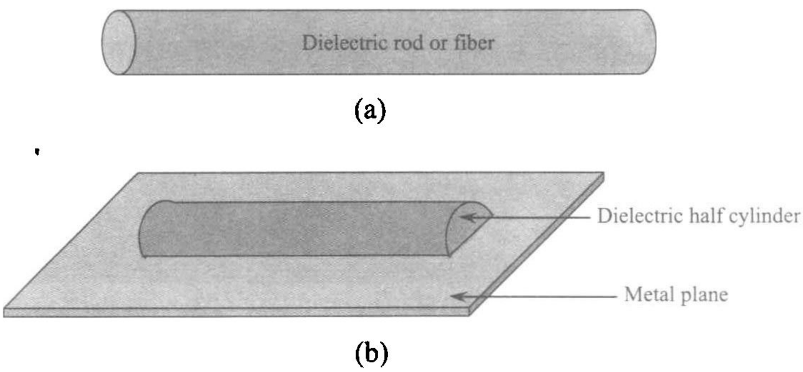

Fig. 1.3. Dielectric rod or fiber and dielectric image line

For transverse magnetic (TM) wave, in Region 0,

$$E_x = -A\mathrm{e}^{-K_1 z}\mathrm{e}^{\mathrm{i}\gamma x}, \tag{1.12}$$

$$E_z = \frac{\mathrm{i}\gamma}{K_1} A\mathrm{e}^{-K_1 z}\mathrm{e}^{\mathrm{i}\gamma x}, \tag{1.13}$$

$$H_y = \frac{\mathrm{i}\omega\varepsilon_0}{K_1} A\mathrm{e}^{-K_1 z}\mathrm{e}^{\mathrm{i}\gamma x}. \tag{1.14}$$

In Region 1,

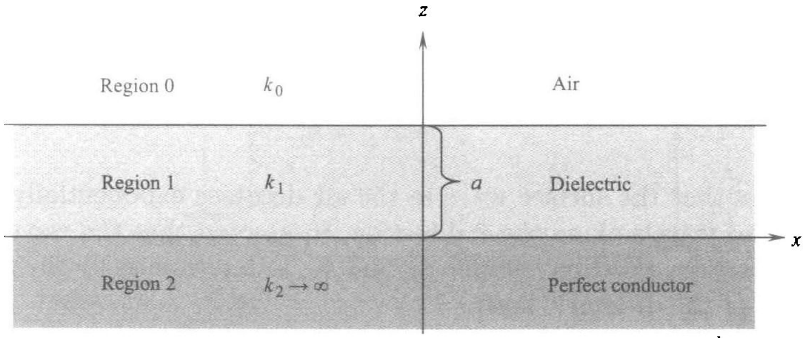

Fig. 1.4. Geometry of three-layered region

$$E_x = -B\sin(K_2 z)\mathrm{e}^{\mathrm{i}\gamma x}, \tag{1.15}$$

$$E_z = -\frac{\mathrm{i}\gamma}{K_2}B\cos(K_2 z)\mathrm{e}^{\mathrm{i}\gamma x}, \tag{1.16}$$

$$H_y = -\frac{\mathrm{i}\omega\varepsilon_0}{K_2}B\cos(K_2 z)\mathrm{e}^{\mathrm{i}\gamma x}, \tag{1.17}$$

where γ, K_1, and K_2 should be satisfied by the following characteristic equations:

$$K_2\tan(K_2 a) = (\varepsilon_1/\varepsilon_0)\,K_1, \tag{1.18}$$

$$K_2^2 + \gamma^2 = k_0^2, \tag{1.19}$$

$$\gamma^2 - K_1^2 = k_1^2. \tag{1.20}$$

For transverse electric (TE) wave, in Region 0,

$$H_x = A\mathrm{e}^{-K_1 z}\mathrm{e}^{\mathrm{i}\gamma x}, \tag{1.21}$$

$$H_z = \frac{\mathrm{i}\gamma}{K_1}A\mathrm{e}^{-K_1 z}\mathrm{e}^{\mathrm{i}\gamma x}, \tag{1.22}$$

$$E_y = -\frac{\mathrm{i}\omega\mu_0}{K_1}A\mathrm{e}^{-K_1 z}\mathrm{e}^{\mathrm{i}\gamma x}. \tag{1.23}$$

In Region 1,

$$H_x = B\sin(K_2 z)\mathrm{e}^{\mathrm{i}\gamma x}, \tag{1.24}$$

$$H_z = \frac{\mathrm{i}\gamma}{K_2}B\cos(K_2 z)\mathrm{e}^{\mathrm{i}\gamma x}, \tag{1.25}$$

$$E_y = -\frac{\mathrm{i}\omega\mu_0}{K_2}B\cos(K_2 z)\mathrm{e}^{\mathrm{i}\gamma x}, \tag{1.26}$$

where

$$K_2 a \tan(K_2 a) = K_1 a, \tag{1.27}$$

$$K_2^2 + \gamma^2 = k_0^2, \tag{1.28}$$

$$\gamma^2 - K_1^2 = k_1^2. \tag{1.29}$$

It follows that the surface wave in the air deceases exponentially in the $\hat{z}$ direction and travels along the $\hat{x}$ direction. It is noted that the wave number in the $\hat{x}$ direction, which is between k_0 and k_1, is determined by the thickness and nature of the dielectric layer.

1.3 Lateral Waves Along the Air-Earth Boundary

The properties of a lateral electromagnetic wave radiated by vertical and horizontal dipoles on the planar boundary between two different media have been investigated extensively. The details are addressed in an excellent summary by King, Owens and Wu (1992).

The region considered consists of the Earth (Region 1) and the air (Region 2), and both the horizontal dipole and the observation point are located on the boundary in Region 1. The analytical formulas were obtained for the electromagnetic field generated by a horizontal electric dipole in the two different media by King and Wu (1983). It is seen that the far field includes direct wave, ideal reflected wave or ideal image wave, and lateral wave.

$$E_1(\rho, z) = E_1^{inc}(\rho, z) + E_1^{ref}(\rho, z) + E_1^{L}(\rho, z), \tag{1.30}$$

where the three terms $E_1^{inc}(\rho, z)$, $E_1^{ref}(\rho, z)$, and $E_1^{L}(\rho, z)$ stand for the direct wave, the ideal reflected wave, and the lateral wave, respectively.

As illustrated in Fig. 1.5, it is seen that the lateral wave travels from the transmitting dipole a distance d in Region 1 (Earth) to the boundary, then radially along the boundary in Region 2 (Air) a distance ρ, and finally vertically a distance z in Region 1 to the receiving dipole. The paths of the direct wave, the ideal reflected wave, and the lateral wave are shown in Fig. 1.6. The lateral-wave characteristics are well known and addressed specifically in the book by King, Owens, and Wu (1992).

1.4 Trapped Surface Wave in the Presence of Three-Layered Region

The electromagnetic fields of vertical and horizontal electric dipoles on or near the boundary between two different media have been well known in terms of closed-form expressions for many years. When the boundary includes a layer of third material with intermediate properties, the properties of the electromagnetic field radiated by a dipole are in general much more complicated.

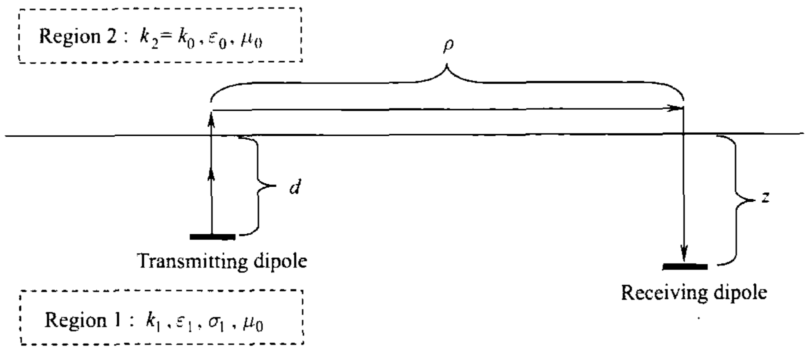

Fig. 1.5. Apparent path of lateral wave

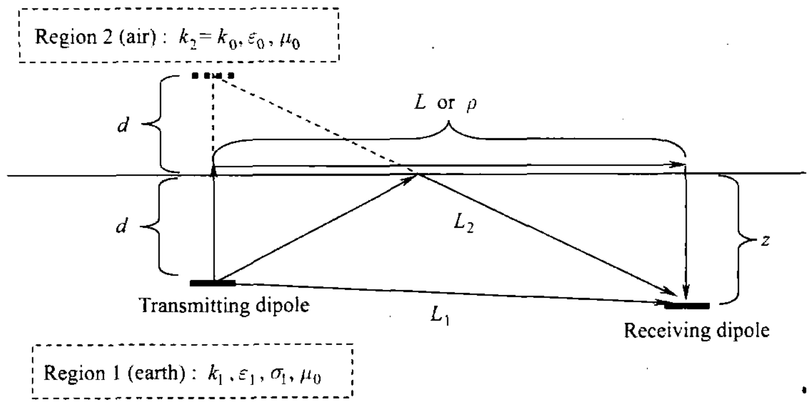

Fig. 1.6. Apparent paths of directed wave, image reflected wave, and lateral wave

In early 1990's, the simply analytical formulas were derived for the electromagnetic fields generated by horizontal and vertical electric dipoles in the presence of a three-layered region (King, 1991; 1992; 1993; King and Sandler, 1994b). It was demonstrated that the total field on or near the air-dielectric boundary is determined primarily by lateral wave, where the amplitude of the field along the boundary is $1/\rho^2$.

In the late 1990's, Wait (1998) and Mahmoud (1999) wrote comments on the work by King and Sandler (1994a, 1994b) and considered that the trapped surface wave, varying as $\rho^{-1/2}$ in the far-field region, should not be overlooked in the three-layered case. In the two papers by Collin (2004a; 2004b), the analysis supports the conclusions reached by Wait and Mahmoud. To clarify the controversies concerning the trapped surface wave, the old problem was revisited again by several investigators in the past several years (Zhang and Pan, 2002; Zhang, Li, and Pan, 2003; Zhang et al., 2001, 2005; Li and Lu, 2005b; Liu and Li, 2007).

From the works by Zhang and Pan (2002), it is concluded that the trapped-surface-wave terms, where the amplitude decreases as $\rho^{-1/2}$ along the air-dielectric boundary in the far-field region, can be excited efficiently by a vertical electric dipole in the presence of a three-layered region. The wave