James Stewart

CALCULUS

Concepts AND Contexts

SINGLE VARIABLE

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SINGLE • VARIABLE

JAMES STEWART
McMaster University

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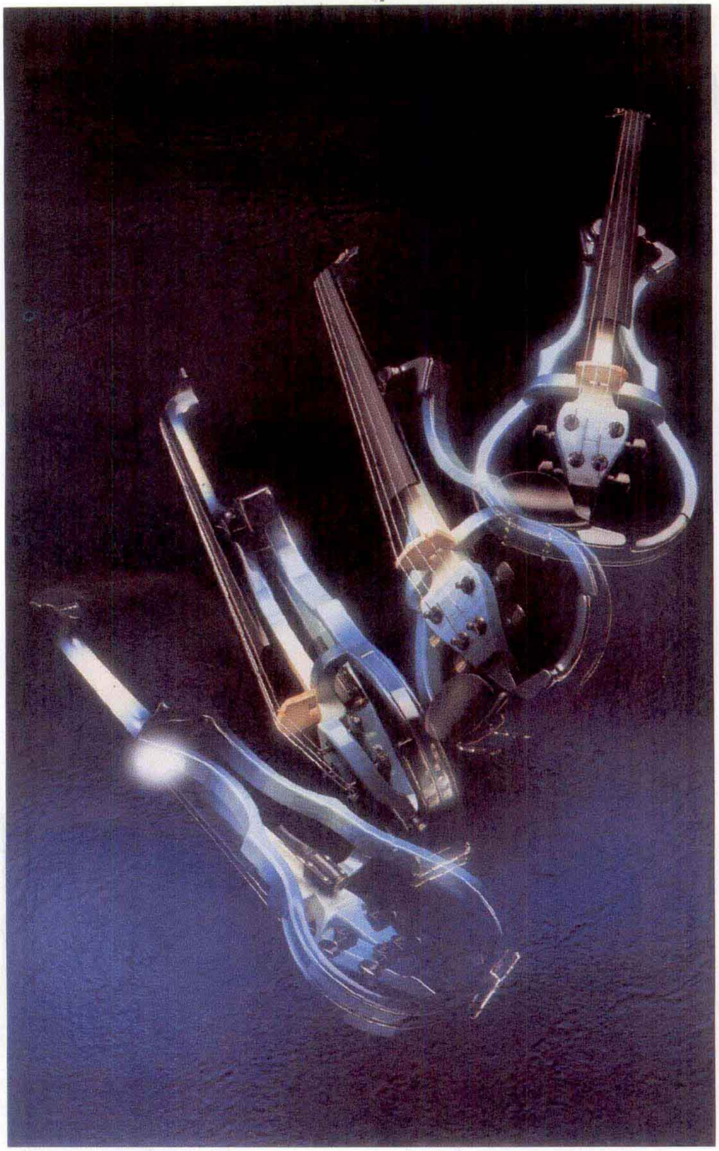

About the Cover

The photograph shows an electric violin made by David Bruce Johnson.

An acoustic violin, with its sound hole in the shape of an integral sign, became a symbol of James Stewart's previous calculus textbooks. Stewart plays both an 18th-century French violin and the blue electric violin that appears on the cover of this book.

The electric violin reflects the increased use of technology in calculus instruction, as well as a more informal approach to the subject. The quadruple image symbolizes the use of the Rule of Four throughout the book—four ways of looking at the same object.

CALCULUS

Concepts AND Contexts

SINGLE • VARIABLE

For Geoff, Brad, Anna, and Jon.

For Debbie, Lorraine, Alan, and Matt.

For absent friends.

Preface

Mathematics departments are engaged in a debate over calculus reform. Our debate, of course, is part of an ongoing one that started 300 years ago when the first calculus textbook was published. But the current debate has sometimes turned acrimonious. Tales abound of departments badly split, with instructors favoring reform and those favoring a traditional approach not speaking to one another.

What is arousing such passions? What do people *mean* when they say they are in favor of calculus reform? As a result of talking with many instructors and reading hundreds of survey responses, I have learned that different people mean different things. They have passionately held opinions; there is common ground on some issues, but instructors are diametrically opposed on other issues. Let's look at some of the suggested key components of calculus reform.

Several survey respondents think that *technology* is the most important issue. Certainly, those of us who have watched our students use graphing calculators or computers know how enlivening this can be. We have seen from the looks on their faces how these devices engage our students' attention and enable them to become active learners. But these machines have been used by many schools with traditional curricula. For example, several traditional calculus texts (including my own *Calculus, Third Edition*) make extensive use of technology. Furthermore, I know of some very innovative reform calculus courses that use virtually no technology. So, while technology can be a critical component for implementing the goals of reform, I don't believe that technology itself characterizes reform.

Many people cite the *Rule of Three* as a key principle: "Topics should be presented geometrically, numerically, and algebraically." The implication is that, in the past, the algebraic point of view has been predominant and the graphical and numerical aspects have been given short shrift. More recently, the Rule of Three has been expanded to become the *Rule of Four* by emphasizing the verbal, or descriptive, point of view as well. But again, I think that my traditional book *Calculus, Third Edition* incorporates visualization and the Rule of Three. So I believe that the Rule of Three (or Four), important guiding principle though it is, still does not capture the most critical aspect of reform by itself.

Some respondents think that the enhanced attention to applications is a key feature and that instructors now have more freedom to choose applications for which they themselves have enthusiasm. While this aspect is certainly true, it is just as important in a traditional course.

So what *do* I think is the essence of calculus reform? In a word: *concepts.* We sometimes forget that the impetus for the current reform movement came from the Tulane Conference in January, 1986. I believe that the primary goal of reform should be what that conference formulated as their first recommendation:

Focus on conceptual understanding

What technology, the Rule of Four, and other aspects of reform have done is to enable instructors to use new tools and approaches to conceptual reasoning and skills. Visualization, numerical and graphical experimentation, and other approaches have changed how we teach conceptual reasoning in fundamental ways.

I think that nearly everybody—from the radical reformer to the staunch traditionalist—supports the central goal of focusing on conceptual understanding. So why are there so many heated discussions in mathematics departments? I believe that the explanation lies in what is involved in implementing this goal. If we are serious about emphasizing conceptual understanding, then we have to expect faculty and students to give clear explanations of what symbols mean and why things work the way they do. That is simply not going to happen unless we take the time to work patiently with students. We need to slow down, provide multiple approaches, and not rush through the material when a new concept is introduced. It follows that some less conceptual traditional calculus topics will not be covered in many courses. And that is where the controversy arises.

Most of the existing reform projects have greatly reduced the coverage of techniques of integration and I agree that this is appropriate. (This book has no full chapter on methods of integration, but substitution and parts are covered in Chapter 5 and partial fractions in Appendix F.) I have also streamlined the coverage of many other topics in order to free up time to achieve conceptual understanding. But I have not gone as far as some other reform texts in deleting traditional topics. In particular, I have decided to retain related rates problems, l'Hospital's Rule, and series of constants. My premise in writing this book has been that it is possible to achieve conceptual understanding and still retain the best traditions of traditional calculus. I hope that this book will support a wider range of approaches to teaching calculus and improving students' conceptual understanding in diverse college and university settings.

Features

Conceptual Exercises

The most important way to foster conceptual understanding is through the problems that we assign. To that end I have devised various types of problems. Some exercise sets begin with requests to explain the meanings of the basic concepts of the section. (See, for instance, the first couple of exercises in Sections 2.2, 2.4, 2.5, 5.3, and 8.2.) Similarly, review sections begin with a Concept Check and a True-False Quiz. Other exercises test conceptual understanding through graphs (see Exercises 1–3 in Section 2.7 and Exercises 29–36 in Section 2.8). Another type of exercise uses verbal description to test conceptual understanding (see Exercise 8 in Section 2.4; Exercise 46 in Section 2.8; Exercises 5, 9, and 10 in Section 2.10; and Exercise 53 in Section 5.9). I particularly value problems that combine and compare graphical, numerical, and algebraic approaches (see Exercise 30 in Section 2.5, Exercise 39 in Section 3.1, and Exercise 2 in Section 7.6).

Pages 109, 128, 140, 380, 577
Pages 156, 170
Pages 129, 171, 180, 437
Pages 141, 200, 544

Real-World Data

My assistants and I spent a great deal of time looking in libraries, contacting companies and government agencies, and searching the Internet for interesting real-world data to introduce, motivate, and illustrate the concepts of calculus. As a result, many of the examples and exercises deal with functions defined by such numerical data or graphs. See, for instance, Figures 1, 11, and 12 in Section 1.1 (seismograms from the Northridge earthquake), Figure 5 in Section 5.3 (San Francisco power consumption), Exercise 10 in Section 5.1 (velocity of the space shuttle *Endeavour*), and Exercise 56 in Section 5.3 (Consumer Price Index).

Pages 12, 16
Pages 379, 360, 381

Projects One way of involving students and making them active learners is to have them work (perhaps in groups) on extended projects that give a feeling of substantial accomplishment when completed. *Applied Projects* involve applications that are designed to appeal to the imagination of students. The project after Section 7.4 asks whether a ball thrown upward takes longer to reach its maximum height or to fall back to its original height. (The answer might surprise you.) *Laboratory Projects* involve technology; the project following Section 3.5 shows how to use Bézier curves to design shapes that represent letters for a laser printer. *Writing Projects* ask students to compare present-day methods with those of the founders of calculus—Fermat's method for finding tangents, for instance. Suggested references are supplied. *Discovery Projects* anticipate results to be discussed later or cover optional topics (hyperbolic functions) or encourage discovery through pattern recognition (see the project following Section 5.7).

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Rigor I include fewer proofs than in my more traditional books, but I think it is still worthwhile to expose students to the idea of proof and to make a clear distinction between a proof and a plausibility argument. The important thing, I think, is to show how to deduce something that seems less obvious from something that seems more obvious. A good example is the use of the Mean Value Theorem to prove the Evaluation Theorem (Part 2 of the Fundamental Theorem of Calculus). I have chosen, on the other hand, not to prove the convergence tests but rather to argue intuitively that they are true.

Technology The availability of technology makes it not less important but more important to clearly understand the concepts that underlie the images on the screen. But, when properly used, graphing calculators and computers are powerful tools for discovering and understanding those concepts. I assume that the student has access to either a graphing calculator or a computer algebra system. The icon indicates an example or exercise that definitely requires the use of such technology, but that is not to say that it can't be used on the other exercises as well. The symbol CAS is reserved for problems in which the full resources of a computer algebra system (like Derive, Maple, Mathematica, or the TI-92) are required. But technology doesn't make pencil and paper obsolete. Hand calculation and sketches are often preferable to technology for illustrating and reinforcing some concepts. Both instructors and students need to develop the ability to decide where the hand or the machine is appropriate.

Problem Solving Students usually have difficulties with problems for which there is no single well-defined procedure for obtaining the answer. I think nobody has improved very much on George Polya's four-stage problem-solving strategy and, accordingly, I have included a version of his problem-solving principles at the end of Chapter 1. They are applied, both explicitly and implicitly, throughout the book. After the other chapters I have placed sections called *Focus on Problem Solving,* which feature examples of how to tackle challenging calculus problems. In selecting the varied problems for these sections I kept in mind the following advice from David Hilbert: "A mathematical problem should be difficult in order to entice us, yet not inaccessible lest it mock our efforts." When I put these challenging problems on assignments and tests I grade them in a different way. Here I reward a student significantly for ideas toward a solution and for recognizing which problem-solving principles are relevant.

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Pages 186, 263, 344, 442, 499, 556, 640

Content

The book begins with *A Preview of Calculus,* which gives an overview of the subject and includes a list of questions to motivate the study of calculus.

Chapter 1 Functions and Models From the beginning, multiple representations of functions are stressed: verbal, numerical, visual, and algebraic. The standard functions, including exponential and logarithmic functions, are reviewed here from these four points of view. Parametric curves are introduced in the first chapter, partly so that curves can be drawn easily, with technology, whenever needed throughout the text. This early placement also enables inverse functions to be graphed in Section 1.6, tangents to parametric curves to be treated in Section 3.5, and graphing such curves to be covered in Section 4.4. All students should read the general discussion of modeling at the beginning of Section 1.7 as a background to the models that pervade the book. The remainder of the section (on curve fitting) is optional, but some instructors may wish to exploit the ability of the newest calculators to model data. A small number of later exercises make use of this material (see, for example, Exercises 59 and 60 in Section 3.5 and Exercise 54 in Section 4.2).

Pages 67, 233, 300

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Chapter 2 Limits and Derivatives The material on limits is motivated by a prior discussion of the tangent and velocity problems. Limits are treated from descriptive, graphical, numerical, and algebraic points of view. (The precise ε-δ definition of a limit is provided in Appendix D for those who wish to cover it.) It is important not to rush through Sections 2.7–2.10, which deal with derivatives (especially with functions defined graphically and numerically) before the differentiation rules are covered in Chapter 3. Here the examples and exercises explore the meanings of derivatives in various contexts. Section 2.10 foreshadows, in an intuitive way and without differentiation formulas, the material on shapes of curves that is studied in greater depth in Chapter 4.

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Chapter 3 Differentiation Rules All the basic functions are differentiated here. When derivatives are computed in applied situations, students are asked to explain their meanings. Optional topics (hyperbolic functions, an early introduction to Taylor polynomials) are explored in Discovery and Laboratory Projects.

Chapter 4 Applications of Differentiation The basic facts concerning extreme values and shapes of curves are derived using the Mean Value Theorem as the starting point. Graphing with technology emphasizes the interaction between calculus and calculators and the analysis of families of curves. Some substantial optimization problems are provided, including an explanation of why you need to raise your head 42° to see the top of a rainbow.

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Chapter 5 Integrals The area problem and the distance problem serve to motivate the definite integral. I have decided to make the definition of an integral easier to understand by using subintervals of equal width. Emphasis is placed on explaining the meanings of integrals in various contexts and on estimating their values from graphs and tables. There is no separate chapter on techniques of integration, but substitution and parts are covered here and partial fractions are treated in Appendix F. The use of computer algebra systems is discussed in Section 5.7.

Pages 410–412

Chapter 6 Applications of Integration General methods, not formulas, are emphasized. The goal is for students to be able to divide a quantity into small pieces, estimate with Riemann sums, and recognize the limit as an integral. There are more applications here than can realistically be covered in a given course. Instructors should select applications suitable for their students and for which they themselves have enthusiasm.

Chapter 7 Differential Equations Modeling is the theme that unifies this introductory treatment of differential equations. Direction fields and Euler's method are studied before separable equations are solved explicitly, so that qualitative, numerical, and analytic approaches are given equal consideration. These methods are applied to the exponential, logistic, and other models for population growth. Predator-prey models are used to illustrate systems of differential equations.

Chapter 8 Sequences and Series Tests for the convergence of series are considered briefly, with intuitive rather than formal justifications. Numerical estimates of sums of series are based on which test was used to prove convergence. The emphasis is on Taylor series and polynomials and their applications to physics. Error estimates include those from graphing devices.

Ancillaries

Calculus: Concepts and Contexts, Single Variable is supported by a complete set of ancillaries developed under my direction. Each piece has been designed to enhance student understanding and to facilitate creative instruction.

The following resources are available, free of charge, to adopters of the text.

Instructor's Guide by Harvey B. Keynes, James Stewart, Douglas Shaw, and Robert Hesse

Offering suggestions on how to implement ideas about reform into your calculus course, this Guide serves as a practical roadmap to topics and projects in the text. Each section of the main text is discussed from several viewpoints and contains suggested time to allot, points to stress, text discussion topics, core materials for lecture, workshop/discussion suggestions, group work exercises in a form suitable for handout, and suggested homework problems.

Complete Solutions Manual by Jeffery A. Cole

Provides detailed solutions to all exercises in the text.

Transparencies by James Stewart

Thirty full-color transparencies featuring 80 of the more complex diagrams from the text for use in the classroom.

Test Items by William Tomhave and Xueqi Zeng

Organized according to the main text, this complete set of Test Items contains both multiple-choice and open-ended questions, offering a range of model problems, including short-answer questions that focus narrowly on one basic concept; items that integrate two or more concepts and require more detailed analysis and written response; and application problems, including situations that use real data generated in laboratory settings.

Electronic Test Items by William Tomhave, Xueqi Zeng, and Charles Heuer

This computerized version of the printed Test Items allows instructors to insert their own questions and customize ones that are provided. Some test items will be algorithmically generated.

A complete range of student ancillaries is also available:

Study Guide by Robert Burton and Dennis Garity

Offering additional explanations and worked-out examples, and formatted to provide guided practice, each section in this Study Guide corresponds to a section in the text. Every section contains a short list of key concepts; a short list of skills to master; a brief introduction to the ideas of the section; an elaboration of the concepts and skills, including extra worked-out examples; and links in the margin to earlier and later material in the text and Study Guide.

Student Solutions Manual by Jeffery A. Cole
Contains detailed solutions to all odd-numbered exercises in the text.

Lab Manuals Each of these comprehensive lab manuals will help students learn to effectively use the technology tools available to them. Each lab contains clearly explained exercises and a variety of labs and projects to accompany the text.

CalcLabs with Maple®
by Al Boggess, David Barrow, Maury Rahe, Jeff Morgan, Samia Massoud, Philip Yasskin, Michael Stecher, Art Belmonte, and Kirby Smith

CalcLabs with Mathematica®
by David Barrow, Art Belmonte, Nancy Blachman, Al Boggess, Samia Massoud, Jeff Morgan, Maury Rahe, Kirby Smith, Michael Stecher, Colin Williams, and Philip Yasskin

CalcLabs with Derive®
by David Barrow, Art Belmonte, Al Boggess, Samia Massoud, Jeff Morgan, Maury Rahe, Kirby Smith, Michael Stecher, and Philip Yasskin

CalcLabs with the TI-82/83 by Jeff Morgan and Selwyn Hollis

CalcLabs with the TI-85/86 by David Rollins

CalcLabs with the TI-92 by Selwyn Hollis

A Companion to Calculus by Dennis Ebersole, Doris Schattschneider, Alicia Sevilla, and Kay Somers

Written to improve algebra and problem-solving skills of students taking a calculus course, every chapter in this companion is keyed to a calculus topic, providing conceptual background and specific algebra techniques needed to understand and solve calculus problems related to that topic. It is designed for calculus courses that integrate the review of precalculus concepts (web site http://www.hvcc.edu/faculty/amm/fipse/fipse.htm) or for individual use.

Scientific Notebook™ by TCI Software Research

Featuring a built-in version of the Maple® computer algebra system, *Scientific Notebook™* provides students with the computational power necessary to solve the most complex homework problems. It is easy to link to resources within a document, in other documents, or in documents on the World Wide Web. This combination gives students a unique tool for exploring, explaining, and understanding key mathematical and scientific concepts.

System requirements: *Scientific Notebook* runs on any Windows® 95 or Windows NT® 4.0 system. It requires 10 MB of hard disk space, a CD-ROM drive, and an Internet connection to access the *Scientific Notebook* Resource Center.

Doing Calculus with Scientific Notebook™ by Darel W. Hardy and Carol L. Walker

Contains activities that will help you develop a clearer understanding of calculus.

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JAMES STEWART

To the Student

Reading a calculus textbook is different from reading a newspaper or a novel, or even a physics book. Don't be discouraged if you have to read a passage more than once in order to understand it. You should have pencil and paper and calculator at hand to sketch a diagram or make a calculation.

Some students start by trying their homework problems and read the text only if they get stuck on an exercise. I suggest that a far better plan is to read and understand a section of the text before attempting the exercises. In particular, you should look at the definitions to see the exact meanings of the terms.

Part of the aim of this course is to train you to think logically. Learn to write the solutions of the exercises in a connected, step-by-step fashion with explanatory sentences—not just a string of disconnected equations or formulas.

The answers to the odd-numbered exercises appear at the back of the book, in Appendix I. Some exercises ask for a verbal explanation or interpretation or description. In such cases there is no single correct way of expressing the answer, so don't worry that you haven't found the definitive answer. In addition, there are often several different forms in which to express a numerical or algebraic answer, so if your answer differs from mine, don't immediately assume you're wrong. There may be an algebraic or trigonometric identity that connects the answers. For example, if the answer given in the back of the book is $\sqrt{2} - 1$ and you obtain $1/(1 + \sqrt{2})$, then you're right and rationalizing the denominator will show that the answers are equivalent.

The icon indicates an example or exercise that definitely requires the use of either a graphing calculator or a computer with graphing software. (Section 1.3 discusses the use of these graphing devices and some of the pitfalls that you may encounter.) But that doesn't mean that graphing devices can't be used to check your work on the other exercises as well. The symbol CAS is reserved for problems in which the full resources of a computer algebra system (like Derive, Maple, Mathematica, or TI-92) are required. You will also encounter the symbol which warns you against committing an error. I have placed this symbol in the margin in situations where I have observed that a large proportion of my students tend to make the same mistake.

Calculus is an exciting subject, justly considered to be one of the greatest achievements of the human intellect. I hope you will discover that it is not only useful but also intrinsically beautiful.

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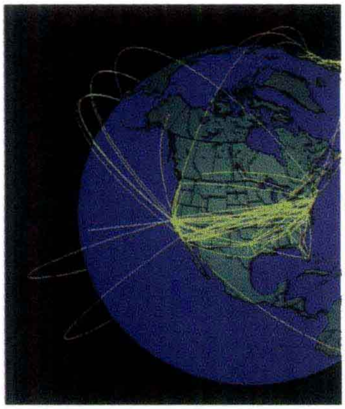

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