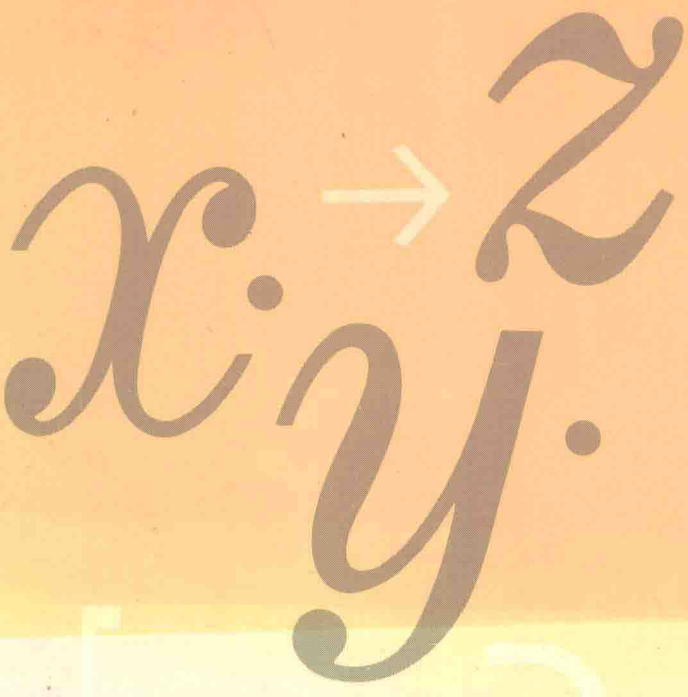
W. V. Quine

Methods of Logic

Fourth Edition



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FOURTH EDITION

W. V. QUINE

Harvard University Press
Cambridge, Massachusetts

Printed in the United States of America
10 9 8

Library of Congress Cataloging in Publication Data

Quine, W. V. (Willard Van Orman)
Methods of logic, fourth edition.

Bibliography: p.
Includes index.
1. Logic. I. Title.
BC71.Q5 1982 160 81-22929
AACR2

ISBN 0-674-57176-2 (pbk.)

METHODS OF LOGIC

TO MARJORIE

PREFACE

This book undertakes both to convey a precise understanding of the formal concepts of modern logic and to develop convenient techniques of formal reasoning. In each successive edition the book has grown. The portions that can be passed over without detriment to an understanding of basic logical theory amount now to about a third of the book. I have bracketed them so that they can conveniently be skipped, entirely or selectively, for purposes of a shorter course. But in my own teaching I have always covered the whole book, in its past editions, in a semester course.

In the second edition, 1959, the main improvements were two: a more readable proof of the soundness of my system of natural deduction and, in an appendix, proofs of completeness and Löwenheim's theorem.

The third edition, 1972, was a more drastic departure: a new book by half. Proof procedures and decision procedures proliferated, offering a sheaf of alternatives. Approaching logical structure thus from a plurality of angles conduced, I felt, to depth of understanding.

My main method, as I called it, in quantification theory was remarkably simple and easily justified and it was strategic as a base from which to develop the array of alternative methods. Also it admitted of a relatively easy completeness proof. Accordingly my system of natural deduction, which had dominated the first two editions, was demoted in the third edition and accorded only brief treatment in the omissible sector. Responding now to teachers who liked that system and regretted its demotion, I have in this new edition restored it to full treatment, though still making it omissible.

Another major departure in that third edition was the postponement of quantifiers by developing monadic logic first along Boolean lines, though without assuming classes. In this new edition I have pressed the idea further, by exploiting also the set-theoretic notation of class abstraction without assuming classes. Abstracts emerge with all the innocence of relative clauses. The device pays its way in the treatment of substitution, it clarifies the pronominal character of bound variables, and it eventuates in a philosophically agreeable perspective on classes. Further illumination of the bound variable is undertaken in an omissible chapter on combinatory functors.

There have been patches where points explained in my earlier books needed to be explained again in essentially the same old way. At these points I have adapted examples and expository passages from *Mathematical Logic, Elementary Logic,* and *O Sentido da nova lógica,* preferring not to obscure genuine points of contact by *ad hoc* shifts of examples or of phrasing. But the places are few. Chapter 1 draws in part on §6 of *O Sentido* and Chapter 5 draws in part on §2 of *Mathematical Logic.* In Chapters 4, 8, 14, and 31, examples are borrowed from *Elementary Logic* but are handled differently.

Out of loyalty to loyal teachers of this book, I have been reluctant to depart from the notations of truth functions and quantifiers that I inherited from Whitehead and Russell. In this edition at last I do so. I switch to other symbols that are increasingly in vogue for the conditional, the biconditional, and universal quantification. I make the move not just as a response to fashion, but for technical reasons that will be noted as the symbols emerge.

Partly because of the changed notation, the copy-editing of this edition was unusually exacting. I am grateful to Katarina Rice of Harvard University Press for a perceptive and meticulous job.

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PART IV. GLIMPSES BEYOND

INTRODUCTION

Logic, like any science, has as its business the pursuit of truth. What are true are certain statements; and the pursuit of truth is the endeavor to sort out the true statements from the others, which are false.

Truths are as plentiful as falsehoods, since each falsehood admits of a negation which is true. But scientific activity is not the indiscriminate amassing of truths; science is selective and seeks the truths that count for most, either in point of intrinsic interest or as instruments for coping with the world.

Strictly speaking, what admit of truth and falsity are not statements as repeatable patterns of utterance, but individual events of statement utterance. For, utterances that sound alike can vary in meaning with the occasion of the utterance. This is due not only to careless ambiguities, but to systematic ambiguities which are essential to the nature of language. The pronoun 'I' changes its reference with every change of speaker; 'here' changes its reference with every significant movement through space; and 'now' changes its reference every time it is uttered.

The crucial point of contact between description and reality is to be sought in the utterance of a statement on the occasion of a stimulation to which that string of words has become associated. Not that the statement will refer to stimulation or sensation; it is apt to refer to physical objects. Language is a social institution, serving, within its limitations, the social end of communication; so it is not to be wondered that the objects of our first and commonest utterances are socially shared physical objects rather than private experiences. Physical objects, if they did not exist, would (to transplant Voltaire's epigram) have had to be invented. They are indispensable as the public common denominators of private sense experience.

The latest scientific pronouncement about positrons and the statement that my pen is in my hand are equally statements about physical objects; and physical objects are known to us only as parts of a systematic conceptual structure which, taken as a whole, impinges at its edges upon observation. We have a network of statements that are variously linked with one another and some of which, out at the periphery of the network, are associated more or less strongly with sensory stimulation. Even the peripheral ones are mostly about physical objects: examples are 'My pen is in my hand', 'The mercury is at 80'.

A sensory stimulation elicits some closely associated statement and the associations then reverberate through the system of statements, activating at length another peripheral statement whose sensory associations make us expect some particular further stimulation. Such, schematically, is the mechanism of prediction. When prediction fails, we question the intervening network of statements. We retain a wide choice as to what statements of the system to preserve and what ones to revise; any one of many revisions will suffice to unmake the particular implication that brought the system to grief.

Normally the peripheral statements, closely associated to stimulations, are to be preserved from revision once the appropriate stimulations have occurred. If revision of the system should become necessary, other statements than these are to suffer. It is only by such an allocation of priority that we can hope to claim any empirical content or objective reference for the system as a whole.

There is also, however, another and somewhat opposite priority: the more fundamental a law is to our conceptual scheme, the less likely we are to choose it for revision. When some revision of our system of statements is called for, we prefer, other things being equal, a revision which disturbs the system least.

Where the two priorities come into conflict, either is capable of prevailing. Statements close to experience and seemingly verified by the appropriate experiences may occasionally be given up, even by pleading hallucination, in the extreme case where their retention would entail a cataclysmic revision of fundamental laws. But to overrule a multiplicity of such statements, if they reinforce one another and are sustained by different observers, would invite criticism.

The priority on law, considered now apart from any competition with the priority on statements verified by experience, admits of many gradations. Conjectures of history and economics will be revised more willingly than laws of physics, and these more willingly than laws of mathematics and logic. Our system of statements has such a thick cushion of indeterminacy, in relation to experience, that vast domains of law can easily be held immune to revision on principle. We can always turn to other quarters of the system when revisions are called for by unexpected experiences. Mathematics and logic, central as they are to the conceptual scheme, tend to be accorded such immunity, in view of our conservative preference for revisions which disturb the system least; and herein, perhaps, lies the "necessity" which the laws of mathematics and logic are felt to enjoy.

In the end it is perhaps the same to say, as one often does, that the laws of mathematics and logic are true simply by virtue of our conceptual scheme. For, it is certainly by virtue of that scheme that those laws are central to it; and it is by virtue of being thus central that the laws are preserved from revision at the expense of statements less strategically situated.

But it must now be remarked that our conservative preference for those revisions which disturb the system least is opposed by a significant contrary force, a force for simplification. Far-reaching revision of the fundamental laws of physics was elected in recent decades, by considerations of simplicity, in preference to the welter of *ad hoc* subsidiary laws which would otherwise have been needed to accommodate the wayward experiences of Michelson and Morley and other experimenters. Continued experiment "confirmed" the fundamental revisions, in the sense of increasing the simplicity differential.

Mathematical and logical laws themselves are not immune to revision if it is found that essential simplifications of our whole conceptual scheme will ensue. There have been suggestions, stimulated largely by quandaries of modern physics, that we revise the true-false dichotomy of current logic in favor of some sort of tri- or *n*-chotomy. Logical laws are the most central and crucial statements of our conceptual scheme, and for this reason the most protected from revision by the force of conservatism; but, because again of their crucial position, they are the laws an apt revision of which might offer the most sweeping simplification of our whole system of knowledge.

Thus the laws of mathematics and logic may, despite all "necessity," be abrogated. But this is not to deny that such laws are true by virtue of the conceptual scheme, or by virtue of meanings. Because these laws are so central, any revision of them is felt to be the adoption of a new conceptual scheme, the imposition of new meanings on old words. No such revolution, by the way, is envisaged in this book; there will be novelties of approach and technique in these pages, but at bottom logic will remain unchanged.

I have been stressing that in large part our statements are linked only remotely to observation. It is only by way of the relations of one statement to another that the statements in the interior of the system can figure at all in the prediction of experience, and can be found deserving of revision when prediction fails. Now of these relations of statements to statements, one of conspicuous importance is the relation of logical implication: the relation of any statement to any that follows logically from it. If one statement is to be held as true, each statement implied by it must also be held as true; and thus it is that statements internal to the system have their effects on statements at the periphery.

But for implication, our system of statements would for the most part be meaningless; nothing but the periphery would make sense. Yet implication is not really an added factor; for, to say that one statement logically implies a second is the same as saying that a third statement of the system, an 'if-then' compound formed from the other two, is logically true or "valid." Logical truths are statements on a par with the rest, but very centrally situated; they are statements of such forms as '*p* or not *p*', 'If *p* then *p*', 'If *p* and *q* then *q*', 'If everything is thus and so then something is thus and so', and others more complex and less quickly recognizable. Their characteristic is that they not only are true but stay true even when we make substitutions upon their component words and phrases as we please, provided merely that the so-called "logical" words '=', 'or', 'not', 'if-then', 'everything', 'something', etc., stay undisturbed. We may write any statements in the '*p*' and '*q*' positions and any terms in the 'thus and so' positions, in the forms cited above, without fear of falsity. All that counts, when a statement is logically true, is its structure in terms of logical words. Thus it is that logical truths are commonly said to be true by virtue merely of the meanings of the logical words.

The chief importance of logic lies in implication, which, therefore, will be the main theme of this book. Techniques are wanted for showing, given two statements, that the one implies the other; herein lies logical deduction. Such techniques will be developed, for increasingly inclusive portions of logic, as the book proceeds. The objects of deduction, the things related by implication, are statements; so statements will constitute not merely the medium of this book (as of most), but the primary subject matter.

Strictly speaking, as urged earlier, what admit of truth and falsity are not the statements but the individual events of their utterance. However, it is a great source of simplification in logical theory to talk of statements in abstraction from the individual occasions of their utterance; and this abstraction, if made in full awareness and subject to a certain precaution, offers no difficulty. The precaution is merely that we must not apply our logical techniques to examples in which one and the same statement recurs several times with changed meanings, owing to variations in immediate context. But such examples are easily enough adjusted to the purposes of logic by some preliminary paraphrasing, by way of bringing the implicit shifts of meaning into explicit form. (See Chapter 8.)

Logic and mathematics were coupled, in earlier remarks, as jointly enjoying a central position within the total system of discourse. Logic as commonly presented, and in particular as it will be presented in this book, seems to differ from mathematics in that in logic we talk about statements and their interrelationships, notably implication, whereas in mathematics we talk about abstract nonlinguistic things: numbers, functions, and the like. This contrast is in large part misleading. Logical truths, e.g., statements of the form 'If p and q then q', are not about statements; they may be about anything, depending on what statements we put in the blanks 'p' and 'q'. When we talk *about* such logical truths, and when we expound implications, we are indeed talking about statements; but so are we when we talk *about* mathematical truths.

But it is indeed the case that the truths of mathematics treat explicitly of abstract nonlinguistic things, e.g., numbers and functions, whereas the truths of logic, in a reasonably limited sense of the word 'logic', have no such entities as specific subject matter. This is an important difference. Despite this difference, however, logic in its higher reaches is found to bring us by natural stages into mathematics. For, it happens that certain unobtrusive extensions of logical theory carry us into a realm, sometimes also called 'logic' in a broad sense of the word, which does have abstract entities of a special kind as subject matter. These entities are classes; and the logical theory of classes, or set theory, proves to be the basic discipline of pure mathematics. From it, as first came to be known through the work of Frege, Dedekind, Weierstrass, and their successors in the late nineteenth century and after, the whole of classical mathematics can be generated. Before the end of the book we shall have ascended through four grades of logic in the narrower sense, and emerged into set theory; and here we shall see, as examples of the derivation of classical mathematics, how the concept of number and various related notions can be defined.

I
TRUTH FUNCTIONS