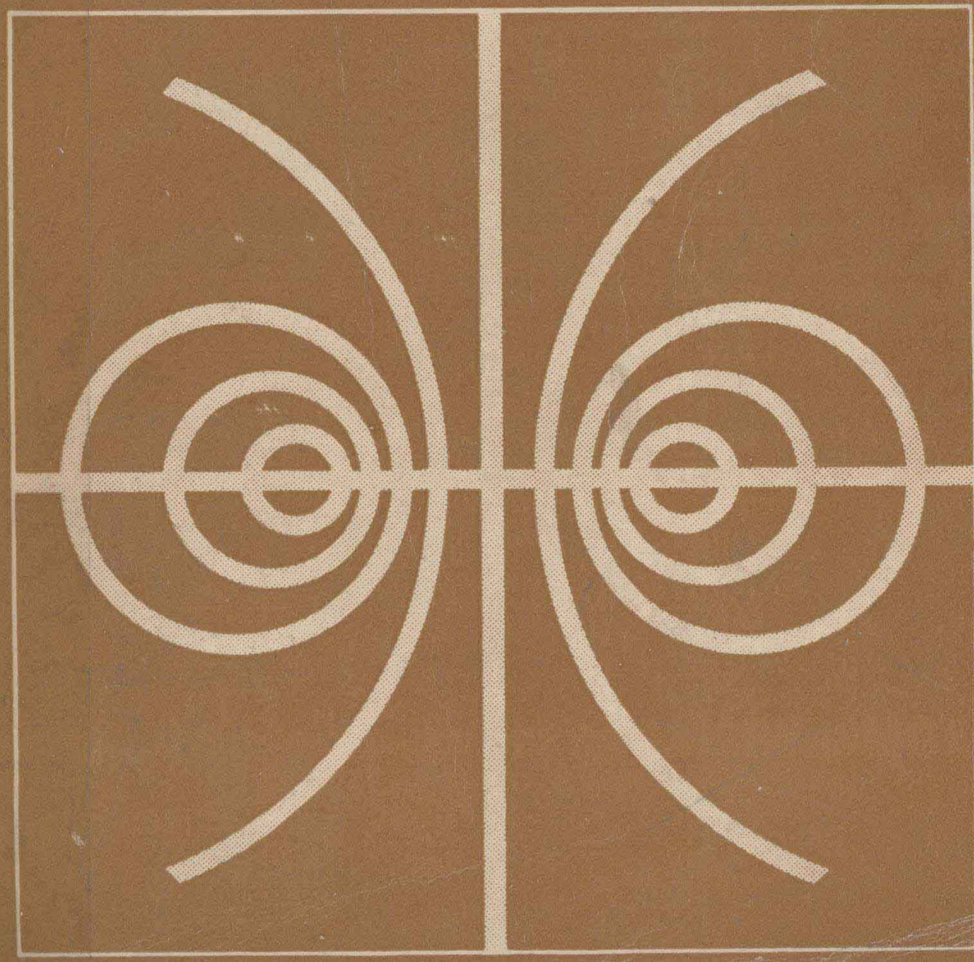
ERWIN KREYSZIG

ADVANCED ENGINEERING MATHEMATICS

Third Edition



Third Edition

Advanced Engineering Mathematics

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Preface

Purpose of the book. This book is intended to introduce students of engineering and physics to those areas of mathematics which, from a modern point of view, seem to be the most important in connection with practical problems. Topics are chosen according to the frequency of occurrence in applications. New ideas of modern mathematical training, as expressed in recent symposia on engineering education, were taken into account. The book should suit those institutions that have offered extended mathematical training for a long time as well as those that intend to follow the general trend of broadening the program of instruction in mathematics.

A course in elementary calculus is the sole prerequisite.

The material included in the book has formed the basis of various lecture courses given to undergraduate and graduate students of engineering, physics, and mathematics in this country, in Canada, and in Europe.

Changes in the third edition. The present edition differs essentially from the first and second editions as follows.

The problems have been changed.

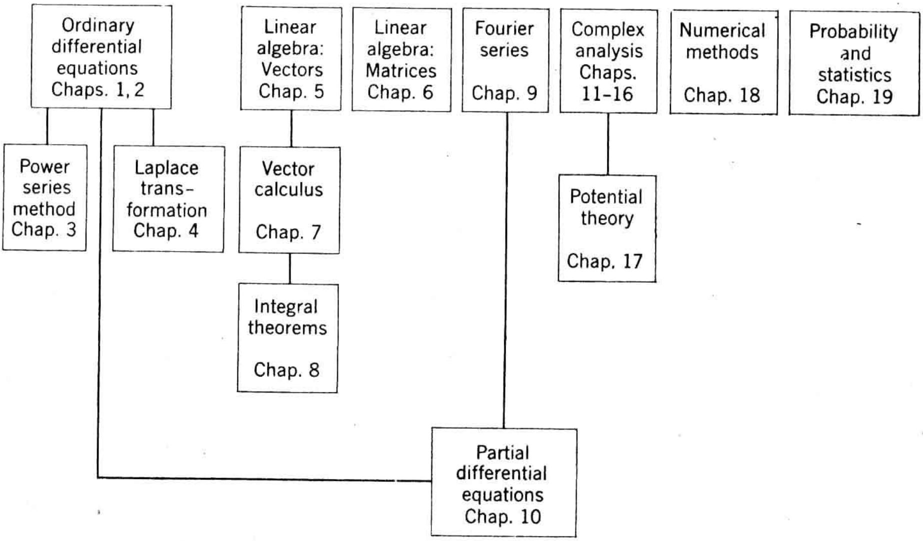
A new chapter on numerical methods for engineers (Chap. 18) has been added. It is suitable for a course meeting three hours a week.

The material on linear algebra and analysis (vectors and matrices in Chaps. 5 to 8) has been revised and amplified. The new presentation emphasizes algebraic as well as geometric aspects. Chapter 5 also contains an elementary introduction to vector spaces, inner product spaces, and some other concepts that are important in functional analysis and its applications.

These major changes are suggested and motivated by the development of engineering mathematics during the past decade, especially the widespread application of automatic computers to the solution of engineering problems and the increasing importance of methods of linear algebra.

Other improvements include a more compact consideration of Bessel functions and Legendre polynomials. In particular, the orthogonality of these functions is now discussed much earlier (in Chap. 3) in connection with the Sturm-Liouville theory. Appendix 1, containing references to the literature, has been updated and extended. A collection of frequently used formulas for elementary and higher functions has been added as Appendix 3; this replaces and extends Chap. 0 in the previous editions. The numerical tables have been placed together in Appendix 4 in order to facilitate the use of the book.

Content and arrangement. The arrangement of the subject matter in major parts can be seen from the diagram.



Much space is devoted to ordinary differential equations, linear algebra and vector analysis, and complex analysis, probably the three most important fields for engineers. But also the length of the other chapters on Fourier series, partial differential equations, numerical methods, etc., is such that the presentation may serve as a text for courses of the usual type.

To facilitate the use of parts of the book, the chapters are kept as independent of one another as possible.

The chapters are subdivided into relatively short sections. Each section includes typical examples and problems illustrating the concepts, methods, and results and their engineering applications.

Historical notes, references to original literature, and about 400 figures are included in the text.

References. A list of some books for reference and further study can be found at the end of the book, on page 789. Some formulas for special functions are included in Appendix 3, on page 826.

Problems and answers. The book contains more than 3500 carefully selected problems, which range from simple routine exercises to practical applications of considerable complexity. Answers to odd-numbered problems are included at the end of the book, starting on page 794.

Tables of functions are included in Appendix 4, on page 833.

Suggestions for a sequence of courses. The material may be taken in sequence and is then suitable for four consecutive semester courses, meeting 3–5 hours a week as follows:

First semester.	Ordinary differential equations (Chaps. 1–4).
Second semester.	Linear algebra and vector analysis (Chaps. 5–8).
Third semester.	Fourier series and partial differential equations (Chaps. 9, 10), Numerical methods (Chap. 18).
Fourth semester.	Complex analysis (Chaps. 11–17).

A course on **engineering statistics** (3–5 hours a week; Chap. 19) may be taught during any of those four semesters or afterwards.

Independent one-semester courses. The book is also suitable for various independent one-semester courses meeting 3 hours a week; for example:

Introduction to ordinary differential equations (Chaps. 1, 2).
Laplace transformation (Chap. 4).
Vector algebra and calculus (Chaps. 5, 7).
Matrices and systems of linear equations (Chap. 8).
Fourier series and partial differential equations (Chaps. 9, 10).
Complex analysis (Chaps. 11–16).
Numerical analysis (Chap. 18).

Shorter courses. Sections that may be omitted in a shorter course are indicated at the beginning of each chapter.

Principles for selection of topics. Which topics should be contained in a book of the present type, and how should these topics be arranged and presented?

To find some answer to these basic questions we may take a look at the historical development of engineering mathematics. This development shows the following two interesting features:

1. Mathematics has become more and more important to engineering science, and it is easy to conjecture that this trend will also continue in the future. In fact, problems in modern engineering are so complex that most of them cannot be solved solely by using physical intuition and past experience. The empirical approach has been successful in the solution of many problems in the past, but fails as soon as high speeds, large forces, high temperatures or other abnormal conditions are involved, and the situation becomes still more critical by the fact that various modern materials (plastics, alloys, etc.) have unusual physical properties. Experimental work has become complicated, time-consuming, and expensive. Here mathematics offers aid in planning constructions and experiments, in evaluating experimental data, and in reducing the work and cost of finding solutions.

2. Mathematical methods which were developed for purely theoretical reasons suddenly became of great importance in engineering mathematics. Examples are the theory of matrices, conformal mapping, and the theory of differential equations having periodic solutions.

What are the reflections of this situation on the teaching of engineering mathematics? Since the engineer will need more and more mathematics, should we try to include more and more topics in our courses, devoting less and less time to each topic? Or should we concentrate on a few carefully selected basic ideas of general practical importance which are especially suitable for teaching the student mathematical thinking and developing his own creative ability?

Sixty or eighty years ago no one was able to predict that conformal mapping or matrices would ever be of importance in the mathematical part of engineering work. Similarly, it is difficult to conjecture which new mathematical theories will have applications to engineering twenty or thirty years from now. But no matter what happens in that respect, if a student has a good training in the fundamentals of mathematics he will meet the future needs because he will be able to get acquainted with new methods by his own further study.

It follows that the most important objective and purpose in engineering mathematics seems to be that the student becomes familiar with mathematical thinking. He should learn to recognize the guiding principles and ideas "behind the scenes," which are more important than formal manipulations. He should get the impression that mathematics is not a collection of tricks and recipes but a systematic science of practical importance, resting on a relatively small number of basic concepts and involving powerful unifying methods. He should soon convince himself of the necessity for applying mathematical procedures to engineering problems, and he will find that the theory and its applications are related to each other like a tree and its fruits.

The student will see that the application of mathematics to an engineering problem consists essentially of three phases:

1. Translation of the given physical information into a mathematical form. In this way we obtain a mathematical model of the physical situation. This model may be a differential equation, a system of linear equations, or some other mathematical expression.

2. Treatment of the model by mathematical methods. This will lead to the solution of the given problem in mathematical form.

3. Interpretation of the mathematical result in physical terms.

All three steps seem to be of equal importance, and the presentation in this book is such that it will help the student to develop skill in carrying out all these steps. In this connection, preference has been given to applications which are of a general nature.

In some considerations it will be unavoidable to rely on results whose proofs are beyond the level of a book of the present type. In any case these points are marked distinctly, because hiding difficulties and oversimplifying matters is no real help to the student in preparing him for his professional work.

These are some of the guiding principles I used in selecting and presenting the material in this book. I made the choice with greatest care, using past and present teaching and research experience and resisting the temptation to include "everything which is important" in engineering mathematics.

Particular efforts have been made in presenting the topics as simply, clearly, and accurately as possible; this refers also to the choice of the notations. In each chapter the level increases gradually, avoiding jumps and accumulations of difficult theoretical considerations.

Acknowledgement. I am indebted to many of my former teachers, colleagues, and students for advice and help in preparing this book. Various parts of the manuscript were distributed to my classes in mimeographed form and returned to me, together with suggestions for improvement. Discussions with various engineers and mathematicians were of great help to me; I want to mention in particular Professors S. Bergman, P. L. Chambré, A. Cronheim, J. W. Dettman, R. G. Helsel, E. C. Klipple, H. Kuhn, H. B. Mann, I. Marx, W. D. Munroe, H.-W. Pu, T. Rado(†), H. A. Smith, J. P. Spencer, J. Todd, H. J. Weiss, A. Wilansky, all in this country, Professor H. S. M. Coxeter of Toronto, and Professors B. Baule, H. Behnke, H. Florian, H. Graf, F. Hohenberg, K. Klotter, M. Pinl, F. Reutter, C. Schmieden, H. Unger, A. Walther(†), H. Wielandt, all in Europe. I can offer here only an inadequate acknowledgment of my appreciation.

Finally, I want to thank John Wiley and Sons for their effective cooperation and their great care in preparing this edition of the book.

Suggestions of many readers were evaluated in preparing the present edition. Any further comment and suggestion for improvement of the book will be gratefully received.

Erwin Kreyszig

Advanced Engineering Mathematics

Contents

CHAPTER 1

Ordinary Differential Equations of the First Order

Differential equations are of fundamental importance in engineering mathematics because many physical laws and relations appear mathematically in the form of such equations. We shall consider various physical and geometrical problems which lead to differential equations and the most important standard methods for solving such equations. These methods will in general involve integration.

We shall pay particular attention to the derivation of differential equations from given physical situations. This transition from the physical problem to a corresponding "mathematical model" is of great practical importance and will be illustrated by typical examples.

The first four chapters of the book are devoted to ordinary differential equations, and in the present chapter we shall start with the simplest of these equations, the so-called equations of the first order. Corresponding **numerical methods** for obtaining approximate solutions are included in Sec. 18.6, which is independent of the other sections in Chap. 18.

Prerequisite for the present chapter. integral calculus.
Sections which may be omitted in a shorter course: 1.8–1.12.
References: Appendix 1, Part B.
Answers to Problems: Appendix 2.

1.1 Basic Concepts and Ideas

In this section we shall define and explain the basic concepts that matter in connection with differential equations and illustrate these concepts by examples. Then we shall consider two simple practical problems taken from physics and geometry. This will give us a first idea of the nature and purpose of differential equations and their application.

By an **ordinary differential equation** we mean a relation which involves one or several derivatives of an unspecified function y of x with respect to x; the relation may also involve y itself, given functions of x, and constants.