



# Scattering Methods in Complex Fluids

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## SCATTERING METHODS IN COMPLEX FLUIDS

Summarising recent research on the physics of complex liquids, this in-depth analysis examines the topic of complex liquids from a modern perspective, addressing experimental, computational and theoretical aspects of the field.

Selecting only the most interesting contemporary developments in this rich field of research, the authors present multiple examples including aggregation, gel formation and glass transition, in systems undergoing percolation, at criticality, or in supercooled states. Connecting experiments and simulation with key theoretical principles, and covering numerous systems including micelles, microemulsions, biological systems and cement pastes, this unique text is an invaluable resource for graduate students and researchers looking to explore and understand the expanding field of complex fluids.

SOW-HSIN CHEN is Professor Emeritus of Applied Radiation Physics, Department of Nuclear Science and Engineering, Massachusetts Institute of Technology, specialising in the dynamics of soft condensed matter using thermal neutron, synchrotron X-ray and laser light spectroscopy methods.

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## Preface

The central theme of this book is ‘slow dynamics in supercooled, glassy liquids and dense colloidal systems’ which has been an intense area of current research for some time. Although it can be well described by the mode-coupling theory of dense liquids, controversial viewpoints persist. Thus, the authors have written about the exciting modern aspects of the physics of liquids by selecting only the most interesting contemporary development in this rich field of research in the last decades.

This book presents and summarises a wide variety of recent research on the physics of complex liquids and suggests that the use of established techniques, essentially neutron, X-ray and light scattering together with theoretical and computer molecular dynamic simulation approaches, can be fruitfully applied to solve many new phenomena. These techniques are also central to investigating new interesting findings in liquid water such as liquid–liquid transition and its associated low-temperature critical point.

Although many materials found in nature can be classed as complex fluids, the authors have chosen to focus on water and colloids in this book for the following reasons:

- Water is the most important liquid for life on Earth. It covers 71% of the Earth’s surface and is probably the most ubiquitous, as well as the most essential, molecule on Earth. It is a vital element controlling not only all aspects of life itself but also the environmental factors that make life enjoyable. Water is a simple molecule yet possesses unique and anomalous properties at low temperatures that have fascinated scientists for many years. Thus in selecting the categories of complex liquids to include in this book, water is the obvious top choice.
- Colloids are another class of complex liquids characterised by the slowing down of the dynamics. They are becoming increasingly studied for their potential applications and the availability of degrees of freedom that are relatively simple

to vary experimentally through physical and chemical control parameters, giving rise to a much larger variety of phenomena compared to simple liquids. Initially a few relevant and classical examples of clustering and percolation in supramolecular colloidal aggregates are treated. Then various aspects of the physics of complex liquids are considered, focusing in particular on glass transition in colloidal systems, emphasising the role of the mode-coupling theory of the kinetic glass transition. The theory predicted and allowed us to study in detail many interesting new phenomena in colloids, such as re-entrant transitions and higher-order singularities in systems where short-range attraction is added to the usual short-distance repulsion between particles.

In order to provide an in-depth analysis examining the topics of complex liquids from a modern perspective, addressing the experimental, computational and theoretical aspect of the field, the book consists of ten chapters divided into three parts:

- Part I with three chapters deals with ‘scattering and liquids’.
- Part II with three chapters deals with ‘structural arrest’ phenomena.
- Part III with four chapters deals mainly with ‘water’.

Setting a good foundation for the rest of the book, the first two chapters cover elements of scattering techniques and theories commonly used in studying the structure and dynamics of liquid state matter. They are the outgrowth of parts of SHC’s lecture notes used in two of his graduate courses at MIT for many years – ‘Photon and neutron scattering spectroscopy and its applications in condensed matter’ and ‘Statistical thermodynamics of complex liquids’. In some of the chapters, certain sections are prefaced ‘Module’ to show that the topics they cover are significant, although they may not be in strict sequential order within the chapter.

Both authors, SHC and PT, have spent a large portion of their lives studying complex liquids, specifically water and colloidal systems, and collectively they have published several hundred research papers on these topics. Furthermore, they have been collaborating on these subjects for over 40 years, which has resulted in more than 50 joint scientific papers. Thus, it is natural for them to want to complement their mutual research interests and summarise their respective research on these topics throughout these long and productive years. A selection of arguments is made in the book, collecting what they consider relevant to the modern physics of liquids, in order to share their knowledge and insights with their readers. The research coverage is very up-to-date to June 2014.

This unique book should be of interest to all scientists who are interested in the dynamical properties of glassy liquids. It will also be an invaluable resource for

science and engineering graduate students and researchers looking to explore and understand the advancing field of complex fluids.

The authors want to thank their colleagues, former Ph.D. students and post-doctoral associates with whom they have shared many research topics reported in this book. They acknowledge in particular the long and fruitful collaboration with, amongst others, Professors Chung-Yuan Mou, Francesco Mallamace, Piero Baglioni and Paola Gallo (for SHC), and Francesco Sciortino (for both PT and SHC). For over a decade, all Chen's research projects have been funded by the Office of Basic Energy Sciences of the US Department of Energy. Their support is gratefully acknowledged. SHC also wants to thank Dr. P. Thiagarajan for his encouragement and discussions.

During the course of preparing and writing this book, for SHC, due to his physical limitation in efficient typing, particularly the equations, he wishes to gratefully acknowledge the persistent help from his Ph.D student, Zhe Wang, for preparing some initial background information and typing revised paragraphs and equations of a few initial chapters during 2011–2013. He appreciates his postdoc Kao-Hsiang Liu for similar help in a few months around late 2012. He also wishes to thank his Ph.D. student, Peisi Le, who has redrawn numerous required graphics from April to August 2014. To his co-author, PT, he has enjoyed greatly the extensive discussions and interactions with PT, and appreciates greatly the additional typesetting work his co-author, PT, had to take on his behalf. Finally, SHC is particularly thankful to his wife, Ching-chih, for her indispensable and time-consuming work in reformatting all his chapters and references, converting and processing many colour graphics to publishable form, as well as performing the final editing, proofreading and indexing of his part of the book, without whose efforts this work could never have been completed.

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January 1, 2015





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