Exciting Electrical Machines

E. R. Laithwaite

Pergamon

EXCITING ELECTRICAL MACHINES

BY

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WHEATON

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Pergamon Press Ltd., Headington Hill Hall, Oxford
Pergamon Press Inc., Maxwell House, Fairview Park, Elmsford, New York 10523
Pergamon of Canada Ltd., 207 Queen's Quay West, Toronto 1
Pergamon Press (Aust.) Pty. Ltd., 19a Boundary Street, Rushcutter's Bay, NSW 2011, Australia
Vieweg & Sohn GmbH, Burgplatz 1, Braunschweig

Selected Topics in Physics
Editor: Brian E. Woolnough

Exciting Electrical Machines

First published 1974

Printed in Great Britain by A. Wheaton & Co, Exeter

08 016863 9

Acknowledgements

Thanks are due to the following, who have kindly provided photographs for use in this book:

C. A. Parsons & Co. Ltd.,
English Electric Diesels Ltd.,
Herbert Morris Ltd.,
The Merlin-Gerin Co.,
Westinghouse Engineering.

Preface

The part of this book which was last to be written was the title! What I wanted to say in the text was never in doubt. I have always enjoyed my engineering and particularly because my subject involved things which moved rather than static pieces of equipment like electronic amplifiers. I hoped to convey some of the thrills of exploring new kinds of electrical machines to young scientists who are inclined to believe that "electrical machines" is a dull subject and one in which the fundamental research "has all been done". I wished to dispel such ideas rapidly.

At the same time I wanted to introduce the subject through the concept of the magnetic circuit which makes many phenomena of electromagnetism easier to appreciate. In this analogue, flux corresponds to current in our electric circuit and reluctance to resistance, whilst the driving element, e.m.f., in the electric circuit is replaced by magnetomotive force (m.m.f.) which is proportional to the product of current and number of turns, called the "ampère-turns". (Nowadays we choose our units so as to make it numerically equal to this product.)

Old fashioned expressions in electrical engineering were often extremely graphic. Thus, a circuit when switched on was said to be "alive". A magnetic circuit when fed with m.m.f. was (and still is) said to be "excited", and the current linkages which make it so can be referred to as "the exciting ampère-turns".

It was only after completing the manuscript that I remembered that in my student days (immediately following the 1939–46 war years) I was lectured by an experienced engineer, Harold Gerrard, who knew much about machines and about people, and to whom I am indebted for many things. The Women's Auxiliary Territorial Service was in those days commonly known as "the ATS". Our Mr Gerrard, lecturing to an all-male audience always referred to m.m.f. as "the exciting ATS", keeping his face so straight that he convinced us that any suggestion of an association of m.m.f. with the opposite sex had never entered his head.

It was this thought which led me to believe that the title "Exciting electrical machines" might express in as few words as possible the two topics which I most wanted to emphasise—the usefulness of m.m.f. as a concept and the fact that I have always found machines exciting.

Contents

1 What do you believe?

First—curiosity

A great deal of man's progress into the age of technology is undoubtedly due to his curiosity. It used to be said, quite wrongly, that science was a striving after the Truth. This was probably a misinterpretation of the search for Knowledge which most of us practise consciously or unconsciously most of our lives.

The fundamental question: "What is matter?" is extremely difficult to answer. The old physics book definition "matter is anything which occupies space" gets you nowhere, for you then have to define space as "something to put matter in". Yet the alert mind wants a more tangible answer than anything which comes out of metaphysics.

A child takes a lump of modelling clay and the likelihood is that he soon gets around to dividing it into smaller and smaller pieces. Soon the question arises as to how small a piece you can make. With more imagination the question becomes: how small can I make a piece in theory? The idea of molecules emerges and then of atoms, then electrons. When you are told what the smallest possible particle is, your first concept of it is as a little blob of something—until you start asking yourself what the density of the blob is, what shape it is, and so on, and a more knowledgeable person tells you that these are meaningless questions, and that if it had a shape it could be divided into parts—which denies the original proposition that it was the smallest particle possible. It seems at this point as if there is no foundation at all on which to build.

Many centuries ago, men thought just as deeply as we do today about the meaning of Creation and the Universe. About 1100 A.D., St Anselm, the first Archbishop of Canterbury, wrote this profound sentence: "*I believe, in order that I may understand.*" This statement is as true today as it was then. It is a statement about science. You must first believe in something, after which you can make deductions. What you choose as a starting point does not affect the logical steps from one subsequent argument to the other.

In an earlier book* I likened this process to the Caucus Race in *Alice in Wonderland* where you could begin running where you liked and leave off where you liked. Naturally, there are some places which, for a given subject, make the understanding of it much easier than do others. These could be said to be "profitable" starting points.

Perhaps a fundamental difference between a physicist and an engineer is that the former tries to identify all his arguments from a single "belief", whilst the latter seeks only to sift what is profitable from what is unprofitable. Certainly an engineer makes more use of analogy, which is the art of likening one system to another and seeing to what extent different parts of the one have corresponding parts in the other.

It is perhaps because of the desire to unify on the part of the physicist that he tends to build on past experience, handing on to each successive generation the state of knowledge at that time so that the story becomes virtually a historical one. Thus it is that many elementary books on electricity and magnetism, as included in school physics, begin with a discussion of lodestones in the Chinese desert in 3000 B.C. and of the curious properties of amber (for which the Greek word was *elektron*). By the time the difficult subject of electromagnetism has been reached the pupil has had the wool pulled over his eyes several times when it has been necessary to jump from one part of the Caucus ring to another, without admitting that there were things between which involved "belief" rather than understanding.

The Phenomenon we call "Electromagnetism"

Electrical machines which make use of the phenomena of electromagnetism likewise appear historically in physics textbooks, so the d.c. machine with its commutator and brushes appears first and tends to dominate to an extent that gives the impression that most electric motors in common use

* The Engineer in Wonderland (E.U.P., 1967) Chapter 3.

have commutators and brushes. The fact that the brushless induction motor which dominates the world of electric power-drives does not attain pride of place in a physics book or syllabus is almost certainly due to its being considered "difficult to explain". Cértainly the explanation is complicated if one is bound by the classical explanations of physics. What is more, there are several jumps in the argument which are not made for any reason that we can think of. The following "explanation" of the action of an induction motor is typical of that used in classical physics. The portions in brackets denote the points where there is apparently no logical argument to support the theory.

"In an induction motor, a travelling magnetic field is set up by a system of primary coils carrying electric current. The secondary member consists of short-circuited loops of conductor. When the conductor is cut by the moving field (whatever that is) an e.m-f. (whatever that is) is induced in the conductor (for a reason not given). The e.m.f. drives a current in the conductor, and this current reacts with the same magnetic field which produced it (for a reason not given) to produce a force."

No wonder it is considered a difficult machine to understand!

This book describes a whole range of electrical machines, the action of each being "explained" in the simplest possible terms in the hope that the electrical machines may be enjoyed. For those who prefer something that moves rather than something that does not (such as an amplifier or computing machine) it is hoped that the account which follows will bring interest, pleasure and curiosity, especially the latter, for we do not yet know all that there is to know about electrical machines, nor have all the kinds of electric motor yet been invented.

Sometimes the most fundamental discoveries may be made with the simplest of apparatus. It is hoped that some of the experiments with iron filings and paper clips, described in Chapter 7 in particular, will encourage those readers who have been misled into believing either that: "all research has been done already" or "only big teams backed by lots of money can produce worthwhile results".

What then are you asked to believe when you begin this account of electric motors and generators? Basically very little more than that you can treat magnetism as you do electricity and regard it as existing only in *circuits*, and that when a magnetic field moves it behaves towards conductors in its path as if it were a viscous liquid flowing like a river. These are surely easier things to believe than induced e.m.fs, magnetic poles or lines of force. Later on we shall learn about these latter analogues also, and how our earlier beliefs can be made to fit the classical physics structure, but for the moment it will suffice to believe only in the concept of a circuit.

Magnetic circuits

Many teachers of physics reject the concept of a magnetic circuit because "flux does not flow" so the analogy is not complete. Very few analogies are, even those of the model of an atom in which electrons revolve in orbits around a nucleus and spin on their own axes (whatever that means). The engineer is concerned only with whether a particular analogy can be profitable *for him*, and we shall find that the concept of a magnetic circuit can be extremely profitable, since it allows us to calculate correctly by simple algebra many quantities which are only calculable in terms of conventional physics by an integration process. The engineer's reply to the challenge that flux does not flow is therefore: "If I say it flows, it flows!" Actually, for the purpose of the following studies it does not matter whether we consider flux to be flowing or not; we are only concerned with measuring its *effect* quantitatively.

Thus, just as we write Ohm's law for an electric circuit:

$$\text{e.m.f.} = \text{current} \times \text{resistance} \quad (1.1)$$

we shall write for a magnetic circuit:

$$\text{m.m.f.} = \text{flux} \times \text{reluctance} \quad (1.2)$$

What is our justification for doing this? In the last resort, only the results of experiments. We can wind a coil A of wire around an iron ring, as shown in fig. 1.1(a). A second coil B acts as detector of flux and is connected to a galvanometer. We can open and close the switch S and observe that the galvanometer deflects in one direction for closure and in the other when S is opened. It is not unreasonable to suggest that something occurs around the iron ring which "transmits" in some way the action of starting and stopping the current in A. We can measure

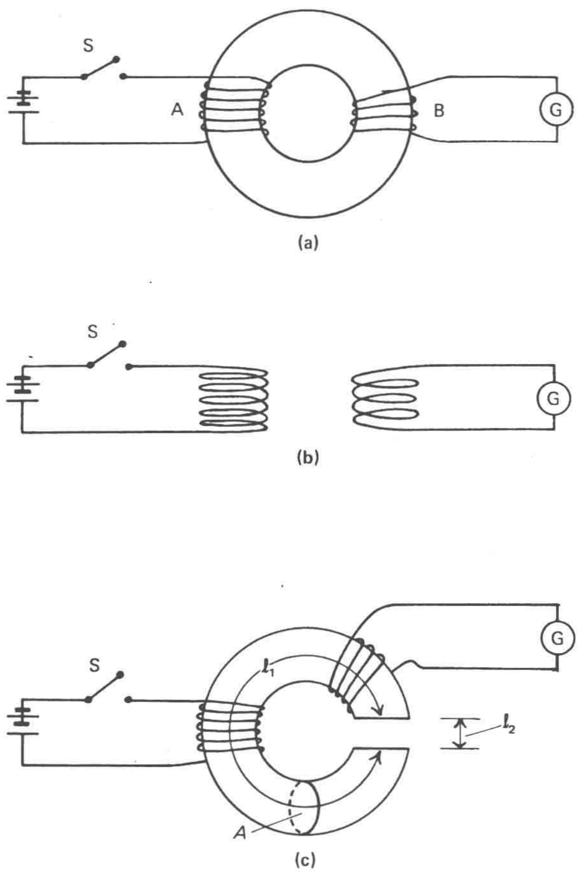

FIG. 1.1 *Some experiments with magnetic circuits.*

the *size* of the effect by observing the throw on the galvanometer. We can experiment with different currents and with a different number of turns of wire in the coil A and soon learn that the size of the effect is proportional to

(i) the number of turns in A
(ii) the current that is switched.

Clearly the *cause* of the action is located in the coil A and, just as e.m.f. is the *cause* and current the *effect* in an electric circuit, we can say that "magneto-motive force" (a word we make up to be the analogue of electromotive force) is proportional to the product of current and number of turns—the ampère turns. Furthermore, since we are developing a new concept, we can choose our units for magnetic flux (the analogue of current) and magnetic resistance (which we call *reluctance*) such that we can write a form of Ohm's law for a magnetic circuit (equation (1.2)) with an "equals" sign rather than one of proportionality, thus: m.m.f. (= ampère turns) = Magnetic flux × Reluctance.

Now, the resistance of an electric wire is proportional to its length l_e and to its resistivity ρ, and inversely proportional to its area A_e thus:

$$R = \frac{\rho\, l_e}{A_e}$$

This expression may be written in terms of conductivity (σ) rather than resistivity, thus:

$$R = \frac{l_e}{\sigma\, A_e} \qquad (1.3)$$

In the same way our magnetic resistance or *reluctance* $\mathscr{R}$ may be calculated in terms of the length, area and conductivity (l_m, A_m, k respectively) of the magnetic circuit, thus:

$$\mathscr{R} = \frac{l_m}{k\, A_m} \qquad (1.4)$$

The idea of a magnetic conductivity may not be as new to you as you think, for if we now write equation (1.2) in terms of all the quantities which we have defined, we get:

$$\text{ampère-turns} = \frac{\Phi\, l_m}{k\, A_m} \qquad (1.5)$$

where Φ is the total flux produced.

The mystic factor μ_0

Equation (1.5) may be re-arranged as

$$\frac{\text{ampère-turns}}{l_m} = \left(\frac{\Phi}{A}\right)\frac{1}{k}$$

But (ampere-turns/length) is the quantity normally called H in classical physics whilst Φ/A_m is the flux density, B.

Thus $H = B/k$, and k is seen to be the quantity we have normally called *magnetic permeability*, μ. This quantity is normally introduced as the constant necessary to balance the equation for the force between magnetic poles.

This is perhaps the main difference between this book and some books of physics. The latter begin with a belief in isolated poles. Here we believe first in magnetic circuits, and later we shall see that poles arise wherever there is a sudden change in reluctance around a magnetic circuit.

One thing is common to the two approaches. μ is the "magic" factor—the expression of all our ignorance about electromagnetism. It is as if we had swept all the dirt in a room into one corner and argued that, although we could not get rid of it, at least the rest of the room was clean! μ, and perhaps more particularly its value μ_0 for empty space, is our interpretation of a 4-dimensional world, our Revelation of God—call it what you like. It is the mystery which we can appreciate but can never hope to understand. Why should free space conduct magnetism? Einstein tried for many years to relate μ_0 to the gravitational constant, a relationship which, if established, would be, as he put it, "the key to the cosmos", but he failed to establish it.

Nowadays it is fashionable to express the permeability of a material as $\mu_r \times \mu_0$ where μ_0 is the permeability of free space and μ_r is the number of times by which the material is more permeable (i.e. the relative permeability). Thus, the value of μ_r for diamagnetics is slightly less than 1·0, for paramagnetics slightly greater than 1·0, and for ferromagnetics may be more than 1000.

RELATIVE PERMEABILITIES OF COMMON MATERIALS (μ_r)

Diamagnetics:	Hydrogen	0·999 999 997 92
	Water	0·999 910
	Glass	0·999 987
	Copper	0·999 990
Paramagnetics:	Oxygen	1·000 018
	Aluminium	1·000 021
	Platinum	1·000 21
Ferromagnetics:	Iron	700
(*Maximum*	Stalloy	6 000
Values)	Mumetal	90 000

Magnetic circuits in practice

If we remove the iron ring from the coils of fig. 1.1(a), leaving the coils in the same position, as shown in fig. 1.1(b), it may still be possible to detect a galvanometer deflection (if the instrument is sufficiently sensitive) when the current is switched, although the effect may be 1 000 times smaller than that in the circuit of fig. 1.1(a).

Generally, the magnetic circuit of a machine consists mostly of steel (for we wish to make the reluctance as low as possible) with a small air-gap, the latter being necessary as mechanical clearance between stationary and moving members. The basic magnetic circuit is thus as shown in fig. 1.1(c), and this is a circuit consisting of two reluctances in series:

(i) the iron part of reluctance $\dfrac{l_1}{\mu \mu_0 A}$

(ii) the air-gap of reluctance $\dfrac{l_2}{\mu_0 A'}$

and these two reluctances may be added arithmetically as in a series electric circuit. If the length l_2 is very small in relation to $\sqrt{A}$, the area A' of the air-gap may be taken as equal to A. For larger gaps the flux lines tend to spread laterally as shown in fig. 1.2(a), giving the air-gap an effective area greater than A. A useful approximation for the value of A' in such cases is to assume that the flux fringing extends only as far as a semi-circle added to the gap cross-section at each side, as shown in fig. 1.2(b), but that, within these semi-circles, the flux density is uniform and equal to that inside the gap itself. Thus, if the area A is a rectangle of dimensions $a \times b$, and the air-gap length is g, the effective value of A' is $(a + g)(b + g)$.

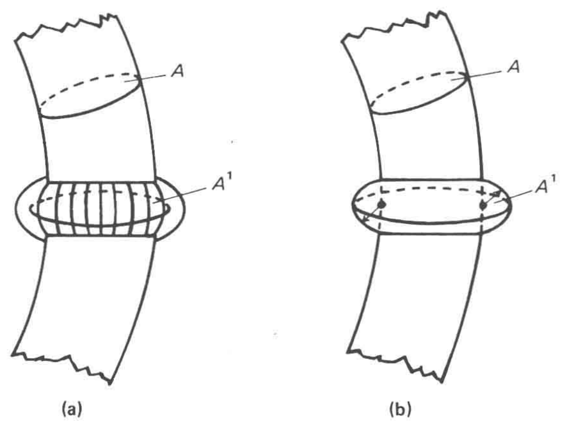

FIG. 1.2 *Flux fringing in the vicinity of an airgap*

A further experiment which may be carried out with the arrangement shown in fig. 1.1(c) consists of moving the coils to different parts of the circuit

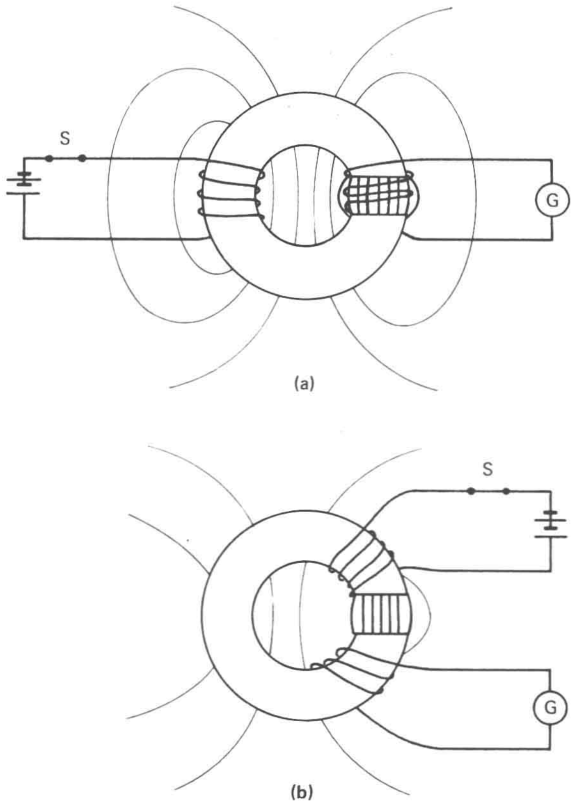

FIG. 1.3 *The positions of coils around a magnetic circuit make little difference to electromagnetic effects.*

as shown in fig. 1.3. If the value of μ for the iron part of the circuit is high there will be hardly any difference between the deflections for a given current change. The circuit is equivalent to an insulated electric circuit (as shown in fig. 1.4) where the position of the battery makes no difference to the current in the circuit.

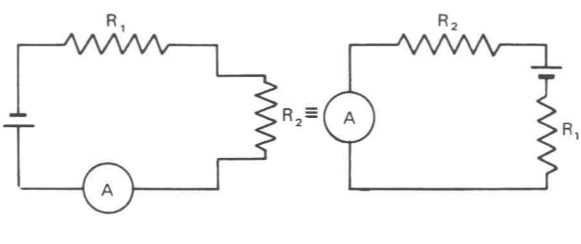

FIG. 1.4 *The position of a battery in a series electric circuit makes no difference to electric effects because the circuit is insulated.*

The machine designer never forgets, however, that there is no material whose value of μ is very much less than 1·0, which can be painted on to the magnetic circuit to prevent flux from "leaking" into the space surrounding the circuit. In an electric circuit such "insulating" materials are readily available, and an electric circuit may therefore consist of many turns of wire, in close proximity to each other, none of which leaks current into the others. In fig. 1.3(a) not all the flux generated by the primary coil threads the secondary. Some of it leaks into space as shown and the circuit is analogous to the electric circuit shown in fig. 1.5 in which the resistor R_L can never be eliminated entirely. In the circuit shown in fig. 1.3(b) almost all the leakage flux threads both coils, as does the flux through the iron, so that a slightly higher deflection is obtainable per unit current change than with the arrangement of fig. 1.3(a).

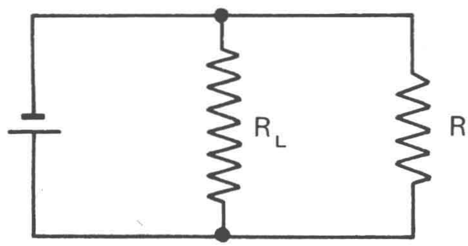

FIG. 1.5 *An electric circuit analogous to a "leaky" magnetic circuit.*

Electric circuits can be insulated, whereas magnetic circuits cannot, and the conductivity of electric circuits is relatively better in terms of the energy that can be transmitted to a load for a given quantity of transmitting material. In the case of a magnetic circuit, the flux is usually made to traverse an air-gap in order to use it for force or power production (the transformer being the exception) and the conductivity of the transmitting material (iron) is only at best 1 000 times more conducting than the air. But an electric transmission system may be effectively millions of times more conducting than its load. For this reason, coupled with the lack of a magnetic insulator, magnetic circuits of machines are short and fat and therefore cannot be tampered with by a service engineer with a pair of pliers. Electric circuits, on the other hand, are long and thin, multi-turn, and open to adjustment by shortening or adding more turns.

Electromagnetism and relativity

Although Einstein failed to relate gravitation and electromagnetism, he did succeed in unifying electricity and magnetism. The classical physicist believes in the electron, and that a flow of such charged electrons constitutes an electric current. Einstein's Special Theory of Relativity is concerned with the force between masses, or between charges which have a *velocity* relative to each other. The modification to the standard formula for the force between charges q_1 and q_2, i.e.:

$$F_1 = \frac{q_1 q_2}{\epsilon_0 d^2} \qquad (1.6)$$

consists in relating the relative velocity v between q_1 and q_2 to the velocity of light c and the resulting formula is

$$F_2 = \frac{q_1 q_2}{\epsilon_0 d^2} \sqrt{1 - \left(\frac{v}{c}\right)^2} \qquad (1.7)$$

where ϵ_0 is the permittivity of free space, using the S.I. system of units.

If the force between two current-carrying electric circuits is calculated from equation (1.7), the difference between this force F_2 and F_1, calculated using equation (1.6), is found to be that predicted by the effect of the *magnetic field* due to one circuit acting on the other. This is the force we use in electrical machines, and the magnitude of it in relation to the force between static charges may be judged by supposing, in the first instance, that the electrons flow at one tenth the velocity of light. The correction factor $\sqrt{1-(v/c)^2}$ thus amounts to $\sqrt{1 - 0{\cdot}01} = \sqrt{0{\cdot}99} = 0{\cdot}995$ or $\frac{1}{2}\%$ of the electrostatic force. Now let us see if we can find the *actual* velocity of electrons in copper when a current flows.

The charge on the electron is $1{\cdot}602 \times 10^{-19}$ coulomb. Since 1 coulomb is the quantity of electricity (i.e. charge) passing a point in one second for a current of one ampère, there are $10^{19}/1{\cdot}602$ electrons passing any point in one second for one ampère, or for I ampères, $10^{19}I/1{\cdot}602$. Another physical fact is that there are roughly 5×10^{28} free electrons per cubic metre of copper. Thus, the volume of copper needed to contain all the electrons which pass in one second is

$$\frac{10^{19}I}{1{\cdot}602 \times 5 \times 10^{28}}\, m^3$$

which is approximately equal to $I \times 1{\cdot}25 \times 10^{-10}$ m^3. If the cross section of a wire is A m^2, the effective length of a column of electrons which passes a given point per second (i.e. the drift velocity) is $1{\cdot}25 \times 10^{-10} \times I/A$ ms^{-1}. Now I/A is the current density. The recommended value of I/A for house wiring is $6{\cdot}5 \times 10^9 A$ m^{-2}, so that the drift velocity is of the order of $0{\cdot}01$ m s^{-1}, or $c/(3 \times 10^{10})$. If we now substitute this value into the correction factor $\sqrt{1-(v/c)^2}$ we see at once that the value of the force due to the "magnetic" component is of the order of one million millionth part of the electrostatic force. Yet this apparently tiny amount is sufficient to drive all the mighty machines in our power stations and rolling mills.

One may therefore ask what potential electrostatic energy is locked in the atoms of copper in the form of free electrons. If we could take all the 5×10^{22} free electrons from one cubic centimetre of copper we should have a charge of about 8 000 coulombs. The force between this and a similar charge placed one metre away from it in air would be $8\,000^2/\epsilon_0$ newtons. The value of the free space permittivity ϵ_0 is $10^{-9}/36\pi$, whence the force is $64 \times 10^6 \times 36\pi \times 10^9 = 7{\cdot}24 \times 10^{18}$ N or roughly 724 million million tonnes—enough to split the earth in half!

It is not unusual for us to use effects which are mere "skimmings off the surface" of what is available. For example, the H-bomb with its enormous potential is but the conversion of an amount of matter which represents the difference between 2 hydrogen nuclei and one helium nucleus. Were a complete nucleus to be converted entirely into energy the present H-bomb would, by comparison, be as gentle as a damp squib. Such is the energy locked in matter. It may be that in other parts of the universe energy is being released at the full level, but this leads us into the wonders of astronomy, and that is another story.

Flux cutting and Flux linking

It is common practice in physics to calculate the e.m.f. E induced in a conductor of length l when it moves in a direction at right angles to a magnetic field of strength B at velocity v as:

$$E = Blv \qquad (1.8)$$

If such an e.m.f. is allowed to produce a current I, the power produced is $BlvI$, which must also represent the rate of mechanical working needed to force the conductor through the flux. This is clearly equal to Fv, where F is the force. Thus:

$$Fv = BlvI \qquad (1.9)$$
$$\text{or} \quad F = BlI$$

This formula, derived by the "action and reaction" principle, is really the one predicted by relativity, since it relates to the force between the current I and the flux B, which somehow or other has been produced by a second current or equivalent current I'. Equation (1.8) is generally known as the "flux cutting rule" to distinguish it from the "flux linking rule" which relates the e.m.f. induced by a *changing* magnetic field through a closed loop at rest. The latter rule should be seen to be *entirely different* from the flux cutting rule, for a rate of change of flux a "($\mathrm{d}B/\mathrm{d}t$)" is set up by a rate of change of current, a "($\mathrm{d}I/\mathrm{d}t$)". A rate of change of current can be produced only by *accelerating* the electrons in a wire and, by a similar action and reaction process, the induced e.m.f., due to flux linkage changes without motion of the conductors, can only be calculated in terms of the forces between charges which have *a relative acceleration*. Such a calculation requires the General Theory of Relativity which starts from quite different premises from the Special Theory and is beyond the scope of this book. Nevertheless we should never confuse the flux cutting and flux linking mechanisms, for, in the more general problems in electrical machines, circuits may have induced e.m.fs. from *both* causes at once, i.e. the conductors may be moving in a magnetic field whose strength is itself varying, for the real formula for induced e.m.f. is:

$$E = k\frac{\mathrm{d}\Phi}{\mathrm{d}t} \text{ and since } \Phi = BA$$

$$E = k\frac{\mathrm{d}(BA)}{\mathrm{d}t} = k\frac{\mathrm{d}B}{\mathrm{d}t}A + k\frac{\mathrm{d}A}{\mathrm{d}t}B$$

The first term of the right hand side is the flux linking term, the second the flux cutting term. No doubt the confusion begins when a conducting rod slides over a pair of rails so as to cut a field of strength B. The e.m.f. can be calculated either as Blv or, since the flux B must close a loop somewhere outside the electric circuit, as a change of linkages $B(lv)$, since $v = \mathrm{d}x/\mathrm{d}t$. Arguments in professional journals have raged as recently as 1963* about flux cutting and linking rules, which is quite surprising, for one can always resolve the problem by remembering whether or not the reaction force, were the e.m.f. to drive a current, is the result of electrons moving with relative velocity or relative acceleration.

The rules of electromagnetism

The approach to electromagnetism by means of circuits can be seen to simplify the analysis of many situations as compared with the magnetic pole approach. The laws of electromagnetism can be expressed in a variety of ways and various physics text books have quoted in one place or another the following rules:

The corkscrew rule
The gripping rule
Fleming's left and right hand rules
Ampère's swimming rule
Faraday's laws of induction
Lenz's law.

What is rarely stated is the number of these rules that are independent of each other. If one accepts the relativistic statement that all forces arise as the result of the interaction of moving charges, then most situations can be resolved by one of two rules together with a somewhat broader expression of Lenz's law. The rule for moving charges (i.e. currents) is that like currents attract each other, unlike currents repel. Lenz's law can be regarded as a special case of Newton's law that action and reaction are equal and opposite. Within such a broader concept, the duality of Fleming's left and right hand rules becomes self evident. When the thumbs and fingers are extended in the manner necessary for Fleming's rules it is possible to point *any two* digits

* Journal of the Institution of Electrical Engineers, Letters to the Editor between Oct. 1962 and Oct. 1963.

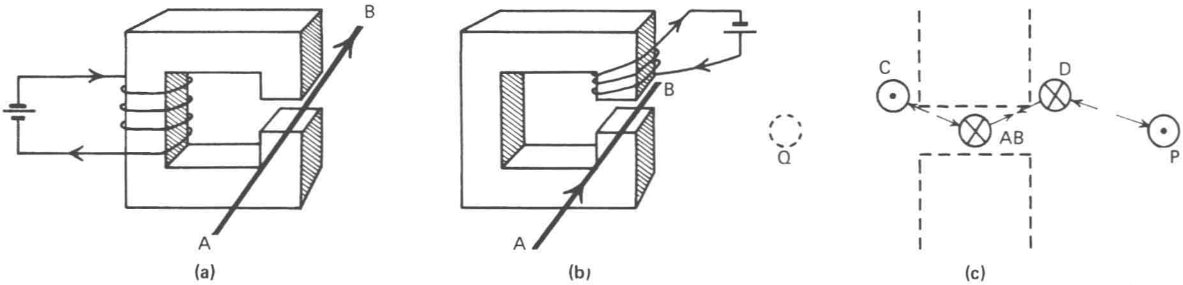

FIG. 1.6 *Prediction of the direction of an electromagnetic force by means of a single rule.*

on one hand in the same directions as the corresponding digits on the other hand, but the remaining digits will always point in *opposite* directions. This expresses the fact that a motor and a generator simply represent the same machine with the direction of power flow reversed.

Consider the situation shown in fig. 1.6(a) in which a wire AB carries a current between the poles of an electromagnet in the direction shown. How many rules is it necessary to use to find the direction of the resulting force between AB and the poles? By classical physics you would require at least two—one to predict the direction of the magnetic field the other to predict the reaction between field and current (or field and field). Now look at fig. 1.6(b). This shows the energising coil of the magnet moved around the magnetic circuit until it is virtually at one of the pole faces. The situation is at once reducible to that shown in fig. 1.6(c) in which only the force between current and current is required. Notice the "safety" of this method. The current in AB must return by some path, for only current in a circuit is meaningful. It does not matter whether we choose to return it at P or at Q. Now, use the rule for the direction of force between currents. C repels AB, D attracts AB, D repels P. Thus, the coil containing AB is forced to the right.

The fields due to some of the simpler arrangements of current-carrying conductors can be predicted by the magnetic circuit concept; for example, the field at distance r from an infinitely long straight wire, as illustrated in fig. 1.7. Infinite though the wire is, it must return its current and by so doing close a circuit which links a magnetic circuit at radius r (shown dotted). Let the area of this

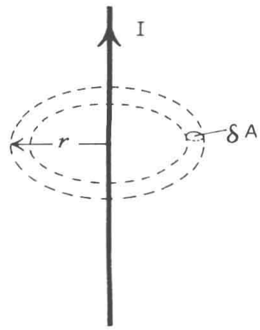

FIG. 1.7 *Calculation of magnetic field due to a long, straight wire.*

circuit be $\triangle A$. By symmetry, the field everywhere at radius r will be the same. The magneto-motive force (m.m.f.) is clearly I (being a single turn) whilst the length of the magnetic circuit is $2\pi r$. For this arrangement equation (1.2) becomes:

$$I = \frac{\Phi\, 2\pi r}{\mu_0 \Delta A} = B\frac{2\pi r}{\mu_0}$$

$$\text{whence } B = \frac{\mu_0 I}{2\pi r}.$$

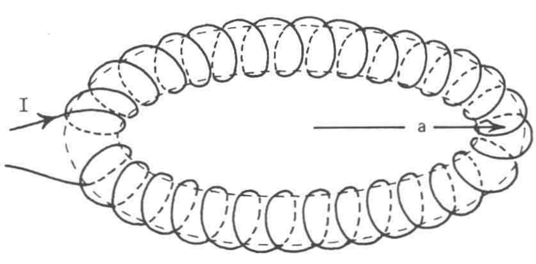

FIG. 1.8 *Calculation of magnetic field due to a toroid.*

Contrast this method with that which begins with the Biot-Savart equation and involves a calculus method.

Next consider the toroid shown in fig. 1.8. Again symmetry considerations allow us to find the field around the mean circle at radius a. If there are N turns per unit length of periphery and I ampères flowing in the coil, equation (1.2) becomes:

$$NI\,(2\pi a) = \frac{B(2\pi a)}{\mu_0}$$

whence $$B = NI\mu_0$$

and this applies even if a is infinite, i.e. for the case of the infinitely long solenoid.

The danger of considering a length of conductor in isolation (i.e. not as part of a circuit) is emphasised by the situation shown in fig. 1.9. The use of classical formulae on each conductor in turn will demonstrate that the forces produced by AB on CD are not equal to the forces produced by CD on AB!

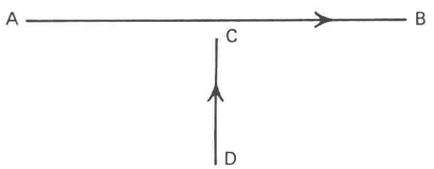

FIG. 1.9 *The danger of considering finite-length conductors in isolation.*

Some conclusions from chapter 1

There are several concepts in science which apply in different sections or disciplines and which are thus more valuable in that the student is called upon to learn fewer rules on the one hand and to obtain a deeper insight into scientific method on the other. The circuit concept is of wider application than when concerned with the flow of water in a pipe or the passage of electric current along wires. The main purpose of this chapter has been to introduce the idea of a magnetic circuit and to show how it can be used. But the circuit concept does not end there. It can be applied equally effectively in calculating heat flow. The four forms of Ohm's law are collected together below for easy reference.

Fluid Circuit

Pressure difference = Rate of flow × Mechanical resistance

Electric Circuit

E.M.F. = Current × Electrical resistance

Magnetic Circuit

M.M.F. = Flux × Reluctance

Thermal Circuit

Temperature difference = Heat flow × Thermal resistance

In the three latter cases the resistance or reluctance term is given by:

$$\left(\frac{\text{length}}{\text{area} \times \text{conductivity}}\right)$$

There is a temptation to try to describe gravitation in terms of a circuit by the concept of a gravitational flux, but it will be found that gravity is much more profitably allied to electrostatics, the gravitational constant G being analogous to free-space permittivity ϵ_0.

2 | The Magic of Electromagnetism

Looking for something new

The fact that we can formulate rules for the behaviour of electric and magnetic structures should never be allowed to lead us to the belief that "it has all been done" and therefore anything we may try has obviously been exploited to the full already. By comparison with what is known about the four-dimensional world there is an almost infinite amount yet to be discovered, for the four-dimensional world is a truly magic world in which force can be exerted on an object a distance away in space for no apparent reason. Objects may be floated, moved or heated without external contact. The music hall "magician" can float a ball across a stage, but we know that we are being deceived and that somewhere there is a quite "rational" explanation for what is apparently impossible. Is not then the phenomenon of electromagnetism, which makes such a feat possible in reality, truly "magic"?

It is worthwhile to repeat some of the early pioneers' experiments, for those workers had none of our background knowledge, their apparatus was not sophisticated. It is only the fact that all our electric motors are enclosed in the same kind of uninteresting metal case which makes them appear dull and "all alike". If only we could see "the works" we should be able to tell which motor worked by change of reluctance, which by inductance or by hysteresis, and the subject of electrical machines would at once come alive and cease to be the dull subject which, alas, is the general impression handed on by successive generations. If you read the scientific papers written between 1880 and 1920 by the men who laid the foundations for the design of our modern electrical machines you will see that they were excited about their discoveries, they argued with each other about the rights and wrongs of particular new features, and they did *their* experiments mainly for fun. Let us repeat some of these in the hope that we may, in the process, observe something which escaped their notice and which gives us a lead into something useful. These are not over-exaggerated hopes. With apparatus no more complicated than iron filings, a magnet and a Meccano set, I discovered how to make a better hysteresis motor than had been possible before. The world of electrical machines is just as open to new ideas as it was 70 years ago. The early experimenters needed patience and accurate manufacture to make their experiments work, because the magnets which could be made at that time were not very effective. The magnets of today are so much better that it is easy to obtain a high success rate in repeating the experiments.

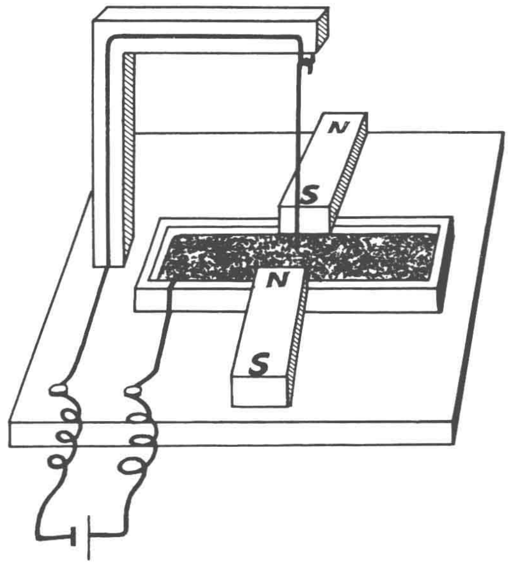

FIG. 2.1 *A very simple electromagnetic experiment —"the swinging wire".*

Elementary d.c. machines

The simple machine shown in fig. 2.1 consists of a swinging wire suspended from a metal hook and dipping into a mercury trough. When current flows in the wire and a pair of magnets (or a horseshoe magnet) are placed with opposite poles on either side of the wire, the latter will be propelled from the mercury by a series of "kicks", falling back each time as the current is broken by the tip of the wire leaving the mercury. Whilst the direction of the force may be predicted from the left hand rule, it is useful to see the apparatus as an interlinked magnetic and electric circuit and to ascribe to the magnetic circuit an equivalent current-carrying coil from which the direction of the force may be deduced in accordance with the method used in Chapter I (fig. 1.6).

Simple oscillators

From this simple apparatus it is only a few short steps to the mechanism of an electric bell, an electric tuning fork or an induction coil movement. Improvements on the swinging wire will soon suggest themselves. It is difficult to persuade the wire to re-enter the mercury pool, for copper tends to float on mercury and the wire is either lifted from the hook (this type of electrical contact is a poor one in any case) or the wire is pushed aside and makes poor contact with the impurities on the surface. A piece of spring steel strip (such as a piece of clock spring), permanently anchored and connected at the top end, would be much better, although its frequency of vibration will be higher than that of the loose wire. Mercury is not the ideal material to have in a piece of apparatus which may be required to be moved about, for it may spill, it requires the apparatus to be level, it collects dirt on its surface, and so on. The mercury contact can clearly be replaced by a solid make and break contact. What further change is now needed to make the household electric bell?

The last step is the most important one, for the apparatus shown in fig. 2.1 is an electromagnetic device in which *both* the electric and magnetic circuits are supplied with ampère-turns (the magnets can be regarded as equivalent to coils carrying an amount of current necessary to produce the same flux). We shall see in Chapter 6 that an electromagnetic device improves as it is made larger and we should find ourselves having to pass large amounts of current to deflect the spring. Furthermore, if the spring be slightly displaced from the centre of the gap between the magnets there will be a tendency for it to be attracted to the nearer pole due to purely magnetic effects. This last effect, although possibly a nuisance in the first experiment, could be the clue we are needing. Why not replace just *one* of the magnets by an electromagnet, remove the other magnet entirely and re-position the electromagnet so that it is best able to attract the spring, i.e. place it with its axis perpendicular to the flat surface of the spring as shown in fig. 2.2. If now the coil of the electromagnet is connected in series with the battery and make-and-break circuit, the oscillating system will be self-sustaining, for as the magnet attracts the steel the contact will be broken, causing the magnet to release the spring which will then re-establish contact, and so on. This fundamental idea of making the effect of something influence its own cause is the basic principle of all closed-loop automatic control or "servo" systems. The fact that the only two possible states for the circuit in this case are "open" and "closed" means that the system can never be at rest for either position excludes the possibility of the strip remaining in that position, so the system is bound to be oscillatory. In the language of control systems, such an arrangement is known as a "bang-bang servo".

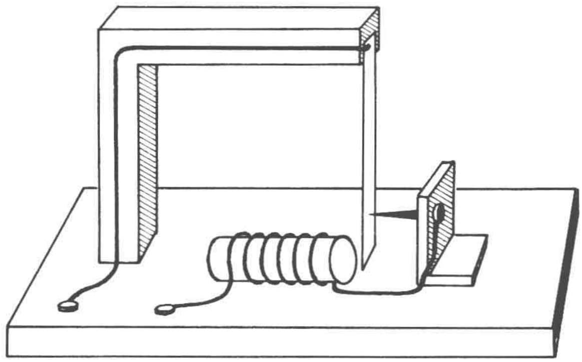

FIG. 2.2 *The self-sustaining oscillator—the mechanism of an electric bell.*

Notice also at this point that whilst we have still both an electric and a magnetic circuit, the latter is not fed by any number of ampère turns *of its own*. The mechanism is therefore a purely "magnetic" machine within the definition proposed in Chapter 6. Magnetic machines are best when built in small sizes, and within this definition an electric bell or buzzer is a "small" machine.

Continuous motion by switching

These first experiments have been concerned with oscillating machines. If we return to fig. 2.1 it is easy to see how continuous motion can be obtained with just a little modification. Suppose that the wire