海外优秀数学类教材系列丛书

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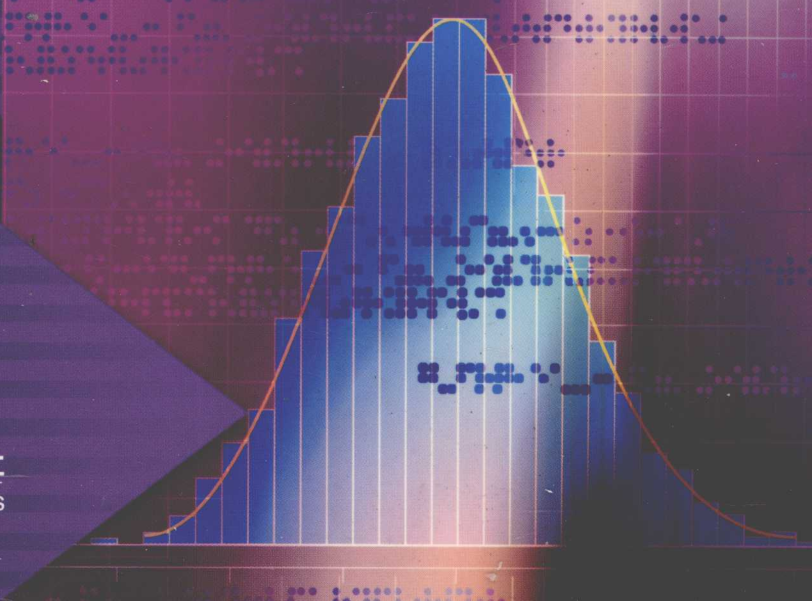
Introduction to Mathematical Statistics

(Fifth Edition)

数理统计学导论

(第 5 版)

□ ROBERT V. HOGG
□ ALLEN T. CRAIG



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出版者的话

在我国已经加入WTO、经济全球化的今天，为适应当前我国高校各类创新人才培养的需要，大力推进教育部倡导的双语教学，配合教育部实施的“高等学校教学质量与教学改革工程”和“精品课程”建设的需要，高等教育出版社有计划、大规模地开展了海外优秀数学类系列教材的引进工作。

高等教育出版社和Pearson Education，John Wiley & Sons，McGraw-Hill，Thomson Learning等国外出版公司进行了广泛接触，经国外出版公司的推荐并在国内专家的协助下，提交引进版权总数100余种。收到样书后，我们聘请了国内高校一线教师、专家、学者参与这些原版教材的评介工作，并参考国内相关专业的课程设置和教学实际情况，从中遴选出了这套优秀教材组织出版。

这批教材普遍具有以下特点：(1) 基本上是近3年出版的，在国际上被广泛使用，在同类教材中具有相当的权威性；(2) 高版次，历经多年教学实践检验，内容翔实准确、反映时代要求；(3) 各种教学资源配套整齐，为师生提供了极大的便利；(4) 插图精美、丰富，图文并茂，与正文相辅相成；(5) 语言简练、流畅、可读性强，比较适合非英语国家的学生阅读。

本系列丛书中，有Finney、Weir等编的《托马斯微积分》（第10版，Pearson），其特色可用“呈传统特色、富革新精神”概括，本书自20世纪50年代第1版以来，平均每四五年就有一个新版面世，长达50余年始终盛行于西方教坛，作者既有相当高的学术水平，又热爱教学，长期工作在教学第一线，其中，年近90的G.B.Thomas教授长年在MIT工作，具有丰富的教学经验；Finney教授也在MIT工作达10年；Weir是美国数学建模竞赛委员会主任。Stewart编的立体化教材《微积分》（第5版，Thomson Learning）配备了丰富的教学资源，是国际上最畅销的微积分原版教材，2003年全球销量约40余万册，在美国，占据了约50%～60%的微积分教材市场，其用户包括耶鲁等名牌院校及众多一般院校600余所。本系列丛书还包括Anton编的经典教材《线性代数及其应用》（第8版，Wiley）；Jay L. Devore编的优秀教材《概率论与数理统计》（第5版，Thomson Learning）等。在努力降低引进教材售价方面，高等教育出版社做了大量和细致的工作，这套引进的教材体现了一定的权威性、系统性、先进性和经济性等特点。

通过影印、翻译、编译这批优秀教材，我们一方面要不断地分析、学习、消化吸收国外优秀教材的长处，吸取国外出版公司的制作经验，提升我们自编

教材的立体化配套标准，使我国高校教材建设水平上一个新的台阶；与此同时，我们还将尝试组织海外作者和国内作者合编外文版基础课数学教材，并约请国内专家改编部分国外优秀教材，以适应我国实际教学环境。

这套教材出版后，我们将结合各高校的双语教学计划，开展大规模的宣传、培训工作，及时地将本套丛书推荐给高校使用。在使用过程中，我们衷心希望广大高校教师和同学提出宝贵的意见和建议，如有好的教材值得引进，请与高等教育出版社高等理科分社联系。

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2004 年 4 月 20 日

Preface

When Allen T. Craig died in late November 1978, I lost my advisor, mentor, colleague, and very dear friend. Due to his health, Allen did nothing on the fourth edition and, of course, this revision is mine alone. There is, however, a great deal of Craig's influence in this book. As a matter of fact, when I would debate with myself whether or not to change something, I could hear Allen saying, "It's very good now, Bob; don't mess it up." Often, I would follow that advice.

Nevertheless, there were a number of things that needed to be done. I have had many suggestions from my colleagues at the University of Iowa; in particular, Jim Broffitt, Jon Cryer, Dick Dykstra, Subhash Kochar (a visitor), Joe Lang, Russ Lenth, and Tim Robertson provided me with a great deal of constructive criticism. In addition, three reviewers suggested a number of other topics to include. I have also had statisticians and students from around the world write to me about possible improvements. Elliot Tanis, my good friend and co-author of our *Probability and Statistical Inference*, gave me permission to use a few of the figures, examples, and exercises used in that book. I truly thank these people, who have been so helpful and generous.

Clearly, I could not use all of these ideas. As a matter of fact, I resisted adding "real" problems, although a few slipped into the exercises. Allen and I wanted to write about the mathematics of statistics, and I have followed that guideline. Hopefully, without those problems, there is still enough motivation to study mathematical statistics in this book. In addition, there are a number of excellent books on applied statistics, and most students have had a little exposure to applications before studying this book.

The major differences between this edition and the preceding one are the following:

- There is a better discussion of assigning probabilities to events, including introducing independent events and Bayes' theorem in the text.
- The consideration of random variables and their expectations is greatly improved.
- Sufficient statistics are presented earlier (as was true in the very early editions of the book), and minimal sufficient statistics are introduced.
- Invariance of the maximum likelihood estimators and invariant location- and scale-statistics are considered.
- The expressions "convergence in distribution" and "convergence in probability" are used, and the delta method for finding asymptotic distributions is spelled out.
- Fisher information is given, and the Rao–Cramér lower bound is presented for an estimator of a function of a parameter, not just for an unbiased estimator.
- The asymptotic distribution of the maximum likelihood estimator is included.
- The discussion of Bayesian procedures has been improved and expanded somewhat.

There are also a number of little items that should improve the understanding of the text: the expressions var and cov are used; the convolution formula is in the text; there is more explanation of p-values; the relationship between two-sided tests and confidence intervals is noted; the indicator function is used when helpful; the multivariate normal distribution is given earlier (for those with an appropriate background in matrices, although this is still not necessary in the use of this book); and there is more on conditioning.

I believe that the order of presentation has been improved; in particular, sufficient statistics are presented earlier. More exercises have been introduced; and at the end of each chapter, there are several additional exercises that have not been ordered by section or by difficulty (several students had suggested this). Moreover, answers have not been given for any of these additional exercises because I thought some instructors might want to use them for questions on examinations. Finally, the index has been improved greatly, another suggestion of students as well as of some of my colleagues at Iowa.

There is really enough material in this book for a three-semester sequence. However, most instructors find that selections from the first five chapters provide a good one-semester background in the probability needed for the mathematical statistics based on selections from the remainder of the book, which certainly would include most of Chapters 6 and 7.

I am obligated to Catherine M. Thompson and Maxine Merrington and to Professor E. S. Pearson for permission to include Tables II and V, which are abridgments and adaptations of tables published in *Biometrika*. I wish to thank Oliver & Boyd Ltd., Edinburgh, for permission to include Table IV, which is an abridgment and adaptation of Table III from the book *Statistical Tables for Biological, Agricultural, and Medical Research* by the late Professor Sir Ronald A. Fisher, Cambridge, and Dr. Frank Yates, Rothamsted.

Finally, I would like to dedicate this edition to the memory of Allen Craig and my wife, Carolyn, who died June 25, 1990. Without the love and support of these two caring persons, I could not have done as much professionally as I have. My friends in Iowa City and my children (Mary, Barbara, Allen, and Robert) have given me the strength to continue. After four previous efforts, I really hope that I have come close to "getting it right this fifth time." I will let the readers be the judge.

R. V. H.

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Introduction to Statistical Inference

CHAPTER 1

Probability and Distributions

1.1 Introduction

Many kinds of investigations may be characterized in part by the fact that repeated experimentation, under essentially the same conditions, is more or less standard procedure. For instance, in medical research, interest may center on the effect of a drug that is to be administered; or an economist may be concerned with the prices of three specified commodities at various time intervals; or the agronomist may wish to study the effect that a chemical fertilizer has on the yield of a cereal grain. The only way in which an investigator can elicit information about any such phenomenon is to perform his experiment. Each experiment terminates with an *outcome*. But it is characteristic of these experiments that the outcome cannot be predicted with certainty prior to the performance of the experiment.

Suppose that we have such an experiment, the outcome of which cannot be predicted with certainty, but the experiment is of such a nature that a collection of every possible outcome can be described prior to its performance. If this kind of experiment can be repeated under the same conditions, it is called a *random experiment*, and the collection of every possible outcome is called the experimental space or the *sample space*.

Example 1. In the toss of a coin, let the outcome tails be denoted by T and let the outcome heads be denoted by H. If we assume that the coin may be repeatedly tossed under the same conditions, then the toss of this coin is an example of a random experiment in which the outcome is one of the two symbols T and H; that is, the sample space is the collection of these two symbols.

Example 2. In the cast of one red die and one white die, let the outcome be the ordered pair (number of spots up on the red die, number of spots up on the white die). If we assume that these two dice may be repeatedly cast under the same conditions, then the cast of this pair of dice is a random experiment and the sample space consists of the following 36 ordered pairs: $(1, 1), \ldots, (1, 6), (2, 1), \ldots, (2, 6), \ldots, (6, 6)$.

Let $\mathscr{C}$ denote a sample space, and let C represent a part of $\mathscr{C}$. If, upon the performance of the experiment, the outcome is in C, we shall say that the *event* C has occurred. Now conceive of our having made N repeated performances of the random experiment. Then we can count the number f of times (the frequency) that the event C actually occurred throughout the N performances. The ratio f/N is called the *relative frequency* of the event C in these N experiments. A relative frequency is usually quite erratic for small values of N, as you can discover by tossing a coin. But as N increases, experience indicates that we associate with the event C a number, say p, that is equal or approximately equal to that number about which the relative frequency seems to stabilize. If we do this, then the number p can be interpreted as that number which, in future performances of the experiment, the relative frequency of the event C will either equal or approximate. Thus, although we *cannot* predict the outcome of a random experiment, we *can*, for a large value of N, predict approximately the relative frequency with which the outcome will be in C. The number p associated with the event C is given various names. Sometimes it is called the *probability* that the outcome of the random experiment is in C; sometimes it is called the *probability* of the event C; and sometimes it is called the *probability measure* of C. The context usually suggests an appropriate choice of terminology.

Example 3. Let $\mathscr{C}$ denote the sample space of Example 2 and let C be the collection of every ordered pair of $\mathscr{C}$ for which the sum of the pair is equal to seven. Thus C is the collection (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), and (6, 1). Suppose that the dice are cast $N = 400$ times and let f, the frequency of a sum of seven, be $f = 60$. Then the relative frequency with which the outcome was in C is $f/N = \frac{60}{400} = 0.15$. Thus we might associate with C a number p that is close to 0.15, and p would be called the probability of the event C.

Remark. The preceding interpretation of probability is sometimes referred to as the *relative frequency approach*, and it obviously depends upon the fact that an experiment can be repeated under essentially identical conditions. However, many persons extend probability to other situations by treating it as a rational measure of belief. For example, the statement $p = \frac{2}{5}$ would mean to them that their *personal* or *subjective* probability of the event C is equal to $\frac{2}{5}$. Hence, if they are not opposed to gambling, this could be interpreted as a willingness on their part to bet on the outcome of C so that the two possible payoffs are in the ratio $p/(1 - p) = \frac{2}{5}/\frac{3}{5} = \frac{2}{3}$. Moreover, if they truly believe that $p = \frac{2}{5}$ is correct, they would be willing to accept either side of the bet: (a) win 3 units if C occurs and lose 2 if it does not occur, or (b) win 2 units if C does not occur and lose 3 if it does. However, since the mathematical properties of probability given in Section 1.3 are consistent with either of these interpretations, the subsequent mathematical development does not depend upon which approach is used.

The primary purpose of having a mathematical theory of statistics is to provide mathematical models for random experiments. Once a model for such an experiment has been provided and the theory worked out in detail, the statistician may, within this framework, make inferences (that is, draw conclusions) about the random experiment. The construction of such a model requires a theory of probability. One of the more logically satisfying theories of probability is that based on the concepts of sets and functions of sets. These concepts are introduced in Section 1.2.

1.2 Set Theory

The concept of a *set* or a *collection* of objects is usually left undefined. However, a particular set can be described so that there is no misunderstanding as to what collection of objects is under consideration. For example, the set of the first 10 positive integers is sufficiently well described to make clear that the numbers $\frac{3}{4}$ and 14 are not in the set, while the number 3 is in the set. If an object belongs to a set, it is said to be an *element* of the set. For example, if A denotes the set of real numbers x for which $0 \le x \le 1$, then $\frac{3}{4}$ is an element of the set A. The fact that $\frac{3}{4}$ is an element of the set A is indicated by writing $\frac{3}{4} \in A$. More generally, $a \in A$ means that a is an element of the set A.

The sets that concern us will frequently be *sets of numbers*. However, the language of sets of *points* proves somewhat more convenient than that of sets of numbers. Accordingly, we briefly indicate how we use this terminology. In analytic geometry considerable emphasis is placed on the fact that to each point on a line (on which an origin and a unit point have been selected) there corresponds one and only one number, say x; and that to each number x there corresponds one and only one point on the line. This one-to-one correspondence between the numbers and points on a line enables us to speak, without misunderstanding, of the "point x" instead of the "number x." Furthermore, with a plane rectangular coordinate system and with x and y numbers, to each symbol (x, y) there corresponds one and only one point in the plane; and to each point in the plane there corresponds but one such symbol. Here again, we may speak of the "point (x, y)," meaning the "ordered number pair x and y." This convenient language can be used when we have a rectangular coordinate system in a space of three or more dimensions. Thus the "point $(x_1, x_2, \ldots, x_n)$" means the numbers $x_1, x_2, \ldots, x_n$ in the order stated. Accordingly, in describing our sets, we frequently speak of a set of points (a set whose elements are points), being careful, of course, to describe the set so as to avoid any ambiguity. The notation $A = \{x : 0 \le x \le 1\}$ is read "A is the one-dimensional set of points x for which $0 \le x \le 1$." Similarly, $A = \{(x, y) : 0 \le x \le 1, 0 \le y \le 1\}$ can be read "A is the two-dimensional set of points (x, y) that are interior to, or on the boundary of, a square with opposite vertices at $(0, 0)$ and $(1, 1)$." We now give some definitions (together with illustrative examples) that lead to an elementary algebra of sets adequate for our purposes.

Definition 1. If each element of a set A_1 is also an element of set A_2, the set A_1 is called a *subset* of the set A_2. This is indicated by writing $A_1 \subset A_2$. If $A_1 \subset A_2$ and also $A_2 \subset A_1$, the two sets have the same elements, and this is indicated by writing $A_1 = A_2$.

Example 1. Let $A_1 = \{x : 0 \le x \le 1\}$ and $A_2 = \{x : -1 \le x \le 2\}$. Here the one-dimensional set A_1 is seen to be a subset of the one-dimensional set A_2; that is, $A_1 \subset A_2$. Subsequently, when the dimensionality of the set is clear, we shall not make specific reference to it.

Example 2. Let $A_1 = \{(x, y) : 0 \le x = y \le 1\}$ and $A_2 = \{(x, y) : 0 \le x \le 1, 0 \le y \le 1\}$. Since the elements of A_1 are the points on one diagonal of the square, then $A_1 \subset A_2$.

Definition 2. If a set A has no elements, A is called the *null set*. This is indicated by writing $A = \varnothing$.

Definition 3. The set of all elements that belong to at least one of the sets A_1 and A_2 is called the *union* of A_1 and A_2. The union of A_1 and A_2 is indicated by writing $A_1 \cup A_2$. The union of several sets $A_1, A_2, A_3, \ldots$ is the set of all elements that belong to at least one of the several sets. This union is denoted by $A_1 \cup A_2 \cup A_3 \cup \cdots$ or by $A_1 \cup A_2 \cup \cdots \cup A_k$ if a finite number k of sets is involved.

Example 3. Let $A_1 = \{x : x = 0, 1, \ldots, 10\}$ and $A_2 = \{x : x = 8, 9, 10, 11,$ or $11 < x \le 12\}$. Then $A_1 \cup A_2 = \{x : x = 0, 1, \ldots, 8, 9, 10, 11,$ or $11 < x \le 12\} = \{x : x = 0, 1, \ldots, 8, 9, 10,$ or $11 \le x \le 12\}$.

Example 4. Let A_1 and A_2 be defined as in Example 1. Then $A_1 \cup A_2 = A_2$.

Example 5. Let $A_2 = \varnothing$. Then $A_1 \cup A_2 = A_1$ for every set A_1.

Example 6. For every set A, $A \cup A = A$.

Example 7. Let

$$A_k = \left\{x : \frac{1}{k+1} \le x \le 1\right\}, \qquad k = 1, 2, 3, \ldots.$$

Then $A_1 \cup A_2 \cup A_3 \cup \cdots = \{x : 0 < x \le 1\}$. Note that the number zero is not in this set, since it is not in one of the sets $A_1, A_2, A_3, \ldots$.

Definition 4. The set of all elements that belong to each of the sets A_1 and A_2 is called the *intersection* of A_1 and A_2. The intersection of A_1 and A_2 is indicated by writing $A_1 \cap A_2$. The intersection of several sets $A_1, A_2, A_3, \ldots$ is the set of all elements that belong to each of the sets $A_1, A_2, A_3, \ldots$. This intersection is denoted by $A_1 \cap A_2 \cap A_3 \cap \cdots$ or by $A_1 \cap A_2 \cap \cdots \cap A_k$ if a finite number k of sets is involved.

Example 8. Let $A_1 = \{(0, 0), (0, 1), (1, 1)\}$ and $A_2 = \{(1, 1), (1, 2), (2, 1)\}$. Then $A_1 \cap A_2 = \{(1, 1)\}$.

Example 9. Let $A_1 = \{(x, y) : 0 \le x + y \le 1\}$ and $A_2 = \{(x, y) : 1 < x + y\}$. Then A_1 and A_2 have no points in common and $A_1 \cap A_2 = \varnothing$.

Example 10. For every set A, $A \cap A = A$ and $A \cap \varnothing = \varnothing$.

Example 11. Let

$$A_k = \left\{x : 0 < x < \frac{1}{k}\right\}, \qquad k = 1, 2, 3, \ldots.$$