ROY D. YATES | DAVID J. GOODMAN

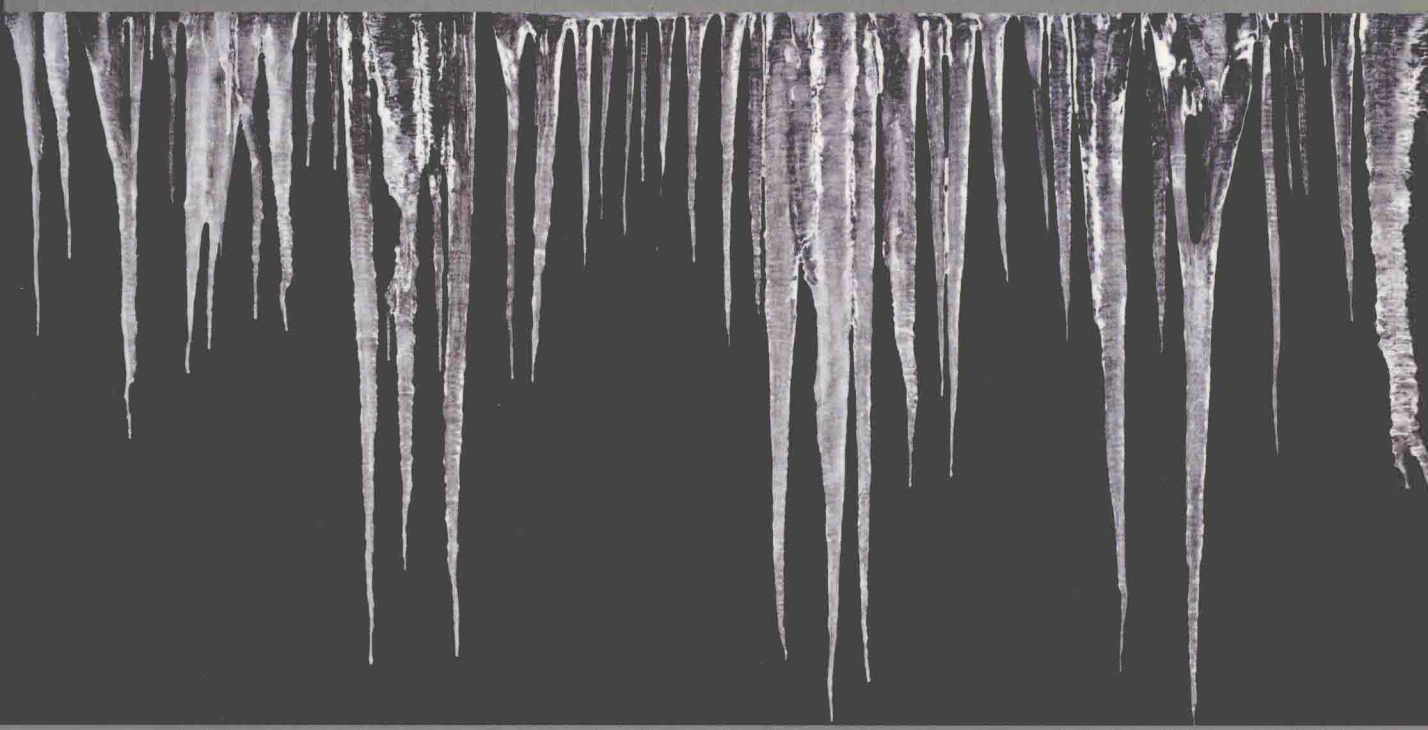
Probability and Stochastic Processes

A Friendly Introduction for Electrical and Computer Engineers

Third Edition

EXCLUSIVE TO THIS VERSION

- Concise Coverage of Topics
- Revised Problem Sets



INTERNATIONAL STUDENT VERSION

WILEY

Probability and Stochastic Processes

A Friendly Introduction for Electrical and Computer Engineers

Third Edition

International Student Version

Roy D. Yates
Rutgers, The State University of New Jersey

David J. Goodman
New York University

WILEY

Cover image © Furtseff/Shutterstock

ISBN: 978-1-118-80871-9

Printed in Asia

10 9 8 7 6 5 4 3 2 1

Features of this Text

Who will benefit from using this text?

This text can be used in Sophomore, Junior or Senior level courses in probability and stochastic processes. The mathematical exposition will appeal to students and practitioners in many areas. The examples, quizzes, and problems are typical of those encountered by practicing electrical and computer engineers. Professionals in the telecommunications and wireless industry will find it particularly useful.

What's New?

This text has been expanded with new introductory material:

- Over 160 new homework problems
- New chapters on *Sequential Trials*, *Derived Random Variables* and *Conditional Probability Models.*
- MATLAB examples and problems give students hands-on access to theory and applications. Every chapter includes guidance on how to use MATLAB to perform calculations and simulations relevant to the subject of the chapter.
- Advanced material online in *Signal Processing* and *Markov Chains* supplements.

Notable Features

The Friendly Approach

The friendly and accessible writing style gives students an intuitive feeling for the formal mathematics.

Quizzes and Homework Problems

An extensive collection of in-chapter quizzes provides checkpoints for readers to gauge their understanding. Hundreds of end-of-chapter problems are clearly marked as to their degree of difficulty from beginner to expert.

Student Companion Website `www.wiley.com/college/yates`

Available for download: All MATLAB m-files in the text, the *Quiz Solutions Manual*, a *Student Solutions Manual*, the *Signal Processing Supplement*, and the *Markov Chains Supplement.*

Instructor Support

Instructors can register for the Instructor Companion Site at `www.wiley.com/college/yates`

To Alissa, Brett, Daniel, Hannah, Leila, Milo, Theresa, Tony, and Zach

Preface

Welcome to the third edition

You are reading the international student version (ISV) of the third edition of our textbook. Although the fundamentals of probability and stochastic processes have not changed since we wrote the first edition, the world inside and outside universities is different now than it was in 1998. Outside of academia, applications of probability theory have expanded enormously in the past 16 years. Think of the 20 billion+ Web searches each month and the billions of daily computerized stock exchange transactions, each based on probability models, many of them devised by electrical and computer engineers.

Universities and secondary schools, recognizing the fundamental importance of probability theory to a wide range of subject areas, are offering courses in the subject to younger students than the ones who studied probability 16 years ago. At Rutgers, probability is now a required course for Electrical and Computer Engineering sophomores.

We have responded in several ways to these changes and to the suggestions of students and instructors who used the earlier editions. The first and second editions contain material found in postgraduate as well as advanced undergraduate courses. By contrast, the printed and e-book versions of this third edition focus on the needs of undergraduates studying probability for the first time.

The more advanced material in the earlier editions, covering random signal processing and Markov chains, is available at the companion website (`www.wiley.com/college/yates`). To promote intuition into the practical applications of the mathematics, we have expanded the number of examples and quizzes and homework problems to about 600, an increase of about 35 percent compared to the second edition. Many of the examples are mathematical exercises. Others are questions that are simple versions of the ones encountered by professionals working on practical applications.

In response to suggestions from international readers of the earlier editions, this third edition ISV is shorter than the United States third edition by about 35 pages. It contains less introductory material and fewer elementary examples. We have also omitted a few topics that are not central to the interests of electrical and computer and engineers. These topics are reliability analysis, confidence interval estimation, and random sums of random variables.

How the book is organized

Motivated by our teaching experience, we have rearranged the sequence in which we present the elementary material on probability models, counting methods, conditional probability models, and derived random variables. In this edition, the first

chapter covers fundamentals, including axioms and probability of events, and the second chapter covers counting methods and sequential experiments. As before, we introduce discrete random variables and continuous random variables in separate chapters. The subject of Chapter 5 is multiple discrete and continuous random variables. The first and second editions present derived random variables and conditional random variables in the introductions to discrete and continuous random variables. In this third edition, derived random variables and conditional random variables appear in their own chapters, which cover both discrete and continuous random variables.

Chapter 8 introduces random vectors. It extends the material on multiple random variables in Chapter 5 and relies on principles of linear algebra to derive properties of random vectors that are useful in real-world data analysis and simulations. Chapter 12 on estimation relies on the properties of random vectors derived in Chapter 8. Chapters 9 through 12 cover subjects relevant to data analysis including Gaussian approximations based on the central limit theorem, estimates of model parameters, hypothesis testing, and estimation of random variables. Chapter 13 introduces stochastic processes in the context of the probability model that guides the entire book: an experiment consisting of a procedure and observations.

Each of the 92 sections of the 13 chapters ends with a quiz. By working on the quiz and checking the solution at the book's website, students will get quick feedback on how well they have grasped the material in each section.

We think that 60- 80% (7 to 10 chapters) of the book would fit into a one semester undergraduate course for beginning students in probability. We anticipate that all courses will cover the first five chapters, and that instructors will select the remaining course content based on the needs of their students. The "roadmap" on page ix displays the thirteen chapter titles and suggests a few possible undergraduate syllabi.

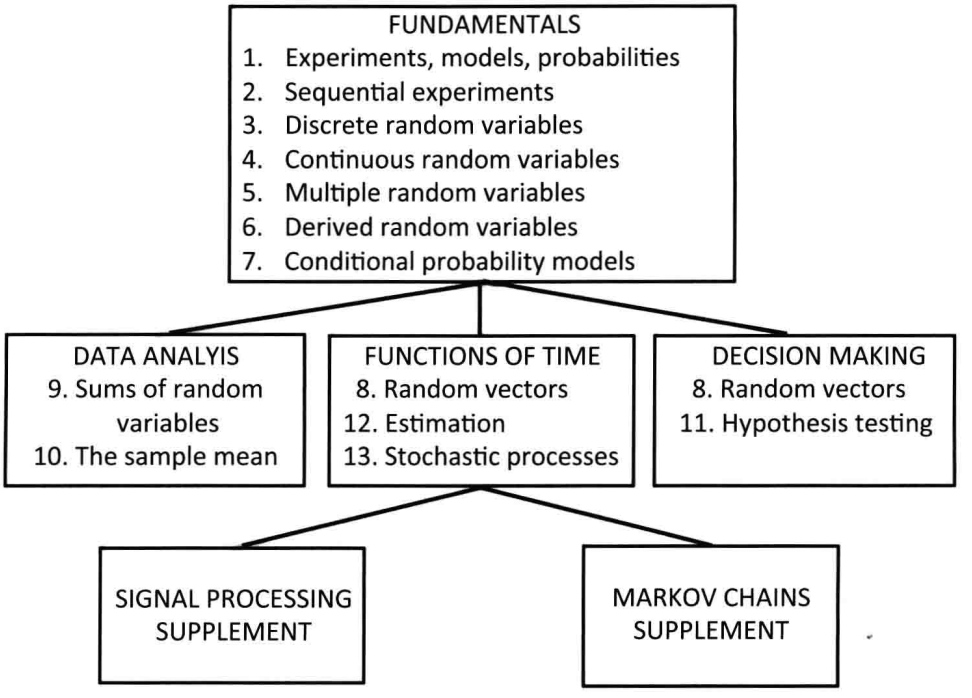
The Signal Processing Supplement (SPS) and Markov Chains Supplement (MCS) are the final chapters of the third edition. They are now available at the book's website. They contain postgraduate-level material. We, and colleagues at other universities, have used these two chapters in graduate courses that move very quickly through the early chapters to review material already familiar to students and to fill in gaps in learning of diverse postgraduate populations.

What is distinctive about this book?

- The entire text adheres to a single model that begins with an experiment consisting of a procedure and observations.

- The mathematical logic is apparent to readers. Every fact is identified clearly as a definition, an axiom, or a theorem. There is an explanation, in simple English, of the intuition behind every concept when it first appears in the text.

- The mathematics of discrete random variables is introduced separately from the mathematics of continuous random variables.



A road map for the text.

- Stochastic processes and statistical inference fit comfortably within the unifying model of the text.
- An abundance of exercises puts the theory to use. New ideas are augmented with detailed solutions of numerical examples.
- Each section begins with a brief statement of the important concepts introduced in the section and concludes with a simple quiz to help students gauge their grasp of the new material.
- Each problem at the end of a chapter is labeled with a reference to a section in the chapter and a degree of difficulty ranging from "easy" to "experts only." For example Problem 3.4.5 requires material from Section 3.4 but not from later sections. Each problem also has a label that reflects our estimate of degree of difficulty. Skiers will recognize the following symbols:

  Easy Moderate Difficult Experts Only

  Every ski area emphasizes that these designations are relative to the trails at that area. Similarly, the difficulty of our problems is relative to the other problems in this text.
- There is considerable support on the World Wide Web for students and instructors, including MATLAB programs and solutions to the quizzes and problems.

Further Reading

Libraries and bookstores contain an endless collection of textbooks at all levels covering the topics presented in this textbook. We know of two in comic book format [GS93, Pos01]. The reference list on page 453 is a brief sampling of books that can add breadth or depth to the material in this text. Most books on probability, statistics, stochastic processes, and random signal processing contain expositions of the basic principles of probability and random variables, covered in Chapters 1–5. In advanced texts, these expositions serve mainly to establish notation for more specialized topics. [LG11] and [Gub06] share our focus on electrical and computer engineering applications. [BT08], [Ros12] and [Dra67] and introduce the fundamentals of probability and random variables to a general audience of students with a calculus background. [KMT12] is a comprehensive graduate level textbook with a thorough presentation of fundamentals of probability, stochastic processes, and data analysis. It uses the basic theory to develop techniques including hidden Markov models, queuing theory, and machine learning used in many practical applications. [Bil12] is more advanced mathematically; it presents probability as a branch of measure theory. [MR10] and [SMM10] introduce probability theory in the context of data analysis. [Dav10] and [HL11] are beginners' introductions to MATLAB. [Ber98] is in a class by itself. It presents the concepts of probability from a historical perspective, focusing on the lives and contributions of mathematicians and others who stimulated major advances in probability and statistics and their application in various fields including psychology, economics, government policy, and risk management.

Acknowledgments

We are grateful for assistance and suggestions from many sources including our students at Rutgers and New York Universities, instructors who adopted the previous editions, reviewers, and the Wiley team.

At Wiley, we are pleased to acknowledge the encouragement and enthusiasm of our executive editor Daniel Sayre and the support of sponsoring editor Mary O'Sullivan, project editor Ellen Keohane, production editor Eugenia Lee, and cover designer Samantha Low.

We also convey special thanks to Ivan Seskar of WINLAB at Rutgers University for exercising his magic to make the WINLAB computers particularly hospitable to the electronic versions of the book and to the supporting material on the World Wide Web.

The organization and content of the second edition has benefited considerably from the input of many faculty colleagues including Alhussein Abouzeid at Rensselaer Polytechnic Institute, Krishna Arora at Florida State University, Frank Candocia at Florida International University, Robin Carr at Drexel University, Keith Chugg at USC, Charles Doering at University of Michigan, Roger Green at North Dakota State University, Witold Krzymien at University of Alberta, Edl Schamiloglu at University of New Mexico, Arthur David Snider at University of South Florida, Junshan Zhang at Arizona State University, and colleagues

Narayan Mandayam, Leo Razumov, Christopher Rose, Predrag Spasojević, and Wade Trappe at Rutgers.

Unique among our teaching assistants, Dave Famolari took the course as an undergraduate. Later as a teaching assistant, he did an excellent job writing homework solutions with a tutorial flavor. Other graduate students who provided valuable feedback and suggestions on the first edition include Ricki Abboudi, Zheng Cai, Pi-Chun Chen, Sorabh Gupta, Vahe Hagopian, Amar Mahboob, Ivana Maric, David Pandian, Mohammad Saquib, Sennur Ulukus, and Aylin Yener.

The first edition also benefited from reviews and suggestions conveyed to the publisher by D.L. Clark at California State Polytechnic University at Pomona, Mark Clements at Georgia Tech, Gustavo de Veciana at the University of Texas at Austin, Fred Fontaine at Cooper Union, Rob Frohne at Walla Walla College, Chris Genovese at Carnegie Mellon, Simon Haykin at McMaster, and Ratnesh Kumar at the University of Kentucky.

Finally, we acknowledge with respect and gratitude the inspiration and guidance of our teachers and mentors who conveyed to us when we were students the importance and elegance of probability theory. We cite in particular Robert Gallager and the late Alvin Drake of MIT and the late Colin Cherry of Imperial College of Science and Technology.

A Message to Students from the Authors

A lot of students find it hard to do well in this course. We think there are a few reasons for this difficulty. One reason is that some people find the concepts hard to use and understand. Many of them are successful in other courses but find the ideas of probability difficult to grasp. Usually these students recognize that learning probability theory is a struggle, and most of them work hard enough to do well. However, they find themselves putting in more effort than in other courses to achieve similar results.

Other people have the opposite problem. The work looks easy to them, and they understand everything they hear in class and read in the book. There are good reasons for assuming this is easy material. There are very few basic concepts to absorb. The terminology (like the word *probability*), in most cases, contains familiar words. With a few exceptions, the mathematical manipulations are not complex. You can go a long way solving problems with a four-function calculator.

For many people, this apparent simplicity is dangerously misleading because it is very tricky to apply the math to specific problems. A few of you will see things clearly enough to do everything right the first time. However, most people who do well in probability need to practice with a lot of examples to get comfortable with the work and to really understand what the subject is about. Students in this course end up like elementary school children who do well with multiplication tables and long division but bomb out on word problems. The hard part is figuring out what to do with the numbers, not actually doing it. Most of the work in this course is that way, and the only way to do well is to practice a lot. Taking the midterm and final are similar to running in a five-mile race. Most people can do it in a respectable time, provided they train for it. Some people look at the runners

who do it and say, "I'm as strong as they are. I'll just go out there and join in." Without the training, most of them are exhausted and walking after a mile or two.

So, our advice to students is, if this looks really weird to you, keep working at it. You will probably catch on. If it looks really simple, don't get too complacent. It may be harder than you think. Get into the habit of doing the quizzes and problems, and if you don't answer all the quiz questions correctly, go over them until you understand each one.

We can't resist commenting on the role of probability and stochastic processes in our careers. The theoretical material covered in this book has helped both of us devise new communication techniques and improve the operation of practical systems. We hope you find the subject intrinsically interesting. If you master the basic ideas, you will have many opportunities to apply them in other courses and throughout your career.

We have worked hard to produce a text that will be useful to a large population of students and instructors. We welcome comments, criticism, and suggestions. Feel free to send us e-mail at *ryates@winlab.rutgers.edu* or *dgoodman@poly.edu*. In addition, the website `www.wiley.com/college/yates` provides a variety of supplemental materials, including the MATLAB code used to produce the examples in the text.

Roy D. Yates
Rutgers, The State University of New Jersey

David J. Goodman
New York University

February 19, 2014

Contents