Russell Monson
Dennis Baldocchi

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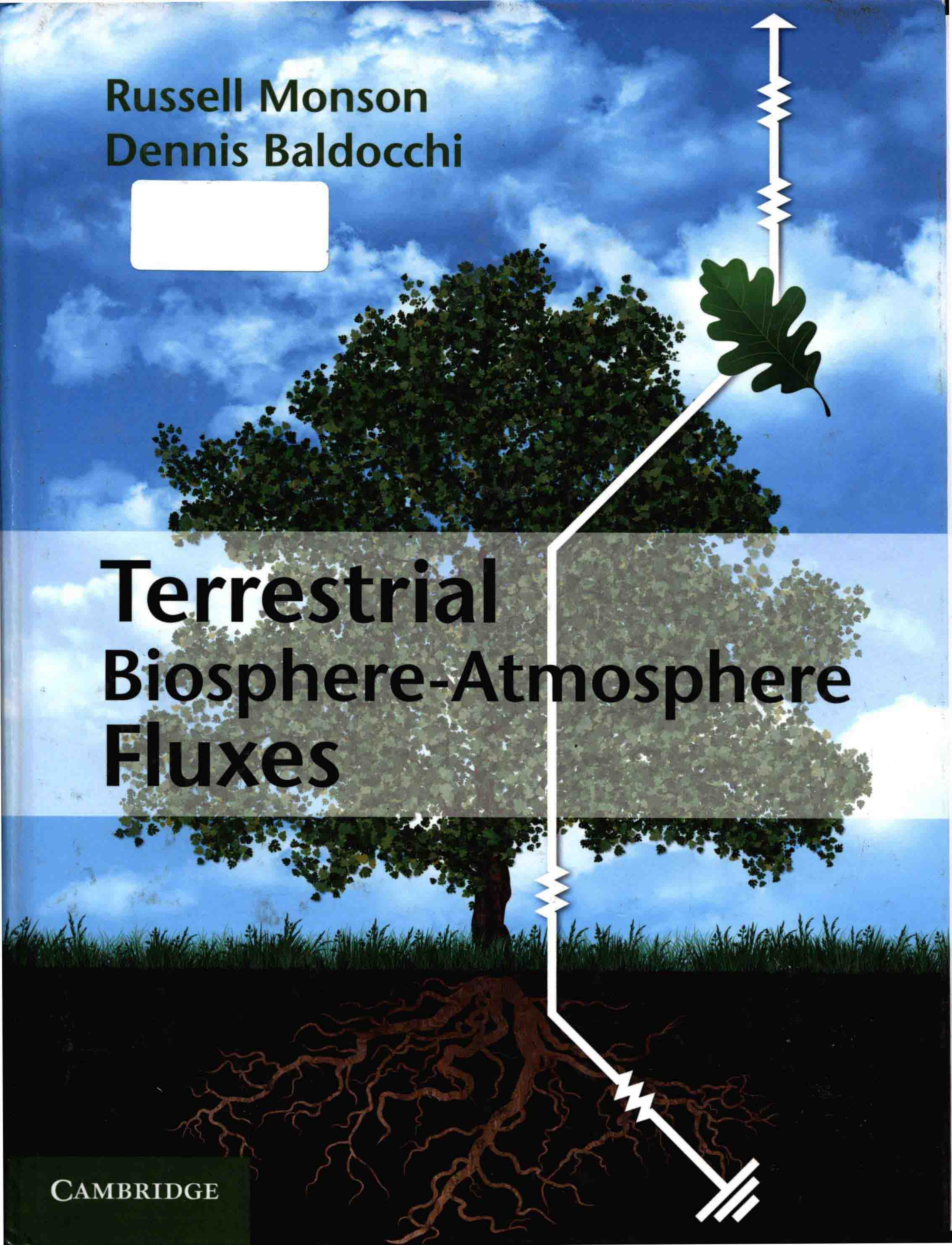

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Terrestrial Biosphere-Atmosphere Fluxes

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Terrestrial Biosphere-Atmosphere Fluxes

Fluxes of trace gases, water, and energy between the terrestrial biosphere and the atmosphere govern the state and fate of these two coupled systems. This "breathing of the biosphere" is controlled by a large number of interacting physical, chemical, biological, and ecological processes. In this integrated and interdisciplinary book, the authors provide the tools to understand and quantitatively analyze fluxes of energy, complex organic compounds such as terpenes, and trace gases including carbon dioxide, water vapor, and methane.

The book first introduces the fundamental principles that affect the supply and demand for energy and trace gas exchange at the leaf and soil scales: thermodynamics, diffusion, turbulence, and physiology. It then builds on these principles to model the exchange of energy, water, carbon dioxide, terpenes, and stable isotopes at the ecosystem scale. Detailed mathematical derivations of commonly used relations in biosphere-atmosphere interactions are provided for reference in appendices.

An accessible introduction for graduate students to this essential component of Earth system science, this book is also a key resource for researchers in many related fields such as atmospheric science, hydrology, meteorology, climate science, biogeochemistry, and ecosystem ecology.

Online resources at www.cambridge.org/monson:

- A short online mathematical supplement guides students through basic mathematical principles, from calculus rules of derivation and integration, to statistical moments and coordinate rotation.

Russell Monson is Louise Foucar Marshall Professor at the University of Arizona, Tucson and Professor Emeritus at the University of Colorado, Boulder. His research focuses on photosynthetic metabolism, the production of biogenic volatile organic compounds and plant water relations from the scale of chloroplasts to the globe. He has received numerous awards, including the Alexander von Humboldt Fellowship, the John Simon Guggenheim Fellowship, and the Fulbright Senior Fellowship, and was also appointed Professor of Distinction in the Department of Ecology and Evolutionary Biology at the University of Colorado. Professor Monson is a Fellow of the American Geophysical Union and has served on advisory boards for numerous national and international organizations and projects. He is Editor-in-Chief of the journal *Oecologia* and has over 200 peer-reviewed publications.

Dennis Baldocchi is Professor of Biometeorology at the University of California, Berkeley. His research focuses on physical, biological, and chemical processes that control trace gas and energy exchange between vegetation and the atmosphere and the micrometeorology of plant canopies. Awards received include the Award for Outstanding Achievement in Biometeorology from the American Meteorological Society (2009), and the Faculty Award for Excellence in Postdoctoral Mentoring (2011). Professor Baldocchi is a Fellow of the American Geophysical Union and is a member of advisory boards for national and international organizations and projects. He is Editor-in-Chief of the *Journal of Geophysical Research: Biogeosciences* and has over 200 peer-reviewed publications.

Preface

This book is about *interactions* – those that occur between the terrestrial biosphere and the atmosphere. Understanding biosphere-atmosphere interactions is a core activity within the discipline of *earth system sciences*. Many of the most pressing environmental challenges that face society (e.g., the anthropogenic forcing of climate change, urban pollution, the production of sustainable energy sources, and stratospheric ozone depletion), and their remedies, can be traced to biosphere-atmosphere interactions within the earth system. Traditionally, biosphere-atmosphere interactions have been studied within a broad range of conventional disciplines, including biology, the atmospheric and geological sciences, and engineering. In this book we take an integrated, interdisciplinary perspective; one that weaves together concepts and theory from all of the traditional disciplines, and organizes them into a framework that we hope will catalyze a new, synergistic approach to teaching university courses in the earth system sciences.

As we wrote the initial outline for the book, we recognized that the interdisciplinary perspective we sought, in a subtle way, had already emerged; it simply had not been formally collated into a synthetic format. For the past several years, biologists have been attending meetings and workshops traditionally associated with meteorology and geochemistry and conversely meteorologists and geochemists have been attending biology meetings. As a result, newly defined and integrative disciplines have already appeared with names such as "biometeorology," "bioclimatology," and "ecohydrology." Thus, the foundations for the book had already been laid. We simply needed to find the common elements and concepts that permeated these emerging disciplines and pull them together into a single treatment.

We have written the book as two colleagues who have migrated from different ends of the biology-meteorology spectrum – one (Monson) from formal training in biology and one (Baldocchi) from formal training in meteorology – but who also have struggled throughout their careers to grasp concepts at these disciplinary interfaces. In many ways this book is autobiographical; it reflects the challenges that both of us faced as we developed collaborations across these disciplines. We actually met for the first time at a conference in Asilomar, California in 1990, which was dedicated to bridging the gaps among biologists, meteorologists, and atmospheric chemists. Thus, the interdisciplinary foundation for the book has deep roots that were initiated over two decades ago. From that initial friendship we developed a collaboration in which we began to compile and combine materials that we extracted from our respective course lectures.

This book is intended to be used as both a textbook and reference book. As a textbook it is intended to support courses for advanced undergraduate students or beginning graduate students. As a reference book it is intended to provide detailed mathematical derivations of some of the most commonly used relations in biosphere-atmosphere interactions. In order to address both aims, we have written the primary text of the chapters to provide what we consider to be *the rudiments*; those concepts essential to an introductory understanding of process interactions and fundamental theory. Detailed mathematical derivations are presented as "appendices" at the end of many chapters. These derivations are intended mostly as reference material; however, in our own experiences we discovered that formal derivations, such as these, also served as an important resource to students. In fact a well-received feature of some of our classes was the "Derivation Derby" held as an evening session in which students were required to use the chalk board to present, in their own words, the foundations of some of the more classic biophysical relations; of course with good food and drink as accompaniment. We have used a second tool to develop advanced topics of more conceptual, rather than quantitative, nature – the "boxes" that are embedded in many chapters. In the boxes we have tried to bring out current topics and issues that appear to have captured the attention of the field at the moment, or we have described studies that have used the concepts under discussion in unique ways. Once again, the boxes will be most effectively used to provide supplementary material that embellishes the rudimentary topics presented in the main text of the chapters. We have tried to use a modest frequency of citations in most chapters. Much of the material we cover is of an elementary nature, and in order to sustain continuity in those discussions we have not interrupted the text with frequent citations. In those cases where we thought that a citation might be useful for further explorations of a topic, especially where a review article or an article of historical significance might be useful, we have provided citations. In the sections that cover contemporary concepts, especially those still being defined through active debate in the literature, we have provided a more complete record of citations. Furthermore, many of the figures were adopted from past studies, and we have provided citations in the figure legends, which will be useful in directing students to primary sources in the literature.

One of the initial decisions we made as we organized material for the book involved the strategy for topical organization. We considered two possible frameworks: chapters that focused on single environmental factors (e.g., a chapter on water, a chapter on light, a chapter on temperature, and so on), or chapters that build in spatiotemporal scale, from processes at smaller scales to those at larger scales (e.g., a chapter on cells and metabolism, a chapter on leaves and diffusion, a chapter on canopies and turbulent transport, and so on). Conventional treatments, especially in texts that deal with environmental physics, have followed the former model, and they have done so with good success. However, we recognized that many of the observations and much of the theory that has emerged in recent years has been framed around hierarchical scaling, and we wanted to develop a treatment that could be used within this framework. After much discussion and deliberation, we decided to follow the second model, though with a bit of introgression from the first model. Thus, the chapters build in scale, beginning with chloroplasts, progressing to leaves and canopies, and culminating with the planetary boundary layer. Each of these scaled chapters is preceded with one or more chapters on the nature of relevant environmental factors as drivers of processes. Thus, the chapter on leaf scale transport is preceded with a chapter on diffusion, and the chapter on turbulent transport is preceded with a chapter on stability in the planetary boundary layer. Exceptions to these patterns are the initial three chapters, which deal with broad topics in thermodynamics and chemical rate theory, and the final three chapters, which deal respectively with soil carbon and nitrogen fluxes, fluxes of volatile reactive compounds and atmospheric chemistry, and fluxes related to stable isotope fractionation. These chapters are intended to provide a framework for understanding the relations among fluxes, sources/sinks, and gradients, in the case of the earliest chapters, and to elaborate on some important recent directions in earth system sciences research, in the case of the latest chapters.

The overall emphasis of the book is on understanding processes that control fluxes. Less emphasis is placed on descriptions of biogeochemical pools and reservoirs. We also pay less attention to instrumentation and experimental protocols. Most of the chapters focus on CO_2, H_2O, and energy fluxes, although we also take up the topic of other trace gases in briefer format. Finally, we note that our book focuses exclusively on terrestrial ecosystems. Our decision not to wade into the oceans was determined by recognition of our strengths and weaknesses as scientists and authors, and this decision does not reflect a bias against the importance of ocean processes to earth system dynamics.

We appreciate the many discussions we have had with generous colleagues as we wrote the book and sought critical feedback. Reviews and discussions of several of the chapters in early form were provided by Dave Bowling, Tom Sharkey, John Finnigan, Rowan Sage, Ray Leuning, Laura Scott-Denton, Peter Harley, Tony Delany, Dan Yakir, Jielun Sun, Mike Weintraub, Dave Moore, Paul Stoy, Dave Schimel, and Keith Mott. Many thanks to all of you! While these colleagues provided many useful insights and suggestions, responsibility for the book's final form belongs with us.

Symbols

In writing a book with as broad a set of mathematical relations as that presented here we had to make decisions as to whether to create new symbols for cases of duplicated usage, or retain those most often used, by convention, in the scientific literature. We tried to use conventional symbols as often as was possible, and we allowed for some overlap in designation, especially when duplicated symbols were used in different chapters.

Uppercase, non-italicized Latin

A	CO_2 assimilation rate (μmol m^{-2} s^{-1})
A_c	canopy net CO_2 assimilation rate
A_n	net CO_2 assimilation rate
A_g	gross CO_2 assimilation rate
E	energy (J) or energy content (J mol^{-1})
E_a	energy of activation (J mol^{-1})
E	surface evaporation or leaf transpiration flux density (mol m^{-2} s^{-1})
E_t	total enzyme protein content (mol l^{-1})
E^o	standard reduction potential (J $coulomb^{-1}$)
F	flux density (mol m^{-2} s^{-1})
F_c	flux density of CO_2
F_w	flux density of H_2O
F_j	flux density of constituent *j*
F_J	photosynthetic electron transport flux density
F_{vm}	vertical atmospheric mean flux density
F_{vt}	vertical atmospheric turbulent flux density
F	Faraday's constant (coulomb mol^{-1})
G	conduction flux density of heat (J m^{-2} s^{-1})
G	free energy (J) or molar free energy content (J mol^{-1})
G^0	standard free energy (J) or molar free energy content (J mol^{-1})
G	rate of biomass increase (g s^{-1})
GPP	gross primary productivity (mol m^{-2} s^{-1} or mol m^{-2} yr^{-1})
H	enthalpy (J) or enthalpy content (J mol^{-1})
H	conduction of heat (W m^{-2})

H_{se}	conduction of sensible heat (from the surface to the atmosphere) (W m^{-2})
H_G	conduction of sensible heat (from the atmosphere to the ground surface) (W m^{-2})
I	photon flux density (mol photons m^{-2} s^{-1})
I_D	direct photon flux density
I_d	diffuse photon flux density
I_s	isoprene emission flux density (nmol m^{-2} s^{-1})
J	joule unit of energy (kg m^2 s^{-1})
LAI	leaf area index (m^2 leaf area m^{-2} ground area)
L	leaf area index (used in equations)
L_e	effective LAI
N	newton unit of force (kg m s^{-1})
N_a	Avogadro's number
NDVI	normalized difference of vegetation index (dimensionless)
NPP	net primary productivity (mol m^{-2} s^{-1} or mol m^{-2} yr^{-1})
P	total atmospheric pressure (N m^{-2}, Pa)
P	statistical probability
P_0	probability of photon penetration to a canopy layer
P_{sf}	probability of a sunfleck in a canopy layer
Q	thermal energy (J) or molar thermal energy content (J mol^{-1})
Q_{10}	respiratory quotient (ratio of R_d at two temperatures separated by 10 °C)
R	radiant energy flux density (J m^{-2} s^{-1} or W m^{-2})
R_S	shortwave radiant energy flux density (J m^{-2} s^{-1} or W m^{-2})
R_L	longwave radiant energy flux density (J m^{-2} s^{-1} or W m^{-2})
R_n	net radiation flux density (J m^{-2} s^{-1} or W m^{-2})
R	isotope abundance ratio
R_d	"dark" (mitochondrial) respiration (μmol m^{-2} s^{-1})
R_e	ecosystem respiration
R_g	growth mitochondrial respiration
R_m	maintenance mitochondrial respiration
S	molar entropy content (J mol^{-1} K^{-1})
S	amount of substrate (moles)
S	sink or source "strength," as a flux density (mol m^{-2} s^{-1})
S_{rel}	relative specificity of Rubisco (unitless)
[S]	enzyme substrate concentration (mol l^{-1} or mol m^{-3})
T	temperature (K or °C)
TKE	turbulence kinetic energy (J)
TPU	triose phosphate utilization flux density (μmol m^{-2} s^{-1})
U	internal energy (J) or molar internal energy content (J mol^{-1})
V	volume (m^3)
V_{max}	Michaelis–Menten velocity coefficient (mol s^{-1})

V_{cmax}	Michaelis–Menten velocity coefficient for Rubisco carboxylation
V_{omax}	Michaelis–Menten velocity coefficient for Rubisco oxygenation
W	work (J) or molar work content (J mol^{-1})
W_p	total plant biomass (g)
Y_g	growth yield (fraction of substrate converted to biomass)

Uppercase, italicized Latin

A	surface area (m^2)
A_G	ground area
A_L	leaf area
B	feedback multiplier (unitless)
B_k	permeability coefficient for viscous flow (m^2)
C_D	drag coefficient (dimensionless)
C_{Ex}	flux control coefficient (unitless)
E_x	radiative transfer extinction function (fraction of total PPFD)
F	force (N)
F_d	molar diffusive force (N mol^{-1})
F_D	drag force (g m s^{-2})
G	fraction of leaf area oriented normal to I_D in radiative transfer models
G	gain of feedback loop (unitless)
G_c	closed-loop feedback gain
G_o	open-loop feedback gain
K_d	molecular diffusion coefficient (m^2 s^{-1})
${}^kK_d$	Knudsen diffusion coefficient (m^2 s^{-1})
K_{dh}	diffusion coefficient for heat (m^2 s^{-1})
K_{dw}	diffusion coefficient for H_2O
K_{dc}	diffusion coefficient for CO_2
K_D	eddy diffusion coefficient (m^2 s^{-1})
K_e	equilibrium constant (unitless)
K_I	canopy PPFD extinction coefficient ($K_I = G/\cos\theta$)
K_m	Michaelis–Menten coefficient (mol l^{-1} or mol m^{-3})
K_c	Michaelis–Menten coefficient for dissolved CO_2
K_o	Michaelis–Menten coefficient for dissolved O_2
K_s	steady state constant (mol^{-1})
Kn	Knudsen number (dimensionless)
L	turbulent length scale (m) (generally used)
L	Obukhov length scale (m) (specifically used)
Nu	Nusselt number (dimensionless)

R	universal gas constant ($J\ K^{-1}\ mol^{-1}$)
Re	Reynolds number (dimensionless)
Ri	Richardson number (dimensionless)
Ri_c	critical Richardson number
Ri_b	bulk Richardson number
S_c	radiative transfer scattering function (fraction of total PPFD)
$S(\kappa)$	spectral density as a function of wavenumber
V	specific volume ($m^3\ kg^{-1}$)
$\bar{V}_w$	partial molal volume of H_2O ($m^3\ mol^{-1}$)

Lowercase, non-italicized Latin

a	radiant or photon absorptance (fractional)
aPAR	fraction of absorbed photosynthetically active radiation
c	concentration as mole fraction
c_{ac}	atmospheric CO_2 mole fraction
c_{aw}	atmospheric H_2O mole fraction
c_{aw}*	atmospheric H_2O mole fraction at saturation
c_{cc}	chloroplast CO_2 mole fraction
c_{co}	chloroplast O_2 mole fraction
c_{ic}	intercellular CO_2 mole fraction in the leaf air spaces
c_{iw}	intercellular H_2O mole fraction in the leaf air spaces
c_{sc}	CO_2 mole fraction at leaf surface
c_{Ex}	mole fraction concentration of enzyme x
fPAR	fraction of absorbed photosynthetically active radiation
g	conductance ($m\ s^{-1}$ or $mol\ m^{-2}\ s^{-1}$)
g_b	boundary layer conductance ($m\ s^{-1}$ or $mol\ m^{-2}\ s^{-1}$)
g_{bw}	boundary layer conductance to H_2O diffusion ($m\ s^{-1}$ or $mol\ m^{-2}\ s^{-1}$)
g_{bc}	boundary layer conductance to CO_2 diffusion ($m\ s^{-1}$ or $mol\ m^{-2}\ s^{-1}$)
g_s	stomatal conductance ($m\ s^{-1}$ or $mol\ m^{-2}\ s^{-1}$)
g_{sw}	stomatal conductance to H_2O vapor diffusion ($m\ s^{-1}$ or $mol\ m^{-2}\ s^{-1}$)
g_{sc}	stomatal conductance to CO_2 diffusion ($m\ s^{-1}$ or $mol\ m^{-2}\ s^{-1}$)
g_{ic}	internal leaf conductance to CO_2 diffusion ($m\ s^{-1}$ or $mol\ m^{-2}\ s^{-1}$)
g_{tw}	total leaf conductance to H_2O vapor diffusion ($m\ s^{-1}$ or $mol\ m^{-2}\ s^{-1}$)
h	height (m)
m	mass (g)
n	molar quantity (mol)
p	pressure or partial pressure of a gas constituent ($N\ m^{-2}$, Pa)
p_r	probability of recollision (secondary collision) of a photon

r	radius (m)
r	reflectance of incident PPFD (fractional)
r	resistance (s m^{-1})
r_a	aerodynamic resistance (s m^{-1})
r_{bl}	boundary layer diffusive resistance (s m^{-1})
r_i	internal leaf diffusive resistance (s m^{-1})
r_s	stomatal diffusive resistance (s m^{-1})
t	transmittance of incident PPFD (fractional)
v	speed or velocity (mol l^{-1} s^{-1} or m s^{-1})
v_c	Rubisco carboxylation rate on leaf area basis (μmol m^{-2} s^{-1})

Lowercase, italicized Latin

a	acceleration (m s^{-2})
c	speed of "light" (m s^{-1})
c	specific heat (J kg K^{-1})
c_p	specific heat of dry air at constant pressure (J kg^{-1} K^{-1})
c_v	specific heat of dry air at a constant volume (J kg^{-1} K^{-1})
d	boundary layer length scale (m)
d_H	canopy displacement height (m)
f	frequency (s^{-1})
f_a	fraction of canopy woody surface area
g	gravitational acceleration (~ 9.8 m s^{-2})
h	Planck's constant (J s)
h_c	heat transfer coefficient (J m^{-2} s^{-1} K^{-1})
k	reaction rate constant (s^{-1} or mol^{-1} s^{-1})
k_{cat}	enzyme catalytic rate constant
k	von Karman's constant (dimensionless)
k_B	Boltzmann constant (J K^{-1})
k_H	Henry's Law partitioning coefficient (kPa liter mol^{-1})
k_N	canopy nitrogen allocation coefficient (dimensionless)
l	length (m)
$\hat{m}$	mechanical advantage of the epidermis (dimensionless)
p	porosity of a soil or leaf volume (fractional)
r_p	radial width of penumbra (cm)
t	time (s)
t_E	Eulerian time scale (s)
t_L	Lagrangian time scale (s)
u	molar flow rate (mol s^{-1})

u longitudinal wind velocity (m s^{-1})
u' turbulent longitudinal wind velocity (m s^{-1})
$\overline{u}$ mean longitudinal wind velocity (m s^{-1})
u_j Einstein–Smoluchowski mobility of constituent j (s kg^{-1})
u_* friction velocity (m s^{-1})
v cross-stream wind velocity (m s^{-1})
w vertical wind velocity (m s^{-1})
w' turbulent vertical wind velocity (m s^{-1})
$\overline{w}$ mean vertical wind velocity (m s^{-1})
z electrical charge
z vertical length (m)
z_{bl} vertical depth of boundary layer (m)
z_0 aerodynamic roughness length (m)
z_p depth of pore (mm)

Lowercase, non-italicized Greek

α isotope effect (unitless)
γ foliar clumping (fraction of LAI)
δ isotope abundance ratio (delta notation) (‰)
ε TKE dissipation rate (s)
ε radiation-use efficiency in remote sensing modeling (g C MJ^{-1})
κ wavenumber (m^{-1})
λ canopy clumping index (dimensionless)
λ_a mean free path of diffusion in air (m)
λ_w latent heat of vaporization for H_2O (J mol^{-1})
$\lambda_w E$ latent heat flux density (J m^{-2} s^{-1})
μ molar chemical potential (J mol^{-1})
μ^* standard molar chemical potential (J mol^{-1})
ν kinematic viscosity (m^2 s^{-1})
ρ density (g m^{-3})
ρ_a mass density of air (g m^{-3})
ρ_m molar density (mol m^{-3})
ρ_{mw} molar density of water (typically of air; mol m^{-3})
ρ_w mass density of water (g m^{-3})
σ standard deviation
τ atmospheric lifetime (s)
τ momentum flux density (N m^{-2})
ϕ fractional leakage of mass from a metabolic pathway
ϕ ratio of the rates of oxygenation and carboxylation for Rubisco

ϕ	molar quantum yield of photosynthesis (mole fraction)
ψ_w	total water potential (Pa)
ψ_g	gravitational component of water potential (Pa)
ψ_m	matric component of water potential (Pa)
ψ_p	pressure component of water potential (Pa)
ψ_π	osmotic component of water potential (Pa)
$\psi_{\pi g}$	osmotic potential of guard cell (Pa)
$\psi_{\pi s}$	osmotic potential of subsidiary cell (Pa)

Lowercase, italicized Greek

α	Kolmogorov constant for turbulent inertial subrange (dimensionless)
α	surface albedo (percentage of incident solar flux density)
ε	radiant emittance (fractional)
ε_L	leaf emittance of longwave radiation
$\varepsilon_j^{V_x}$	elasticity coefficient of reaction x with respect to metabolite *j* (unitless)
θ	solar zenith angle (degrees or radians)
θ_t	potential temperature (K)
θ_{vt}	virtual potential temperature (K)
κ	thermal conductivity (J s^{-1} m^{-1} K^{-1})
κ_E	Eyring transmission coefficient (fractional)
λ	wavelength (m)
μ	dynamic viscosity (kg m^{-1} s^{-1})
v	frequency of electromagnic wave
σ	Stefan–Boltzmann constant (5.673×10^{-8} J s^{-1} m^{-2} K^{-4})
τ	tortuosity of a pore system (dimensionless)
ϕ	Monin–Obukhov scaling coefficient (dimensionless)
ϕ	solar azimuth angle (degrees or radians)
ϕ	Bunsen solubility coefficient for gases (m^3 gas m^{-3} solution)
φ_E	electrical potential (J coloumb^{-1})
χ	stomatal mechanical coefficient (mmol H_2O m^{-2} s^{-1} MPa^{-1})
ω	photon scatter coefficient (dimensionless)

Uppercase, non-italicized Greek

Γ	CO_2 compensation point (μmol mol^{-1})
Γ_*	CO_2 photocompensation point (μmol mol^{-1})
Δ	isotope discrimination (‰)

Δc_j finite difference in mole fraction of chemical species j

Λ_E Eulerian length scale (m)

Λ_L Lagrangian length scale (m)

Ω angle of solar photon interactions with a surface (degrees or radians)

Ω_L angle of leaf surface orientation

A Note on the Parenthetical Formatting of Function Relations and Collected Sums or Differences

Conventional algebraic notation indicates that a dependent variable is a 'function of' an independent variable through use of parenthetical formatting. Thus, dependent variable y is related to independent variable x according to $y = f(x)$. However, other symbols can be used to designate dependent and independent variables using parenthetical notation. Take the example of atmospheric vapor pressure (often designated as e_s) determined as a function of air temperature (often designated as T_a). We can write an equation with e_s expressed as a function of T_a, and related to surface temperature (T_s), and a linear slope (s), as: $e_s\ [T_a] \approx e_s\ [T_s] + s\ (T_a - T_s)$. This relation is read as 'e_s' evaluated as a function of 'T_a' is approximated by 'e_s' as a function of 'T_s' plus the product between a linear slope 's' and the difference between T_a and T_s. The terms containing e_s on the left and right sides of the equation *should not* be read as "e_s multiplied by T_a or T_s"; rather, the reader should be aware from the context of the equation that the notation is referring to e_s as a function of T_a or T_s. The mathematical difference between T_a and T_s on the right side of the equation is gathered as a "collected difference" within parentheses. Similar parenthetical nomenclature is used to indicate "collected sums". Both *collected differences* and *collected sums*, unlike the terms indicated as *parenthetical functions*, are indeed active variables of the relation. We have tried to assist the reader in making these distinctions by using squared brackets around those terms intended as functional relations (e.g., $[T_a]$), and rounded parentheses around those terms intended as collected sums or differences (e.g., $(T_a - T_s)$).