

RESONANCE

Applications in Physical Science



Michael M. Woolfson

Imperial College Press

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University of York, UK



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Introduction

Many centuries ago, an intelligent individual could master a large proportion of the prevailing knowledge of the time and there were no accepted boundaries in science, as we know them today. Thus, Isaac Newton was not only a mathematician and physicist but also spent much of his later life in the practice of alchemy, a kind of chemistry not held in high regard, even in those days. Robert Hooke, a contemporary of Newton, had even wider interests, covering gravitation, astronomy, biology, palaeontology and the design of timepieces.

With the explosive expansion of scientific knowledge, particularly in the last few decades, it has become increasingly difficult to keep up-to-date even in one area of science — be it physics, chemistry, biology or any other well-defined subject area. Indeed, it is a well-rounded physicist who can understand even 50% of the titles in the publication *Physics Abstracts*, let alone the contents of the papers to which they refer.

It has been traditional to structure university science lecture courses in terms of mainstream topics (e.g. physical optics, structure and bonding, atomic physics, spectroscopy etc.) and to many students the links between different strands of their course never become clear. Not only do they not see the essential unity of science, but also some even lack a sense of unity in their own specialist area of study.

It is clearly impossible to return to the era of the universal scientist, but what *is* possible is to explore some topics which act like

weft in binding together different strands within a single subject area or even, to some extent, different subject areas. Various branches of mathematics serve this purpose: as an example, matrices can be used to analyse complex lens systems; to represent the Lorentz transformation in relativity theory; to express electron spin in quantum mechanics; and to study both vibrating systems and biological populations.

At the level of the trained scientist, the appreciation of these cross-links can give insights that lead to the better practice of science. X-ray crystallography is an example of a topic that binds together vast tracts of science such as mathematics, physics, chemistry, biology, geology and mineralogy — and there may be others. The American pure mathematician Herbert Hauptman (1917–2011), who became a theoretical crystallographer, eventually won a Nobel Prize in Chemistry!

Another of these linking topics is *resonance*, which occurs in various guises: in everyday life; in artistic contexts such as music; and in many forms in the physical sciences. The purpose of this textbook is to explore a variety of scientific areas in which resonance occurs, ranging from concerns about the safety of bridges to the confirmation of a prediction of Einstein's Theory of General Relativity.

Interspersed within the chapters are *exercises*, which are usually simple numerical applications of the immediately preceding material and are designed to instil a feeling for the magnitudes of the quantities of interest. At the end of chapters there are *problems*, which are more demanding than the exercises and whose purpose is to test the student's understanding of the material within each chapter. A solutions section provides a check on the student's success (or otherwise) in completing these tests, but where the student has difficulties, they also provide a guide that reinforces the teaching role of the chapter.

Before the advent of the computer age there were two branches of every science: *experimental* and *theoretical*. To these we must now add *computational*; there are many areas of science where both experimental and theoretical solutions to problems are impossible and the only solution is through computation. An outstanding example is the solution of many-body problems in astronomy;

there are some theoretical solutions for special three-body problems (see Section 4.4.4) but, in general, the motion of more than two bodies under their mutual gravitational forces requires a computational approach. Appendices II and III give programs to solve astronomical many-body problems, which are required for the complete solution of some of these problems.¹ They are in very basic FORTRAN77 code, but for those unfamiliar with FORTRAN the program will guide them in inserting the correct data, if they have access to a compiler.

A useful ability for most modern scientists is to be able quickly to write short programs to carry out straightforward calculations that would be tedious any other way. There are problems in this text where the value of a function of a single variable is required for a large number of values of the variable and simple programs consisting of few lines of code enable this to be done quickly. By outputting data to a file, a graphical solution can then be found with the use of a graphics package.

¹Copies of these programs are available for free download from:
<http://www.worldscientific.com/worldscibooks/10.1142/p963#t=suppl>

Contents

<i>Introduction</i>	x
Chapter 1. Simple Harmonic Motion, Damping and Resonance	1
1.1 Simple Harmonic Motion	1
1.1.1 A mass on a vertical spring	1
1.1.2 The simple pendulum	4
1.1.3 The energy of simple harmonic motion	5
1.2 Damped Simple Harmonic Motion	7
1.2.1 Light damping	8
1.2.2 Heavy damping	9
1.2.3 Critical damping	10
1.3 Forced Vibration and Resonance	11
Problems 1	14
Chapter 2. Resonance in Everyday Life	17
2.1 A Girl on a Swing	17
2.2 The Opera Singer and the Wine Glass	18
2.3 Bridges	20
2.3.1 The Broughton Suspension Bridge	21
2.3.2 The Millennium Bridge, London	22
2.4 Washboard Roads	23
2.5 Buildings and Earthquakes	25
2.6 Resonance and Musical Instruments	27
2.6.1 Resonance of air in a pipe	27
2.6.2 Resonance in a string	31
2.6.3 The violin	32

Chapter 3.	Electrical Circuits and Resonance	35
3.1	Direct-Current Circuits	35
3.2	Expressing an Alternating Potential Difference	36
3.3	Complex Impedances	37
3.3.1	Inductors	37
3.3.2	Capacitors	39
3.4	A Series LCR Resonance Circuit	41
3.5	A Parallel LCR Resonance Circuit	43
	Problems 3	47
Chapter 4.	Resonance in the Solar System	49
4.1	Kirkwood Gaps	49
4.2	Saturn's Rings	54
4.3	Volcanoes on the Satellite Io	58
4.3.1	Elastic hysteresis and Q values	59
4.3.2	Elliptical orbits	60
4.3.3	The generation of energy in Io by tidal stressing	62
4.4	Commensurabilities of Planetary and Satellite Orbits	65
4.4.1	Planetary commensurabilities	65
4.4.2	The commensurabilities of the Galilean satellites	70
4.4.3	The commensurabilities of some of Saturn's satellites	72
4.4.4	The Trojan asteroids	74
	Problems 4	78
Chapter 5.	Nuclear Magnetic Resonance	81
5.1	A Brief Review of the Structure of Atoms	81
5.2	Intrinsic Spins and Magnetic Moments	82
5.2.1	Orientation of nuclei in a magnetic field	84
5.3	Magnetic Resonance Imaging (MRI)	87

5.3.1	Larmor precession	88
5.3.2	The basic physics of the MRI process	91
5.3.3	Fourier transforms	95
5.3.4	The MRI procedure	97
5.4	Other Applications of NMR	102
	Problems 5	102
Chapter 6. Electron Spin Resonance		105
6.1	The Electronic Structure of Molecules and Free Radicals	105
6.1.1	The electronic structure of atoms . . .	105
6.1.2	The electronic structure of molecules	108
6.1.3	The electronic structure of free radicals	110
6.2	The Basic Theory of Electron Spin Resonance	111
6.3	The Form and Use of an ESR Spectrometer	114
6.4	ESR Spectra	115
6.4.1	ESR spectra for a single neighbouring nucleus with nuclear spin	116
6.4.2	Many nuclei in equivalent positions	119
6.4.3	Two sets of non-equivalent nuclei . . .	120
	Problems 6	122
Chapter 7. Resonance with Electromagnetic Radiation		123
7.1	Fraunhofer Lines	123
7.1.1	Energy levels in atoms	123
7.1.2	Atomic spectra	124
7.1.3	Formation of Fraunhofer lines	126
7.2	Lasers	126
7.2.1	Spontaneous and stimulated emission	127

7.2.2	A simple laser system and uses of lasers	130
7.3	Radar	131
7.3.1	The klystron amplifier	133
7.3.2	The cavity magnetron	135
7.4	The Anomalous Scattering of X-rays	136
7.4.1	Scattering from a free electron	137
7.4.2	Scattering from a bound electron	138
	Problems 7	143
Chapter 8.	Nuclear Physics, Radiation and Particle Physics	145
8.1	The Beginning of Nuclear Physics	145
8.2	The Cockcroft–Walton Experiment	147
8.3	The Cyclotron	149
8.3.1	Maintaining resonance	153
8.3.2	Overcoming special relativity effects	154
8.4	Linear Particle Accelerators	155
8.5	Synchrotrons	157
8.6	Particles and Particle Colliders	160
8.6.1	The neutrino	161
8.6.2	Leptons	161
8.6.3	Quarks and sub-atomic particles	162
8.6.4	Particle colliders	164
	Problems 8	166
Chapter 9.	The Mössbauer Effect	169
9.1	The Basis of the Mössbauer Effect	169
9.2	Natural Line-Widths	169
9.3	Doppler Broadening	170
9.4	The Effect of Recoil	172
9.5	Nuclear Emission of γ -rays	174
9.6	Factors Affecting γ -Ray Emission	175
9.6.1	Natural line-width for γ -rays	175
9.6.2	Doppler line broadening for γ -rays	176

9.6.3	The recoil frequency shift for γ -rays	176
9.7	Mössbauer Spectroscopy	177
9.7.1	Experimental equipment	178
9.8	Spectral Features	179
9.8.1	Isomer shift	180
9.8.2	Quadrupole splitting	180
9.8.3	Magnetic splitting	182
9.9	Information from Mössbauer Spectroscopy . .	183
	Problem 9	184
Appendix I	The Binomial Theorem and Approximations	187
Appendix II	A Program for Simulating Kirkwood Gap Formation	191
Appendix III	A Program for Finding the Orbits of Trojan Asteroids	197
	Physical Constants and Useful Data	205
	Solutions to Examples and Problems	207
	Index	239

Chapter 1

Simple Harmonic Motion, Damping and Resonance

1.1 Simple Harmonic Motion

In 1582 the Italian mathematician and astronomer Galileo Galilei (1564–1642) observed the swinging motion of a chandelier in Pisa Cathedral. He noticed that the period of its swing, which it is said that he timed using his pulse, did not change as the amplitude of the swing died down. In 1602 he carried out a series of experiments on the properties of a pendulum and later, towards the end of his life, he conceived the idea of using a pendulum to regulate a mechanical clock. Such clocks were made after he died and were the most precise timepieces available until comparatively recently.

1.1.1 *A mass on a vertical spring*

The type of motion performed by the pendulum is known as *simple harmonic motion*. However, although Galileo observed no variation of period with the amplitude of the swing there *is* a dependence that would only be measurable for moderately large swings with instruments better than a pulse. For that reason we shall consider another system that gives simple harmonic motion: a mass, m , attached to the end of a light vertical spring, as shown in Figure 1.1.

The downward force on the spring due to the mass is its weight, mg , (where g is the acceleration due to gravity) and this stretches the

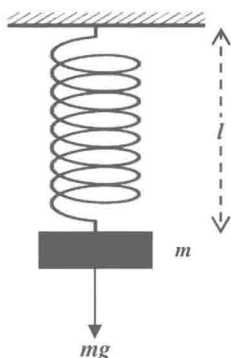


Figure 1.1 A mass on a spring.

spring until the upward force due to the spring balances the weight and the mass is then stationary at a point of equilibrium. A normal spring, not under excessive strain, satisfies Hooke's law that the force required either to extend or compress it will be proportional to the extension or compression. This means that if we displace the mass in Figure 1.1 by a distance x , which can be either positive or negative, the net force on it will be

$$F = -\kappa x, \quad (1.1)$$

in which κ , the *stiffness* of the spring, is the force per unit displacement and the negative sign indicates that the force is in the opposite direction to the displacement. This force will give an acceleration of the mass so we can write

$$m \frac{d^2 x}{dt^2} = -\kappa x$$

or

$$\frac{d^2 x}{dt^2} = -\frac{\kappa}{m} x = -\omega^2 x, \quad (1.2)$$

in which we have introduced $\omega = \sqrt{\kappa/m}$.

The general solution to the differential equation (1.2) is

$$x = a_1 \cos(\omega t) + a_2 \sin(\omega t), \quad (1.3a)$$