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Mathematical Modeling in Continuum Mechanics

Second Edition

连续介质力学中的数学模型 第2版

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MATHEMATICAL MODELING IN CONTINUUM MECHANICS

SECOND EDITION

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Temam and Miranville present core topics within the general themes of fluid and solid mechanics. The brisk style allows the text to cover a wide range of topics including viscous flow, magnetohydrodynamics, atmospheric flows, shock equations, turbulence, nonlinear solid mechanics, solitons, and the nonlinear Schrödinger equation.

This second edition will be a unique resource for those studying continuum mechanics at the advanced undergraduate and beginning graduate level whether in engineering, mathematics, physics, or the applied sciences. Exercises and hints for solutions have been added to the majority of chapters, and the final part on solid mechanics has been substantially expanded. These additions have now made it appropriate for use as a textbook, but it also remains an ideal reference book for students and anyone interested in continuum mechanics.

Preface

This book is an extended version of a course on continuum mechanics taught by the authors to junior graduate students in mathematics. Besides a thorough description of the fundamental parts of continuum mechanics, it contains ramifications in a number of adjacent subjects such as magnetohydrodynamics, combustion, geophysical fluid dynamics, and linear and nonlinear waves. As is, the book should appeal to a broad audience: mathematicians (students and researchers) interested in an introduction to these subjects, engineers, and scientists.

This book can be described as an "interfacial" book: interfaces between mathematics and a number of important areas of sciences. It can also be described by what it is not: it is not a book of mathematics: the mathematical language is simple, only the basic tools of calculus and linear algebra are needed. This book is not a treatise of continuum mechanics: although it contains a thorough but concise description of many subjects, it leaves aside many developments which are fundamental but not needed in practical applications and utilizations of mechanics, e.g., the intrinsic – frame invariance – character of certain quantities or the coherence of certain definitions. The reader interested by these issues is referred to the many excellent mechanics books which are available, such as those quoted in the list of references to Part I. Finally, by its size limitations, this book cannot be encyclopedic, and many choices have been made for the content; a number of subjects introduced in this book can be developed themselves into a full book. All in all, we believe that this book, benefiting from prolonged efforts and teaching experience of the authors, can be very useful to scientists who want to reduce the gap between mathematics and sciences, a gap usually due to the language barrier and the differences in thinking and reasoning.

The core of the book contains the fundamental parts of continuum mechanics: description of the motion of a continuous body, the fundamental law of dynamics, the Cauchy and the Piola-Kirchhoff stress tensors, the constitutive laws, internal energy and the first principle of thermodynamics, shocks and the Rankine–Hugoniot relations, an introduction to fluid mechanics for inviscid and viscous Newtonian fluids, an introduction to linear elasticity and the variational principles in linear elasticity, and an introduction to nonlinear elasticity.

Besides the core of continuum mechanics, this book also contains more or less detailed introductions to several important related fields that could be themselves the subjects of separate books: magnetohydrodynamics, combustion, geophysical fluid dynamics, vibrations, linear acoustics, and nonlinear waves and solitons in the context of the Korteweg–de Vries and the nonlinear Schrödinger equations. The whole book is suitable for a one-year course at the advanced undergraduate or beginning graduate level. Parts of it are suitable for a one-semester course either on the fundamentals of continuum mechanics or on a combination of selected topics.

This second edition of the book has been augmented by the introduction of exercises and hints at solutions making it more suitable for class utilization, by a new chapter on nonlinear elasticity, and by several additions and corrections suggested by the readers of the first edition. In particular it has benefited from the comments of the anonymous and non-anonymous reviewers of the first edition, especially J. Dunwoody and J.J. Telega. The authors want also to thank P. G. Ciarlet for his comments; the new chapter on nonlinear elasticity borrowed very much from his classical book on the subject. Finally they gratefully acknowledge essential help in the production of the volume from Teresa Bunge, Jacques Laminie, Eric Simonnet, and Djoko Wirosoetisno.

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Alain Miranville
June 2004

A few words about notations

The notations in this book are not uniform; this is partly done on purpose and partly because we had no choice. Indeed modelers usually have to comply or at least adapt to the notations common in a given field, and thus they must be trained to some flexibility. Another reason for having non-uniform notations is that different fields are present in this volume, and it was not possible to find notations fitting "all the standards."

Another objective while deciding the notations was to choose notations that can be easily reproduced by handwriting, thus avoiding as much as possible arrows, boldfaced type, and simple and double underlining with bars or tildes; in general, in a given chapter of this book, in a given context, it is clear what a given symbol represents.

Although the notations are not rigid, there are still some repeated patterns in the notations, and we indicate hereafter notations used in several chapters:

Ω or $\mathcal{O}$, possibly with indices: domain in $\mathbb{R}^2$ or $\mathbb{R}^3$

$x = (x_1, x_2)$ or (x_1, x_2, x_3): generic point in $\mathbb{R}^2$ or $\mathbb{R}^3$. Also denoted (x, y) or (x, y, z)

$a = (a_1, a_2)$ or (a_1, a_2, a_3): initial position in Lagrangian variables

t: time

$u = (u_1, u_2)$ or (u_1, u_2, u_3), or v or w: vectors in $\mathbb{R}^2$ or $\mathbb{R}^3$. Also denoted (u, v) or (u, v, w)

AB (or $\overrightarrow{AB}$ to emphasize): vector from A to B

u or U: velocity

u: displacement vector

γ: acceleration

m: mass

f, F: forces; usually f for volume forces and F for surface forces
ρ: density
g: gravity constant. Also used for equation of state for fluids
T or θ: temperature
σ: Cauchy stress tensor (in general)
n: unit outward normal on the boundary of an open set Ω or $\mathcal{O}$, $n = (n_1, n_2)$ or $n = (n_1, n_2, n_3)$

We will use also the following classical symbols and notations:

δ_{ij}: the Kronecker symbol equal to 1 if $i = j$ and to 0 if $i \neq j$
$\varphi_{,i}$ will denote the partial derivative $\partial\varphi/\partial x_i$.

The Einstein summation convention will be used: when an index (say j) is repeated in a mathematical symbol or within a product of such symbols, we add these expressions for $j = 1, 2, 3$. Hence

$$\sigma_{ij,j} = \sum_{j=1}^{3} \frac{\partial\sigma_{ij}}{\partial x_j}, \qquad \sigma_{ij} \cdot n_j = \sum_{j=1}^{3} \sigma_{ij} n_j.$$

Contents

PART I

FUNDAMENTAL CONCEPTS IN CONTINUUM MECHANICS

CHAPTER ONE

Describing the motion of a system: geometry and kinematics

1.1. Deformations

The purpose of mechanics is to study and describe the motion of material systems. The language of mechanics is very similar to that of set theory in mathematics: we are interested in material bodies or systems, which are made of material points or matter particles. A material system fills some part (a subset) of the ambient space ($\mathbb{R}^3$), and the position of a material point is given by a point in $\mathbb{R}^3$; a part of a material system is called a subsystem.

We will almost exclusively consider material bodies that fill a domain (i.e., a connected open set) of the space. We will not study the mechanically important cases of thin bodies that can be modeled as a surface (plates, shells) or as a line (beams, cables). The modeling of the motion of such systems necessitates hypotheses that are very similar to the ones we will present in this book, but we will not consider these cases here.

A material system fills a domain Ω_0 in $\mathbb{R}^3$ at a given time t_0. After deformation (think, for example, of a fluid or a tennis ball), the system fills a domain Ω in $\mathbb{R}^3$. A material point, whose initial position is given by the point $a \in \Omega_0$, will be, after transformation, at the point $x \in \Omega$.

The deformation can thus be characterized by a mapping as follows (see Figure 1.1):

$$\Phi : a \in \Omega_0 \mapsto x \in \Omega .$$

Assuming that matter is conserved during the deformation, we are led to make the following natural hypothesis:

The function Φ is one – to – one from Ω_0 onto Ω.

We will further assume that the deformation Φ is a smooth application of class $\mathcal{C}^1$ at least, from Ω_0 into Ω, as well as its inverse ($\mathcal{C}^1$ from Ω onto Ω_0). In fact we assume that Φ is as smooth as needed.

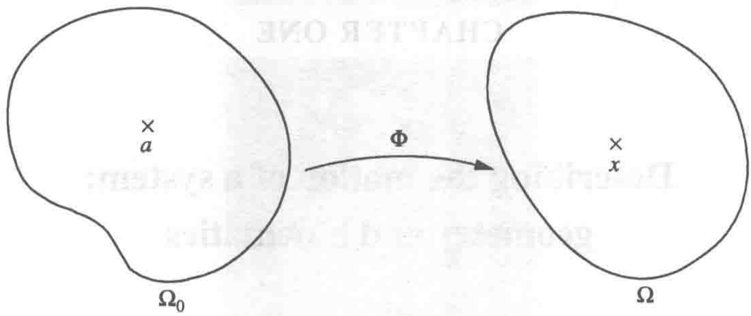

Figure 1.1 The mapping Φ.

Regularity assumption

The regularity assumption made on Φ will actually be general; we will assume that all the functions we introduce are as regular as needed for all the mathematical operations performed to be justified (e.g., integration by parts, differentiation of an integral depending on a parameter, etc.). This hypothesis, which will be constantly assumed in the following, will only be weakened in Chapter 6 for the study of shock waves, which correspond to the appearance of discontinuity surfaces. In that case, we will assume that the map Φ is piecewise C^1. This assumption must be weakened also for the study of other phenomena which will not be considered here, such as singular vortices for fluids, dislocations for solids, or collisions of rigid bodies.

Let grad $\Phi(a) = \nabla\Phi(a)$ be the matrix whose entries are the quantities $(\partial\Phi_i/\partial a_j)(a)$; this is the Jacobian matrix of the mapping $a \mapsto x$ also denoted sometimes Dx/Da. Because Φ^{-1} is differentiable, the Jacobian $\det(\nabla\Phi)$ of the transformation $a \mapsto x$ is necessarily different from zero. We will assume in the following that it is strictly positive; the negative sign corresponds to the nonphysical case of a change of orientation (a left glove becoming a right glove). We will later study the role played by the linear tangent map at point a in relation to the Taylor formula

$$\Phi(a) = \Phi(a_0) + \nabla\Phi(a_0) \cdot (a - a_0) + o(|a - a_0|).$$

We will also introduce the dilation tensor to study the deformation of a "small" tetrahedron.

Displacement

Definition 1.1. *The map $u : a \mapsto x - a = \Phi(a) - a$ is called the displacement; u(a) is the displacement of the particle a.*