



**FAITH A. MORRISON**

AN INTRODUCTION TO

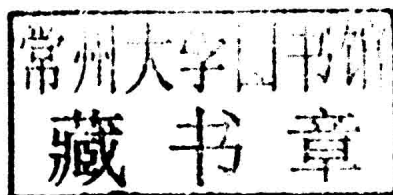
**FLUID**  
**MECHANICS**

# AN INTRODUCTION TO FLUID MECHANICS

---

**Faith A. Morrison**

Department of Chemical Engineering  
Michigan Technological University



CAMBRIDGE UNIVERSITY PRESS  
Cambridge, New York, Melbourne, Madrid, Cape Town,  
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press  
32 Avenue of the Americas, New York, NY 10013-2473, USA

[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9781107003538](http://www.cambridge.org/9781107003538)

© Faith A. Morrison 2013

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2013

Printed in the United States of America

*A catalog record for this publication is available from the British Library.*

*Library of Congress Cataloging in Publication data*

Morrison, Faith A.

An introduction to fluid mechanics / Faith A. Morrison.

p. cm.

ISBN 978-1-107-00353-8 (hardback)

1. Fluid mechanics. I. Title.

QA901.M67 2012

532-dc23 2011049511

ISBN 978-1-107-00353-8 Hardback

Additional resources for this publication at [www.cambridge.org/us/knowledge/isbn/item6684157/](http://www.cambridge.org/us/knowledge/isbn/item6684157/).

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party Internet Web sites referred to in this publication and does not guarantee that any content on such Web sites is, or will remain, accurate or appropriate.

Cover photo: The Naruto Whirlpools, Japan, as seen from a tourist cruise boat. Photo taken by Hellbuny.

## Equations Summary

Unit-conversions: [www.chem.mtu.edu/~fmorriso/convert.pdf](http://www.chem.mtu.edu/~fmorriso/convert.pdf).

Mechanical Energy Balance  $\frac{\Delta p}{\rho} + \frac{\Delta \langle v \rangle^2}{2\alpha} + g\Delta z + F_{friction} = -\frac{W_{s,by\ fluid}}{m} \quad \begin{cases} \alpha_{laminar} = 0.5 \\ \alpha_{turbulent} \approx 1 \end{cases}$

$$F_{friction} = \left[ 4f \frac{L}{D} + \sum_{fittings_i} n_i K_{f,i} \right] \frac{\langle v \rangle^2}{2}$$

Fanning Friction Factor (pipe flow)  $f = \frac{\mathcal{F}_{drag}}{\frac{1}{2}\rho \langle v \rangle^2 \pi R^2} = \frac{\Delta p D}{2L\rho \langle v \rangle^2}$

Drag Coefficient (sphere drop)  $C_D = \frac{\mathcal{F}_{drag}}{\frac{1}{2}\rho v_\infty^2 \pi R^2} = \frac{4gD(\rho_{body} - \rho)}{3\rho v_\infty^2}$

Momentum Balance on a CV (Reynolds transport theorem)  $\frac{d\underline{\mathbf{P}}}{dt} + \iint_{CS} (\hat{n} \cdot \underline{v}) \rho \underline{v} dS = \sum_{\text{on CV}} \underline{f}$

Hydrostatic Pressure  $p_{bottom} = p_{top} + \rho gh$

Hagen-Poiseuille Equation (steady, laminar tube flow, incompressible)  $Q = \frac{\pi(p_0 - p_L)R^4}{8\mu L}$

Prandtl Equation (steady, turbulent tube flow)  $\frac{1}{\sqrt{f}} = -4.0 \log \left( \frac{4.67}{\text{Re}\sqrt{f}} \right) + 2.28$

Stokes-Einstein-Sutherland Equation (steady, slow flow around a sphere)  $\mathcal{F}_{drag} = 6\pi R\mu v_\infty$

Macroscopic Momentum Balance on a CV

$$\begin{aligned} \frac{d\underline{\mathbf{P}}}{dt} + \sum_{i=1}^{\#streams} \left[ \frac{\rho A \cos \theta \langle v \rangle^2}{\beta} \hat{v} \right] \Big|_{A_i} \\ = \sum_{i=1}^{\#streams} [-pA\hat{n}]_{A_i} + \underline{R} + M_{CV}\underline{g} \quad \begin{cases} \beta_{laminar} = 0.75 \\ \beta_{turbulent} \approx 1 \end{cases} \end{aligned}$$

Navier-Stokes Equation (microscopic momentum balance, incompressible, Newtonian fluids)  $\rho \left( \frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} \right) = -\nabla p + \mu \nabla^2 \underline{v} + \rho \underline{g}$

Continuity Equation (microscopic mass balance, incompressible fluids)  $\nabla \cdot \underline{v} = 0$

Total Stress Tensor  $\underline{\underline{\tilde{\Pi}}} = -p\underline{\underline{I}} + \underline{\underline{\tilde{\tau}}}$

$$\begin{pmatrix} \tilde{\Pi}_{11} & \tilde{\Pi}_{12} & \tilde{\Pi}_{13} \\ \tilde{\Pi}_{21} & \tilde{\Pi}_{22} & \tilde{\Pi}_{23} \\ \tilde{\Pi}_{31} & \tilde{\Pi}_{32} & \tilde{\Pi}_{33} \end{pmatrix}_{123} = \begin{pmatrix} \tilde{\tau}_{11} - p & \tilde{\tau}_{12} & \tilde{\tau}_{13} \\ \tilde{\tau}_{21} & \tilde{\tau}_{22} - p & \tilde{\tau}_{23} \\ \tilde{\tau}_{31} & \tilde{\tau}_{32} & \tilde{\tau}_{33} - p \end{pmatrix}_{123}$$

Dynamic Pressure  $\mathcal{P} \equiv p + \rho gh$

Newtonian Constitutive Equation  $\underline{\underline{\tilde{\tau}}} = \mu (\nabla \underline{\underline{v}} + (\nabla \underline{\underline{v}})^T)$

$$= \mu \begin{pmatrix} 2\frac{\partial v_1}{\partial x_1} & \frac{\partial v_2}{\partial x_1} + \frac{\partial v_1}{\partial x_2} & \frac{\partial v_3}{\partial x_1} + \frac{\partial v_1}{\partial x_3} \\ \frac{\partial v_2}{\partial x_1} + \frac{\partial v_1}{\partial x_2} & 2\frac{\partial v_2}{\partial x_2} & \frac{\partial v_2}{\partial x_3} + \frac{\partial v_3}{\partial x_2} \\ \frac{\partial v_3}{\partial x_1} + \frac{\partial v_1}{\partial x_3} & \frac{\partial v_2}{\partial x_3} + \frac{\partial v_3}{\partial x_2} & 2\frac{\partial v_3}{\partial x_3} \end{pmatrix}_{123}$$

Total Molecular Fluid Force on a Finite Surface  $\mathcal{S}$   $\underline{\underline{\mathcal{F}}} = \iint_{\mathcal{S}} [\hat{n} \cdot \underline{\underline{\tilde{\Pi}}}]_{\text{at surface}} dS$

Stationary Fluid  $[\hat{n} \cdot \underline{\underline{\tilde{\Pi}}}] = -p\hat{n}$

Moving Fluid  $[\hat{n} \cdot \underline{\underline{\tilde{\Pi}}}] = -p\hat{n} + \hat{n} \cdot \underline{\underline{\tilde{\tau}}}$

Total Fluid Torque on a Finite Surface  $\mathcal{S}$   $\underline{\underline{\mathcal{T}}} = \iint_{\mathcal{S}} [\underline{\underline{R}} \times (\hat{n} \cdot \underline{\underline{\tilde{\Pi}}})]_{\text{at surface}} dS$

Total Flow Rate Out Through a Finite Surface  $\mathcal{S}$   $Q = \dot{V} = \iint_{\mathcal{S}} [\hat{n} \cdot \underline{\underline{v}}]_{\text{at surface}} dS$

Average Velocity Across a Finite Surface  $\mathcal{S}$   $\langle v \rangle = \frac{Q}{S}$

## **AN INTRODUCTION TO FLUID MECHANICS**

This is a modern and elegant introduction to engineering fluid mechanics enriched with numerous examples, exercises, and applications. The goal of this textbook is to introduce the reader to the analysis of flows using the laws of physics and the language of mathematics. The approach is rigorous, but mindful of the student. Emphasis is on building engagement, competency, and problem-solving confidence that extends beyond a first fluids course.

This text delves deeply into the mathematical analysis of flows, because knowledge of the patterns fluids form and why they are formed and the stresses fluids generate and why they are generated is essential to designing and optimizing modern systems and devices. Inventions such as helicopters and lab-on-a-chip reactors would never have been designed without the insight brought by mathematical models.

Faith A. Morrison is Professor of Chemical Engineering at Michigan Technological University, where she has taught for 22 years. Morrison's expertise is in polymer rheology, in particular focusing on materials with structure, including high-molecular-weight polymers, block copolymers, hydrogels, and composites. She is the Past President of the Society of Rheology and Editor of the *Rheology Bulletin*. Morrison is the author of *Understanding Rheology* (2001).



*This book is dedicated to my mother Frances P. Morrison,  
my father Philip W. Morrison, and my elder brother  
Professor Philip W. Morrison, Jr.*





# Preface

This book forms the basis of a one-semester introductory course in fluid mechanics for engineers and scientists. Students working with this text are expected to have a background in multivariable calculus, linear algebra, and differential equations; review of these topics as applied to fluid mechanics is provided in Chapter 1. Problem solving is taught by example throughout the text. We include numerous solved examples and end-of-chapter problems, and a complete solution manual is available for instructors.

Fluid mechanics can be a difficult subject. Nonlinear physics governs flow, and thus we often resort to a variety of simplifications to obtain solutions. Different simplifications are used under different conditions, making fluid mechanics intimidating, at least to a beginner. *An Introduction to Fluid Mechanics* presents the topic through a discovery process, as described in this preface, that mimics engineering practice. The process used seeks solutions by answering the following questions:

1. *What is the problem?*
2. *What do we need to know, and do, to address the problem?*
3. *What is the solution to the problem?*
4. *What other problems/opportunities may be addressed now that we have solved this problem?*

This organizational choice builds critical thinking skills by emphasizing the thought processes that lead to model development. The book is divided into four parts that answer these four questions for the study of fluid mechanics.

1. **What is the problem?** [*Part I: Preparing to Study Flow*]  
*Chapter 1: Why Study Fluid Mechanics*  
*Chapter 2: How Fluids Behave*

The problem addressed in this book is how to bring readers to an understanding of flow behavior and to mastery of flow-modeling calculations. To accomplish this objective, students must come to the task with skills in mathematics and simple flow calculations. In Chapter 1 we introduce the problem, cover needed background calculations (i.e., the macroscopic mass balance and the mechanical engineering balance), and review mathematics that is prerequisite to the study of fluid mechanics (i.e., calculus and differential equations). In Chapter 2, we showcase the diversity and complexity of fluid behaviors—showing readers that the mechanical energy balance is insufficient to explain flow patterns and making

the case that effort spent learning fluid mechanics is worth it. The presentation in Chapter 2 is at the survey level and spans from the introduction of viscosity to discussions of magnetohydrodynamics and vorticity. Overall, the text follows a path inspired by the spiral learning curve [Bruner, 1966], with the topics of Chapter 2 revisited at the end of the book (*Chapter 10: How Fluids Behave (Redux)*). That final chapter demonstrates how the intervening presentation leads to the ability to solve complex flow problems.

2. **What do we need to know, and do, to address the problem?** [*Part II: The Physics of Flow*]  
*Chapter 3: Modeling Fluids*  
*Chapter 4: Molecular Fluid Stresses*  
*Chapter 5: Stress-Velocity Relationships*

Having clarified our objectives in Part I, we seek methods to address the objectives in Part II. The continuum and the control volume are introduced in Chapter 3, and the stress components, fluid statics, and surface tension are presented in Chapter 4. To apply momentum conservation to a continuum, we need the stress constitutive equations, developed in Chapter 5 (Newtonian and non-Newtonian). These three chapters introduce the complete continuum model.

It can be a challenge to maintain student focus when covering background material, and we address this issue in a unique way: we provide a storyline. At the end of Chapter 3 we introduce two flow calculations and follow them longitudinally throughout Part II. These two problems (flow down an incline plane and flow in a 90-degree bend) are addressed in a just-in-time format, beginning before readers know enough fluid mechanics to be able to solve them. The solution develops gradually, incorporating new model pieces as they are covered. The repeated appearance of the two highlighted problems focuses readers on new developments, demonstrating the utility of the most recent step. Both highlighted problems are completed in Chapter 5, and Part II closes with the continuum model in place.

3. **What is the solution to the problem?** [*Part III: Flow Field Calculations*]  
*Chapter 6: Microscopic Balance Equations*  
*Chapter 7: Internal Flows*  
*Chapter 8: External Flows*

Model in hand, we turn to flows of interest. In Chapter 6 we develop the microscopic momentum balance (i.e., the Navier-Stokes equation), which represents an adaptation of the methods of Part II to the general case. We introduce the expressions for flow rates, fluid forces on walls, and fluid torques and show how to use these. In Chapter 7 a range of internal flows is discussed (pipes and ducts); in Chapter 8 external flows and boundary-layer flows are presented in detail (drag and lift).

The reader's path through Chapters 7 and 8 follows once again a storyline of a pair of highlighted flow problems. Chapter 7 begins with the quest to determine the extent of a home flood. Although not transparently related to the continuum model, the home flood problem is readily associated with pipe flow and motivates

the examination of pressure drop/flow rate relationships, laminar and turbulent flow, and other internal-flow topics. We repeat this structure in Chapter 8, asking about a skydiver, which raises the question of flow past an obstacle in general, leading to discussion of drag, lift, and boundary layers.

Throughout Part III we employ dimensional analysis when the models we develop are too difficult to solve. Dimensional analysis is presented as a natural step in a problem-solving methodology that begins with addressing simplified versions of a real problem (because those are the problems we can solve and they give us insight), progresses to solving mathematically complex models, and turns ultimately to obtaining practical data correlations.

**4. What other problems/opportunities may be addressed now that we have solved this problem?** [Part IV: Advanced Flow Calculations]

*Chapter 9: Macroscopic Balance Equations*

*Chapter 10: How Fluids Behave (Redux)*

The final two chapters of *An Introduction to Fluid Mechanics* guide readers through advanced modeling calculations on a variety of flows. In Chapter 9 the macroscopic balances, including the mechanical energy balance and the macroscopic momentum balance, are derived and applied. Although simple uses of the mechanical energy balance are covered in Chapter 1, in Chapter 9 the applications are more involved, including pump sizing and open-channel flow. Applying the macroscopic momentum balance is generally considered to be a difficult topic; we systemize macroscopic momentum solutions, making them more accessible. In Chapter 10, the learning spiral returns us to the more complex flows introduced in Chapter 2, and we apply the now-familiar continuum model to begin to understand these flows. Chapter 10 discusses numerical solutions, statistical aspects of turbulence, lift, circulation, vorticity, and supersonic flow.

The text includes reference materials provided to aid the student. The appendices contain a glossary of terms and mathematical tables. There is additional mathematical assistance available on the Internet in the Web Appendix. Finally, key equations are presented on the inside covers as an aid to problem solving.

## REFERENCE

---

Bruner, Jerome S., *The Process of Education* (Harvard University Press: Cambridge, MA, 1966).

## Acknowledgments

My path to choosing this presentation method for my fluids class began in 1998 when I was first asked to teach fluid mechanics. I looked at the texts available, and, given the goals of both my course and my students, I had difficulty choosing a text. Although I did not find a book that satisfied my needs, I did find notes from a colleague, Professor Davis W. Hubbard, that got me started in the right direction. Professor Hubbard passed away in 1994, before this text was conceived, but his contribution to pedagogy lives on through his influence on this book.

I would like to thank many colleagues, friends, and family members for their assistance, encouragement, and support during the time spent working on this project. A partial list includes Tomas Co, Susan Muller, Scott Chesna, Denise Lorson, Pushpalatha Murthy, Madhukar Vable, Frances Morrison, Rosa Co, Tommy Co, and my colleagues and students in the Department of Chemical Engineering at Michigan Technological University and in the Society of Rheology.

In 2005–6 I spent a sabbatical year at Korea University in Seoul, Korea, teaching and working on this text. Many thanks to my hosts and colleagues for their welcome and for creating such a productive atmosphere in which to work. I would like to thank particularly Jae Chun Hyun, Chongyoun Kim, Joung Sook Hong, Jun Hee Sung, Kwan Young Lee, Jae Sung Lee (Postech), and my students in CBE614 Rheology, especially Yang Soo Son, Wun-gwi Kim, and Seoung Hyun Park. Final edits on this manuscript were prepared during another sabbatical year in 2012–13 as the William R. Kenan, Jr., Visiting Professor for Distinguished Teaching at Princeton University. I would like to thank my hosts at Princeton for this opportunity, especially Robert K. Prud'homme and Richard Register.

# Contents

Preface *page* xiii

## **PART I PREPARING TO STUDY FLOW**

<b>1 Why Study Fluid Mechanics?</b>	<b>3</b>
1.1 Getting motivated	3
1.2 Quick start: The mechanical energy balance	8
1.2.1 MEB with no friction, no work: Macroscopic Bernoulli equation	15
1.2.2 MEB with shaft work	26
1.2.3 MEB with friction	34
1.3 Connecting mathematics to fluid mechanics	49
1.3.1 Calculus of continuous functions	50
1.3.1.1 Derivatives	50
1.3.1.2 Integrals	54
1.3.2 Vector calculus	58
1.3.2.1 Coordinate systems	61
1.3.2.2 Tensors	67
1.3.2.3 Differential operations	70
1.3.2.4 Curvilinear coordinates	74
1.3.3 Substantial derivative	84
1.3.4 Practical advice	91
1.4 Problems	93
<b>2 How Fluids Behave</b>	<b>106</b>
2.1 Viscosity	106
2.2 Drag	113
2.3 Boundary layers	118
2.4 Laminar versus turbulent flow: Reynolds number	127
2.5 Aerodynamics: Lift	137
2.6 Supersonic flow	143
2.7 Surface tension	145
2.8 Flows with curved streamlines	149
2.9 Magnetohydrodynamics	153

2.10	Particulate flow	154
2.11	Summary	157
2.12	Problems	158

## **PART II THE PHYSICS OF FLOW**

<b>3</b>	<b>Modeling Fluids</b>	<b>167</b>
3.1	Motion of rigid bodies	167
3.2	Motion of deformable media	172
3.2.1	The continuum model	175
3.2.1.1	Field variables	176
3.2.1.2	The continuum hypothesis	181
3.2.1.3	Fluid particles	184
3.2.2	Control-volume approach	187
3.2.2.1	Momentum balance on a control volume	190
3.2.2.2	The convective term	194
3.2.3	Problem solving with control volumes	206
3.2.3.1	Microscopic control-volume problem	207
3.2.3.2	Macroscopic control-volume problem	212
3.3	Summary	218
3.4	Problems	218
<b>4</b>	<b>Molecular Fluid Stresses</b>	<b>228</b>
4.1	Forces on a control volume	229
4.2	Stationary fluids: Hydrostatics	236
4.2.1	Gases	237
4.2.2	Liquids	241
4.2.3	Pascal's principle	261
4.2.4	Static fluid devices	271
4.2.4.1	Manometers	271
4.2.4.2	Hydraulic lifts	277
4.3	Fluids in motion	283
4.3.1	Total molecular stress	284
4.3.1.1	Stress tensor	286
4.3.1.2	Stress sign convention	298
4.3.2	Isotropic and anisotropic stress	302
4.4	Free-surface stress effects	320
4.5	Problems	333
<b>5</b>	<b>Stress-Velocity Relationships</b>	<b>346</b>
5.1	Simple shear flow	348
5.1.1	Velocity field	350
5.1.2	Stress field	351
5.1.3	Viscosity	360
5.2	Newtonian fluids	364
5.2.1	The constitutive equation	369
5.2.2	Using the constitutive equation	379

5.3	Non-Newtonian fluids	393
5.3.1	Non-Newtonian viscosity	394
5.3.2	Shear-induced normal stresses	397
5.3.3	Inelastic constitutive equations	402
5.3.4	Viscoelastic constitutive equations	414
5.4	Summary	418
5.5	Problems	418

### **PART III FLOW FIELD CALCULATIONS**

#### **6 Microscopic Balance Equations** 429

---

6.1	Deriving the microscopic balance equations	430
6.1.1	Gauss-Ostrogradskii divergence theorem	432
6.1.2	Mass balance	433
6.1.3	Momentum balance	438
6.1.3.1	General fluids	438
6.1.3.2	Newtonian fluids	441
6.1.4	Energy balance	442
6.2	Using microscopic-balance equations	445
6.2.1	Solution methodology	446
6.2.1.1	The equations	447
6.2.1.2	Applying the equations	452
6.2.2	Boundary conditions	464
6.2.3	Engineering quantities from velocity and stress fields	472
6.2.3.1	Total force on a wall	472
6.2.3.2	Torque	478
6.2.3.3	Flow rate and average velocity	481
6.2.3.4	Velocity and stress extrema	483
6.3	Summary	485
6.4	Problems	486

#### **7 Internal Flows** 494

---

7.1	Circular pipes	494
7.1.1	Laminar flow in pipes	497
7.1.2	Turbulent flow in pipes	511
7.1.2.1	Momentum balance in turbulent flow	517
7.1.2.2	Dimensional analysis	518
7.1.2.3	Data correlations	529
7.2	Noncircular conduits	540
7.2.1	Laminar flow in noncircular ducts	541
7.2.1.1	Poisson equation	541
7.2.1.2	Poiseuille number and hydraulic diameter	554
7.2.2	Turbulent flow in noncircular ducts	570
7.3	More complex internal flows	572
7.3.1	Unsteady-state solutions	573
7.3.2	Quasi-steady-state solutions	577



7.3.3	Geometrically complex flows (including lubrication approximation, converging flows, and entry flows)	580
7.4	Problems	585
<b>8</b>	<b>External Flows</b>	<b>600</b>
8.1	Flow around a sphere	601
8.1.1	Creeping flow around a sphere	604
8.1.2	Noncreeping flow around a sphere	622
8.1.2.1	Dimensional analysis of noncreeping flow	628
8.1.2.2	Flow patterns	647
8.1.2.3	Potential flow	650
8.2	Boundary layers	673
8.2.1	Laminar boundary layers	678
8.2.2	Turbulent boundary layers	696
8.2.3	Flow past blunt objects	705
8.3	More complex external flows	718
8.3.1	Vorticity	718
8.3.2	Dimensional analysis redux	726
8.4	Problems	733
<b>PART IV ADVANCED FLOW CALCULATIONS</b>		
<b>9</b>	<b>Macroscopic Balance Equations</b>	<b>741</b>
9.1	Deriving the macroscopic balance equations	741
9.1.1	Macroscopic mass-balance equation	742
9.1.2	Macroscopic momentum-balance equation	745
9.1.3	Energy balance	750
9.1.3.1	Closed systems	751
9.1.3.2	Open systems	753
9.1.3.3	Mechanical energy balance	759
9.2	Using the macroscopic balance equations	766
9.2.1	Pressure-measurement devices	769
9.2.2	Flow-rate-measurement devices	772
9.2.3	Valves and fittings	779
9.2.4	Pumps	800
9.2.4.1	Pump sizing	801
9.2.4.2	Net positive suction head	814
9.2.5	Open-channel flow	823
9.3	Problems	830
<b>10</b>	<b>How Fluids Behave (Redux)</b>	<b>838</b>
10.1	Viscosity, drag, and boundary layers	838
10.2	Numerical solution methods	840
10.2.1	Strategy	840
10.2.2	Software packages	842
10.2.3	Accuracy	843
10.3	Laminar flow, turbulent flow	845