FAITH A. MORRISON

AN INTRODUCTION TO FLUID MECHANICS

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press
32 Avenue of the Americas, New York, NY 10013-2473, USA

www.cambridge.org
Information on this title: www.cambridge.org/9781107003538

First published 2013

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication data

Morrison, Faith A.
An introduction to fluid mechanics / Faith A. Morrison.
p. cm.
ISBN 978-1-107-00353-8 (hardback)
1. Fluid mechanics. I. Title.
QA901.M67 2012
532–dc23 2011049511

ISBN 978-1-107-00353-8 Hardback

Additional resources for this publication at www.cambridge.org/us/knowledge/isbn/item6684157/.

Cover photo: The Naruto Whirlpools, Japan, as seen from a tourist cruise boat. Photo taken by Hellbuny.

Equations Summary

Unit-conversions: www.chem.mtu.edu/~fmorriso/convert.pdf.

Mechanical Energy Balance

$$\frac{\Delta p}{\rho} + \frac{\Delta \langle v \rangle^2}{2\alpha} + g\Delta z + F_{friction} = -\frac{W_{s,by\ fluid}}{m} \qquad \begin{cases} \alpha_{laminar} = 0.5 \\ \alpha_{turbulent} \approx 1 \end{cases}$$

$$F_{friction} = \left[4f\frac{L}{D} + \sum_{fittings_i} n_i K_{f,i} \right] \frac{\langle v \rangle^2}{2}$$

Fanning Friction Factor (pipe flow)

$$f = \frac{\mathcal{F}_{\text{drag}}}{\frac{1}{2}\rho \langle v \rangle^2 \pi R^2} = \frac{\Delta p D}{2L\rho \langle v \rangle^2}$$

Drag Coefficient (sphere drop)

$$C_D = \frac{\mathcal{F}_{\text{drag}}}{\frac{1}{2}\rho v_\infty^2 \pi R^2} = \frac{4gD\left(\rho_{\text{body}} - \rho\right)}{3\rho v_\infty^2}$$

Momentum Balance on a CV (Reynolds transport theorem)

$$\frac{d\underline{\mathbf{P}}}{dt} + \iint_{CS} (\hat{n} \cdot \underline{v})\, \rho \underline{v}\, dS = \sum_{\substack{\text{on} \\ \text{CV}}} \underline{f}$$

Hydrostatic Pressure $\quad p_{bottom} = p_{top} + \rho g h$

Hagen-Poiseuille Equation (steady, laminar tube flow, incompressible)

$$Q = \frac{\pi (p_0 - p_L) R^4}{8\mu L}$$

Prandtl Equation (steady, turbulent tube flow)

$$\frac{1}{\sqrt{f}} = -4.0 \log\left(\frac{4.67}{\text{Re}\sqrt{f}}\right) + 2.28$$

Stokes-Einstein-Sutherland Equation (steady, slow flow around a sphere)

$$\mathcal{F}_{\text{drag}} = 6\pi R \mu v_\infty$$

Macroscopic Momentum Balance on a CV

$$\frac{d\underline{\mathbf{P}}}{dt} + \sum_{i=1}^{\#streams} \left[\frac{\rho A \cos\theta \langle v \rangle^2}{\beta}\, \hat{v} \right] \Bigg|_{A_i}$$

$$= \sum_{i=1}^{\#streams} \left[-pA\hat{n} \right]_{A_i} + \underline{R} + M_{CV}\underline{g} \qquad \begin{cases} \beta_{laminar} = 0.75 \\ \beta_{turbulent} \approx 1 \end{cases}$$

Navier-Stokes Equation (microscopic momentum balance, incompressible, Newtonian fluids)

$$\rho \left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} \right) = -\nabla p + \mu \nabla^2 \underline{v} + \rho \underline{g}$$

Continuity Equation (microscopic mass balance, incompressible fluids)

$$\nabla \cdot \underline{v} = 0$$

Total Stress Tensor $\underline{\underline{\tilde{\Pi}}} = -p\underline{\underline{I}} + \underline{\underline{\tilde{\tau}}}$

$$\begin{pmatrix} \tilde{\Pi}_{11} & \tilde{\Pi}_{12} & \tilde{\Pi}_{13} \\ \tilde{\Pi}_{21} & \tilde{\Pi}_{22} & \tilde{\Pi}_{23} \\ \tilde{\Pi}_{31} & \tilde{\Pi}_{32} & \tilde{\Pi}_{33} \end{pmatrix}_{123} = \begin{pmatrix} \tilde{\tau}_{11} - p & \tilde{\tau}_{12} & \tilde{\tau}_{13} \\ \tilde{\tau}_{21} & \tilde{\tau}_{22} - p & \tilde{\tau}_{23} \\ \tilde{\tau}_{31} & \tilde{\tau}_{32} & \tilde{\tau}_{33} - p \end{pmatrix}_{123}$$

Dynamic Pressure $\mathcal{P} \equiv p + \rho g h$

Newtonian Constitutive Equation

$$\underline{\underline{\tilde{\tau}}} = \mu \left(\nabla \underline{v} + (\nabla \underline{v})^T \right)$$

$$= \mu \begin{pmatrix} 2\frac{\partial v_1}{\partial x_1} & \frac{\partial v_2}{\partial x_1} + \frac{\partial v_1}{\partial x_2} & \frac{\partial v_3}{\partial x_1} + \frac{\partial v_1}{\partial x_3} \\ \frac{\partial v_2}{\partial x_1} + \frac{\partial v_1}{\partial x_2} & 2\frac{\partial v_2}{\partial x_2} & \frac{\partial v_2}{\partial x_3} + \frac{\partial v_3}{\partial x_2} \\ \frac{\partial v_3}{\partial x_1} + \frac{\partial v_1}{\partial x_3} & \frac{\partial v_2}{\partial x_3} + \frac{\partial v_3}{\partial x_2} & 2\frac{\partial v_3}{\partial x_3} \end{pmatrix}_{123}$$

Total Molecular Fluid Force on a Finite Surface $\mathcal{S}$

$$\underline{\mathcal{F}} = \iint_{\mathcal{S}} [\hat{n} \cdot \underline{\underline{\tilde{\Pi}}}]_{\text{at surface}} \, dS$$

Stationary Fluid $[\hat{n} \cdot \underline{\underline{\tilde{\Pi}}}] = -p\hat{n}$

Moving Fluid $[\hat{n} \cdot \underline{\underline{\tilde{\Pi}}}] = -p\hat{n} + \hat{n} \cdot \underline{\underline{\tilde{\tau}}}$

Total Fluid Torque on a Finite Surface $\mathcal{S}$

$$\underline{\mathcal{T}} = \iint_{\mathcal{S}} [\underline{R} \times (\hat{n} \cdot \underline{\underline{\tilde{\Pi}}})]_{\text{at surface}} \, dS$$

Total Flow Rate Out Through a Finite Surface $\mathcal{S}$

$$Q = \dot{V} = \iint_{\mathcal{S}} [\hat{n} \cdot \underline{v}]_{\text{at surface}} \, dS$$

Average Velocity Across a Finite Surface $\mathcal{S}$

$$\langle v \rangle = \frac{Q}{\mathcal{S}}$$

AN INTRODUCTION TO FLUID MECHANICS

This is a modern and elegant introduction to engineering fluid mechanics enriched with numerous examples, exercises, and applications. The goal of this textbook is to introduce the reader to the analysis of flows using the laws of physics and the language of mathematics. The approach is rigorous, but mindful of the student. Emphasis is on building engagement, competency, and problem-solving confidence that extends beyond a first fluids course.

This text delves deeply into the mathematical analysis of flows, because knowledge of the patterns fluids form and why they are formed and the stresses fluids generate and why they are generated is essential to designing and optimizing modern systems and devices. Inventions such as helicopters and lab-on-a-chip reactors would never have been designed without the insight brought by mathematical models.

Faith A. Morrison is Professor of Chemical Engineering at Michigan Technological University, where she has taught for 22 years. Morrison's expertise is in polymer rheology, in particular focusing on materials with structure, including high-molecular-weight polymers, block copolymers, hydrogels, and composites. She is the Past President of the Society of Rheology and Editor of the *Rheology Bulletin*. Morrison is the author of *Understanding Rheology* (2001).

This book is dedicated to my mother Frances P. Morrison,
my father Philip W. Morrison, and my elder brother
Professor Philip W. Morrison, Jr.

Preface

This book forms the basis of a one-semester introductory course in fluid mechanics for engineers and scientists. Students working with this text are expected to have a background in multivariable calculus, linear algebra, and differential equations; review of these topics as applied to fluid mechanics is provided in Chapter 1. Problem solving is taught by example throughout the text. We include numerous solved examples and end-of-chapter problems, and a complete solution manual is available for instructors.

Fluid mechanics can be a difficult subject. Nonlinear physics governs flow, and thus we often resort to a variety of simplifications to obtain solutions. Different simplifications are used under different conditions, making fluid mechanics intimidating, at least to a beginner. *An Introduction to Fluid Mechanics* presents the topic through a discovery process, as described in this preface, that mimics engineering practice. The process used seeks solutions by answering the following questions:

1. *What is the problem?*
2. *What do we need to know, and do, to address the problem?*
3. *What is the solution to the problem?*
4. *What other problems/opportunities may be addressed now that we have solved this problem?*

This organizational choice builds critical thinking skills by emphasizing the thought processes that lead to model development. The book is divided into four parts that answer these four questions for the study of fluid mechanics.

1. **What is the problem?** *[Part I: Preparing to Study Flow]*
   Chapter 1: Why Study Fluid Mechanics
   Chapter 2: How Fluids Behave

The problem addressed in this book is how to bring readers to an understanding of flow behavior and to mastery of flow-modeling calculations. To accomplish this objective, students must come to the task with skills in mathematics and simple flow calculations. In Chapter 1 we introduce the problem, cover needed background calculations (i.e., the macroscopic mass balance and the mechanical engineering balance), and review mathematics that is prerequisite to the study of fluid mechanics (i.e., calculus and differential equations). In Chapter 2, we showcase the diversity and complexity of fluid behaviors—showing readers that the mechanical energy balance is insufficient to explain flow patterns and making the case that effort spent learning fluid mechanics is worth it. The presentation in Chapter 2 is at the survey level and spans from the introduction of viscosity to discussions of magnetohydrodynamics and vorticity. Overall, the text follows a path inspired by the spiral learning curve [Bruner, 1966], with the topics of Chapter 2 revisited at the end of the book (*Chapter 10: How Fluids Behave* (*Redux*)). That final chapter demonstrates how the intervening presentation leads to the ability to solve complex flow problems.

2. **What do we need to know, and do, to address the problem?** [*Part II: The Physics of Flow*]
   Chapter 3: Modeling Fluids
   Chapter 4: Molecular Fluid Stresses
   Chapter 5: Stress-Velocity Relationships

Having clarified our objectives in Part I, we seek methods to address the objectives in Part II. The continuum and the control volume are introduced in Chapter 3, and the stress components, fluid statics, and surface tension are presented in Chapter 4. To apply momentum conservation to a continuum, we need the stress constitutive equations, developed in Chapter 5 (Newtonian and non-Newtonian). These three chapters introduce the complete continuum model.

It can be a challenge to maintain student focus when covering background material, and we address this issue in a unique way: we provide a storyline. At the end of Chapter 3 we introduce two flow calculations and follow them longitudinally throughout Part II. These two problems (flow down an incline plane and flow in a 90-degree bend) are addressed in a just-in-time format, beginning before readers know enough fluid mechanics to be able to solve them. The solution develops gradually, incorporating new model pieces as they are covered. The repeated appearance of the two highlighted problems focuses readers on new developments, demonstrating the utility of the most recent step. Both highlighted problems are completed in Chapter 5, and Part II closes with the continuum model in place.

3. **What is the solution to the problem?** [*Part III: Flow Field Calculations*]
   Chapter 6: Microscopic Balance Equations
   Chapter 7: Internal Flows
   Chapter 8: External Flows

Model in hand, we turn to flows of interest. In Chapter 6 we develop the microscopic momentum balance (i.e., the Navier-Stokes equation), which represents an adaptation of the methods of Part II to the general case. We introduce the expressions for flow rates, fluid forces on walls, and fluid torques and show how to use these. In Chapter 7 a range of internal flows is discussed (pipes and ducts); in Chapter 8 external flows and boundary-layer flows are presented in detail (drag and lift).

The reader's path through Chapters 7 and 8 follows once again a storyline of a pair of highlighted flow problems. Chapter 7 begins with the quest to determine the extent of a home flood. Although not transparently related to the continuum model, the home flood problem is readily associated with pipe flow and motivates the examination of pressure drop/flow rate relationships, laminar and turbulent flow, and other internal-flow topics. We repeat this structure in Chapter 8, asking about a skydiver, which raises the question of flow past an obstacle in general, leading to discussion of drag, lift, and boundary layers.

Throughout Part III we employ dimensional analysis when the models we develop are too difficult to solve. Dimensional analysis is presented as a natural step in a problem-solving methodology that begins with addressing simplified versions of a real problem (because those are the problems we can solve and they give us insight), progresses to solving mathematically complex models, and turns ultimately to obtaining practical data correlations.

4. **What other problems/opportunities may be addressed now that we have solved this problem?** [*Part IV: Advanced Flow Calculations*]
   Chapter 9: Macroscopic Balance Equations
   Chapter 10: How Fluids Behave (*Redux*)

The final two chapters of *An Introduction to Fluid Mechanics* guide readers through advanced modeling calculations on a variety of flows. In Chapter 9 the macroscopic balances, including the mechanical energy balance and the macroscopic momentum balance, are derived and applied. Although simple uses of the mechanical energy balance are covered in Chapter 1, in Chapter 9 the applications are more involved, including pump sizing and open-channel flow. Applying the macroscopic momentum balance is generally considered to be a difficult topic; we systemize macroscopic momentum solutions, making them more accessible. In Chapter 10, the learning spiral returns us to the more complex flows introduced in Chapter 2, and we apply the now-familiar continuum model to begin to understand these flows. Chapter 10 discusses numerical solutions, statistical aspects of turbulence, lift, circulation, vorticity, and supersonic flow.

The text includes reference materials provided to aid the student. The appendices contain a glossary of terms and mathematical tables. There is additional mathematical assistance available on the Internet in the Web Appendix. Finally, key equations are presented on the inside covers as an aid to problem solving.

REFERENCE

Bruner, Jerome S., *The Process of Education* (Harvard University Press: Cambridge, MA, 1966).

Acknowledgments

My path to choosing this presentation method for my fluids class began in 1998 when I was first asked to teach fluid mechanics. I looked at the texts available, and, given the goals of both my course and my students, I had difficulty choosing a text. Although I did not find a book that satisfied my needs, I did find notes from a colleague, Professor Davis W. Hubbard, that got me started in the right direction. Professor Hubbard passed away in 1994, before this text was conceived, but his contribution to pedagogy lives on through his influence on this book.

I would like to thank many colleagues, friends, and family members for their assistance, encouragement, and support during the time spent working on this project. A partial list includes Tomas Co, Susan Muller, Scott Chesna, Denise Lorson, Pushpalatha Murthy, Madhukar Vable, Frances Morrison, Rosa Co, Tommy Co, and my colleagues and students in the Department of Chemical Engineering at Michigan Technological University and in the Society of Rheology.

In 2005–6 I spent a sabbatical year at Korea University in Seoul, Korea, teaching and working on this text. Many thanks to my hosts and colleagues for their welcome and for creating such a productive atmosphere in which to work. I would like to thank particularly Jae Chun Hyun, Chongyoup Kim, Joung Sook Hong, Jun Hee Sung, Kwan Young Lee, Jae Sung Lee (Postech), and my students in CBE614 Rheology, especially Yang Soo Son, Wun-gwi Kim, and Seoung Hyun Park. Final edits on this manuscript were prepared during another sabbatical year in 2012–13 as the William R. Kenan, Jr., Visiting Professor for Distinguished Teaching at Princeton University. I would like to thank my hosts at Princeton for this opportunity, especially Robert K. Prud'homme and Richard Register.

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