



EMIL WOLF

EDITOR



PROGRESS IN OPTICS

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PREFACE

This volume presents four articles dealing with topics of considerable current research activities. The first article by C. Aegerter and G. Maret is concerned with localization of classical waves by strong multiple scattering, with emphasis on propagation of visible light in optically turbid media. When the multiple scattering is weak, there is a twofold enhancement of the intensity of the scattered light in the back direction, an effect known as coherent backscattering. The origin of this effect is discussed, as well as experimental investigations of multiple scattering of light in various media.

The second article by Y. V. Kartashov, V. A. Vysloukh, and L. Torner gives an account of recent theoretical and experimental investigations concerning soliton manipulation of lattices. Optical lattices make it possible to control diffraction of light beams in media with periodically-modulated optical properties to control reflection and transmission bands. This leads to a rich variety of new families of nonlinear stationary waves and solitons, offering novel opportunity for all-optical shaping, switching and transmitting of optical signals. This technique offers new possibilities of producing non-diffracting light patterns.

The article which follows, by P. Gallion, F. Mendieta and S. Jiang, deals with basic quantum noise manifestations in optical amplification, optical direct detection and coherent detection systems. Applications to optical communications and quantum cryptography are also discussed.

The concluding article by M. Yan, W. Yan, and M. Qiu entitled Invisibility Cloaking by Coordinate Transformation explains how recent developments of new optical materials make it possible to produce perfect invisibility cloaks. A review of recent theoretical and experimental researchers for producing such cloaks is also given.

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Coherent Backscattering and Anderson Localization of Light

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1. INTRODUCTION

Most of the time, we obtain information on an object by looking at it, that is, we exploit the light that is scattered from it. The spectral and angular distribution of the backscattered (and reflected) light gives us information about the nature of the particles making up the object. For instance, the reddish color of copper is determined by the absorption

properties (in the green) of the d electrons in the partially filled shell. On the other hand, the blue color of the sky is well known to originate from the scattering properties of the air molecules, which follows Rayleigh scattering with a cross-section proportional to $1/\lambda^4$. This tells us that the molecules are much smaller than the wavelength of light. In fact, a more thorough analysis allows a characterization of the density fluctuations of the air from the scattering properties of the sky. As a final example, we mention the ‘Glory’, the halo sometimes seen around the shadow of an airplane on clouds it is flying over, which will be discussed further below. In the following, we will be concerned with instances of such enhanced backscattering in nature, where the intensity is enhanced in the direction of backscattering. As we will see below, one such effect is due to the interference of multiple scattering paths in disordered media like clouds, milk or white paint. Due to the reciprocity of light propagation, such paths will always have a counterpart of exactly the same length, which implies that they will always interfere constructively in the backward direction.

We will also discuss how this effect can lead to a marked change in the transport behaviour of the light waves in a disordered system, where diffuse transport comes to a halt completely. This transition is known as Anderson localization, and has been of great influence in the development of the theory of electrons in metals and condensed-matter physics. However, as will be seen in the discussion of backscattering enhancement below, the effect is also present in classical waves such as light, and there have been great efforts to try and experimentally observe the transition to Anderson localization of light.

In the rest of the introduction, we will discuss the different instances of enhanced backscattering in nature and their possible connection to coherent backscattering. Then we will discuss the connection of coherent backscattering to Anderson localization in more detail, before discussing the main predictions of Anderson localization in order to guide the experimental search for the effect.

Section 2 will return to coherent backscattering and will discuss in detail the different experimental observations connected to recurrent scattering, the influence of absorption and finite size of the medium, as well as the problem of energy conservation. In that section we will also discuss other instances of coherent backscattering, that is, with light scattered by cold atoms as well as with waves other than light.

In Section 3 we will discuss the quest for Anderson localization of light, describing the different experimental approaches used in the past, as well as their advantages and disadvantages. Finally, we will concentrate on our studies of time-resolved transmission and the corresponding determination of critical exponents of Anderson localization of light.

1.1 Instances of Enhanced Backscattering

As first realized by Descartes (1637), the rainbow is an enhancement in intensity (different for different colors due to dispersion) due to refraction of light in the rain drops, which, due to the dispersion of water, is highest at different angles for different colors. However, this is a purely geometric effect, which does not yield information on the size of the rain drops reflecting the light. Something akin to a rainbow can be seen when flying in an airplane over an overcast sky. When the sun is low and the cloud cover not too thick, one can see a beautiful halo around the shadow of the plane on the clouds. The effect is also well-known to alpinists who can observe this halo around their own shadow on a day that is hazy in the valley. In contrast to what one might think, this 'Glory' as it is called, is not in fact a rainbow. One can see this for instance by considering the angle of this colorful enhancement, which is usually only a few degrees and hence much smaller than the 42° corresponding to a rainbow. Therefore another mechanism has to be at work. It has been shown that the size of the scattering droplets influences the angle of the glory (Bryant and Jarmie, 1974). It turns out that this is due to the Mie-scattering properties (Mie, 1908) of the droplets. With a typical size of $10\text{ }\mu\text{m}$, the droplets in a cloud are large compared to the wavelength of light. Furthermore, as illustrated by experiments on a levitating droplet of water, Glory is the property of a single drop (Lenke, Mack and Maret, 2002).

Enhanced backscattering is also commonly observed in forests, where the leaves of dew-covered trees, or the blades of dew-covered grass, have a halo. This effect is called sylvanshine (see e.g. Fraser (1994)) and is due to the focusing action of the droplet on the reflecting surface of the leaf. By the same principle, the diffuse reflection from the leaf is channeled back through the lens (i.e. the drop) which decreases the angle of reflection. Hence the leaves or the grass blades are brighter than the background. The grass does not even need to be dew-covered to observe a halo, as there is an additional effect increasing the intensity in the direct backscattering direction. Exactly opposite to the incidence, any ensemble of rough objects will be brightest. This is because in this direction, we see the reflected light directly and none is lost due to shadows of other objects (Fraser, 1994). This is known as the corn-field effect.

As a final instance of enhanced backscattering, let us mention observation of the intensity of objects in the solar system, such as the moon or other satellites of planets, when the earth and the sun are in opposition to the moon. In that case, it was observed by Gehrels (1956) for the moon and subsequently for many other satellites (Oetking, 1966) that the intensity of the satellite is in fact increased over its usual value. Due to the arrangement of sun and satellite when the effect is observed, this was called the 'opposition effect'. In this effect, coherent backscattering

as we will discuss below, works in concert with analogues of the effects described above, such as the corn-field effect. The presence of coherent backscattering in the opposition effect was discovered (Hapke, Nelson and Smythe, 1993). With this knowledge it was then possible to actually study the surface properties (e.g. the granularity) of these satellites from remote observations.

1.2 Coherent Backscattering

Among instances of enhanced backscattering, here we will be concerned mostly with coherent backscattering, an interference effect that survives all averages in a random medium. Fundamentally, the enhancement is due to the fact that, because of time-reversal symmetry, every path through a random medium has a counterpropagating partner. Light elastically scattered on these two paths interferes constructively, because the path-lengths are necessarily the same. This leads to an enhancement of exactly a factor of two in the direction directly opposite to the incidence. In contrast to Glory or other effects discussed above (Lenke, Mack and Maret, 2002), coherent backscattering is not an interference due to the properties of a single scatterer, but relies fundamentally on multiple scattering. In fact, in the single-scattering regime there cannot be a coherent backscattering cone as there cannot be a counterpropagating light path. The entry- and exit-points of a multiple-scattering path can then be seen as the two points of a double slit, which, due to the coherence of the time-reversed paths, necessarily interfere with each other. The different interference patterns corresponding to different light paths in the disordered medium have to be averaged over, which will lead to the shape of the backscattering cone discussed in Section 1.3 below. What can be seen from this picture is that in the exact backscattering direction, the averaging will always lead to an enhancement factor of two.

These principles behind the origin of the backscattering cone will strongly influence the transport through a random system. Taking the end points of the counterpropagating paths to coincide somewhere inside the sample, there will be a two-fold enhancement at this point on such a closed loop. This in turn leads to a decreased probability of transport through the system. This effect is what causes Anderson localization of light (Anderson, 1958), i.e. the loss of diffuse transport due to increasing disorder. As disorder increases, the probability of forming closed loops on which intensity is enhanced increases. At a certain critical amount of disorder, these closed loops start to be macroscopically populated, which leads to a loss of diffuse transport. This critical amount of disorder has been estimated using dimensional arguments by Ioffe and Regel (1960) to be when the mean free path roughly equals the inverse wavenumber, i.e. when $kl^* \sim 1$. Such a mechanism was first proposed for the transport of

electrons in metals, where it was found that an increase in disorder can turn a metal into an insulator (see e.g. Bergmann (1984)).

Historically, the first instances of localization were discussed in the context of electron transport in metals, and thus localization was thought to be a quantum effect. Moreover, due to the fact that localization should always be present in two dimensions (see scaling theory below) and is not influenced too much by the presence of correlations, these studies were carried out in thin films. A review of these experiments can be found in Altshuler and Lee (1988) and Bergmann (1984) and these studies of localization in lower dimensions have had a big influence on the study of other quantum effects in low-dimensional electron systems, such as the quantum Hall effect (Klitzing, Dorda and Pepper, 1980; Laughlin, 1983).

Eventually however, it was realized that the quantum nature of electrons is not a necessary ingredient for the occurrence of Anderson localization as, in fact, this is purely a wave effect. Thus, it should also be possible to observe localization effects with classical waves, such as light, as was proposed by John (1984) and Anderson (1985). As we will see below, coherent backscattering, that is, weak localization, was observed with light shortly thereafter; subsequently, there was a vigorous programme to also observe signs of strong localization of light, because the study of photon transport in disordered media has many advantages over the study of electrons in metals. This is because in the latter case there are alternatives that may also lead to localization: in the case of electrons, a random potential can lead to a trapping of particles, which also strongly affects transport, while not being connected to localization. On the other hand, electrons also interact with each other via Coulomb interaction, so that correlations in electron transport are again not necessarily due to localization effects, but may more likely be explained by electron–electron interactions. In fact, it can be shown that in the presence of particle interactions, the effects of localization vanish (Lee and Ramakrishnan, 1985).

However, as we will discuss below, the photonic system is not completely free either of possible artifacts masking as localization. For instance, light will be absorbed by materials to a certain extent, which leads to a loss of energy transport similar to localization. Furthermore, resonant scattering can lead to a time delay in the scattering process, which leads to a slowing down of transport, which again may be mistaken for localization. In Section 3 we will discuss in detail how these possible artifacts can be circumvented and localization can in fact be observed.

1.3 Theoretical Predictions

As discussed above, the enhanced backscattering from turbid samples, known as coherent backscattering, is a manifestation of weak localization

of light. Localization has been studied intensely in electronic systems, and many of the predictions found there can be applied also to optics. Here we will discuss the most important predictions, which will also serve as a guiding line in the quest to observe Anderson localization of light. Most prominent in these are the predictions of the change in static transmission (Anderson, 1985; John, 1984) which turned out to be difficult to observe experimentally due to the presence of absorption in real samples. The critical prediction for Anderson localization concerns the fact that there should be a phase transition to a state where diffusion comes to a halt. This is described by scaling theory (Abrahams, Anderson, Licciardello and Ramakrishnan, 1979), which can also be investigated by a self-consistent diagrammatic theory (Vollhardt and Wölfle, 1980). This version of the theory can also be extended to describe open systems with absorption, a situation much more suitable for experiments (Skipetrov and van Tiggelen, 2004, 2006). First of all, however, we will describe the shape of the backscattering cone as calculated by Akkermans, Wolf and Maynard (1986) and van der Mark, van Albada and Lagendijk (1988).

1.3.1 The Cone Shape

Given the nature of the backscattering cone due to interference of photons on time-reversed paths, one can explicitly calculate the shape of the enhancement as a function of angle. In order to do this, the interference patterns, corresponding to two counterpropagating paths with end-to-end distance ρ , need to be averaged weighted by the probability distribution of such an end-to-end distance occurring. Like in a double-slit experiment with slit separation ρ , each of these interference patterns will contribute a factor $1 + \cos(q\rho)$, such that the enhancement above the incoherent background is simply given by the real part of the Fourier transform of the end-to-end distance distribution:

$$\alpha(q) = \int p(\rho) \cdot \cos(q\rho) d\rho. \quad (1)$$

In the diffusion approximation, this probability distribution can be calculated (Akkermans, Wolf, Maynard and Maret, 1988; van der Mark, van Albada and Lagendijk, 1988) to be $1/a(1 - \rho/\sqrt{\rho^2 + a^2})$ in the case of a semi-infinite planar half-space. Here, the length scale $a = 4\gamma l^*$ describes how the diffuse intensity penetrates the sample as described by the Milne parameter γ and the transport mean free path l^* . The parameter γ can be calculated from the radiative transfer equation to be ~ 0.71 and in the diffusion approximation is exactly $\gamma = 2/3$. In the following, we will always use the value of $\gamma = 2/3$. This leads to the following expression

for the backscattering enhancement:

$$\alpha(q) = \int \left(1 - \frac{\rho}{\sqrt{\rho^2 + a^2}} \right) \cdot \cos(q\rho) \, d\rho, \quad (2)$$

which can be solved to give (Akkermans, Wolf and Maynard, 1986; Akkermans, Wolf, Maynard and Maret, 1988; van der Mark, van Albada and Lagendijk, 1988):

$$\alpha(q) = \frac{3/7}{(1 + ql^*)^2} \left(1 + \frac{1 - \exp(-4/3ql^*)}{ql^*} \right). \quad (3)$$

This gives a cone shape in very good agreement with the experiments that will be discussed in Section 2. As can be seen from an investigation of the angle dependence, the cone tip is triangular with an enhancement of 1 in the exact backscattering direction. The enhancement then falls off on an angular scale proportional to $1/(kl^*)$; in fact the full width at half maximum of the curve is given by $0.75/(kl^*)$. Thus the investigation of the backscattering cone is a very efficient method of determining the turbidity of a sample as given by $1/l^*$.

A similar description following diagrammatic theory, where the most crossed diagrams have to be added up, was given by Tsang and Ishimaru (1984). The main features of the curve remain the same, however the different theories use different approximations for the Milne parameter.

1.3.2 Static Transmission

One of the main predictions of Anderson localization in electronic systems is the transition from a conducting to an insulating state. This of course has strong implications for the transmission properties of localized and non-localized samples. For a conducting sample, the transmission is described by Ohm's law, which describes diffusive transport of particles and hence a decrease of transmission with sample thickness as $1/L$. This is also the case in turbid optical samples, where the transmission in the diffuse regime is simply given by $T(L) = T_0 l^*/L$ (see e.g. Akkermans and Montambaux (2006)). In the presence of absorption, this thickness dependence of the total transmission will change to an exponential decay for thick samples according to e.g. Genack (1987)

$$T(L) = T_0 \frac{l^*/L_a}{\sinh(L/L_a)}, \quad (4)$$

where $L_a = \sqrt{l^* l_a / 3}$ is the sample absorption length corresponding to an attenuation length l_a of the material, which describes the absorption of the light intensity along a random scattering path.

The localization of photons will similarly affect the transmission properties of a sample. As the diffusion coefficient of light becomes scale dependent close to the transition to localization, the total transmission will decrease. Scaling theory of localization, to be discussed below, predicts that the diffusion coefficient at the transition will decrease as $1/L$ (John, 1984, 1985, 1987). This should then be inserted into the expression for the diffuse transmission of the sample, resulting in a different thickness dependence $T(L) \propto 1/L^2$. Again, this ignores the effects of absorption, and Berkovits and Kaveh (1987) have calculated the effects of absorption in the presence of a renormalized diffusion coefficient, finding

$$T(L) = T_0 \exp(-1.5L/L_a). \quad (5)$$

Again, this leads to an exponential decrease of the transmitted intensity for very thick samples, where, however, the length scale of the exponential decrease has changed. When photons are fully localized, the transport is exponentially suppressed, as only the tails of the localized intensity can leave the sample. Thus Anderson (1985) has predicted the transmission in the localized case to be given by $T(L) = T_0 \exp(-L/L_{\text{loc}})$, where L_{loc} describes the length scale of localization. As was the case above, this derivation again does not take into account absorption, and a fuller description would be given by

$$T(L) = T_0 \frac{l^*/L_a}{\sinh(L/L_a)} \exp(-L/L_{\text{loc}}). \quad (6)$$

Again, this gives an exponential decrease of the transmitted intensity for thick samples with an adjusted length scale not solely given by the absorption length L_a . In an experimental investigation of Anderson localization therefore, static transmission measurements will have to find an exponential decrease of the transmission that is faster than that given by absorption alone. This implies that the absorption length must be determined independently for such an investigation to be able to indicate localization of light.

1.3.3 Scaling Theory

When studying the thickness dependence of the conductance (i.e. the transmission), its dependence on disorder has to be taken into account. Abrahams, Anderson, Licciardello and Ramakrishnan (1979) produced the first version of such a theory, where they introduce the 'dimensionless

conductance' g as the relevant parameter to study. In electronic systems, this simply is the measured conductance normalized by the quantum of conductance, e^2/h . In optics, the conductance is naturally dimensionless and can be defined simply via the transmission properties of the sample. In fact, g can be calculated in three dimensions from the ratio of the sample volume to that occupied by a multiple scattering path. This volume of the multiple scattering path is given by $\lambda^2 s$, where s is the length of the path, which in the case of diffusion is $s \propto L^2/l^*$. Thus one obtains $g \approx (W/L)(kW)(kl^*)$, where W is the width of the illumination, which could also be obtained from the static transmission discussed above. In the case of a localized sample, the transmission decreases exponentially with L , which has to be reflected in a renormalization of the path-lengths in order to give an exponentially decreasing g . The main ansatz of Abrahams, Anderson, Licciardello and Ramakrishnan (1979), in treating the problem of the localization transition in the following, is to suppose that the logarithmic derivative $\beta = d(\ln g)/d(\ln L)$ can be expressed as a function of g only.

The transition to a localized state is then given by the criterion that β changes from a positive value to a negative one. Ohm's law as a function of dimensionality states that the conductance scales as $g \propto L^{d-2}$. Therefore, making a sample larger and larger in low-dimensional systems will in fact lead to a reduction of the conductance and hence be associated with localization. Actually Ohm's law straightforwardly implies that $\beta = d - 2$ for large L (and thus g), such that $d = 2$ is the lower critical dimension for a transition to localization to occur. In fact, for low-dimensional systems the waves are always localized (Abrahams, Anderson, Licciardello and Ramakrishnan, 1979).

Where there is a transition to localization (i.e. in $d > 2$), more details about that transition can be obtained by assuming the dependence of β on g to be linear at the crossing of the null-line. In this case, the scaling function β describes how one arrives from a diffuse conductance to one which is exponentially suppressed in the localization length. This transition is a function of the disorder in the system, such that one can describe it in terms of a diverging length scale of localization at the transition. This would be given by an exponent ν , such that $L_{\text{loc}} \propto |(g - g_c)/g_c|^{-\nu}$. With the assumption that close to the transition, β can be approximated by a linear function in $\ln g$, this exponent is simply given by the inverse slope of β at the transition. In the framework of scaling theory, no exact value can be given for this exponent, however extrapolating β from its known dependencies at large and small disorder, Abrahams, Anderson, Licciardello and Ramakrishnan (1979) obtain an upper bound of $\nu < 1$. As a matter of fact, John (1984) has shown that expanding the treatment around the lower critical dimension, the exponent should be given by $\nu = 1/2$ in $d = 2 + \epsilon$