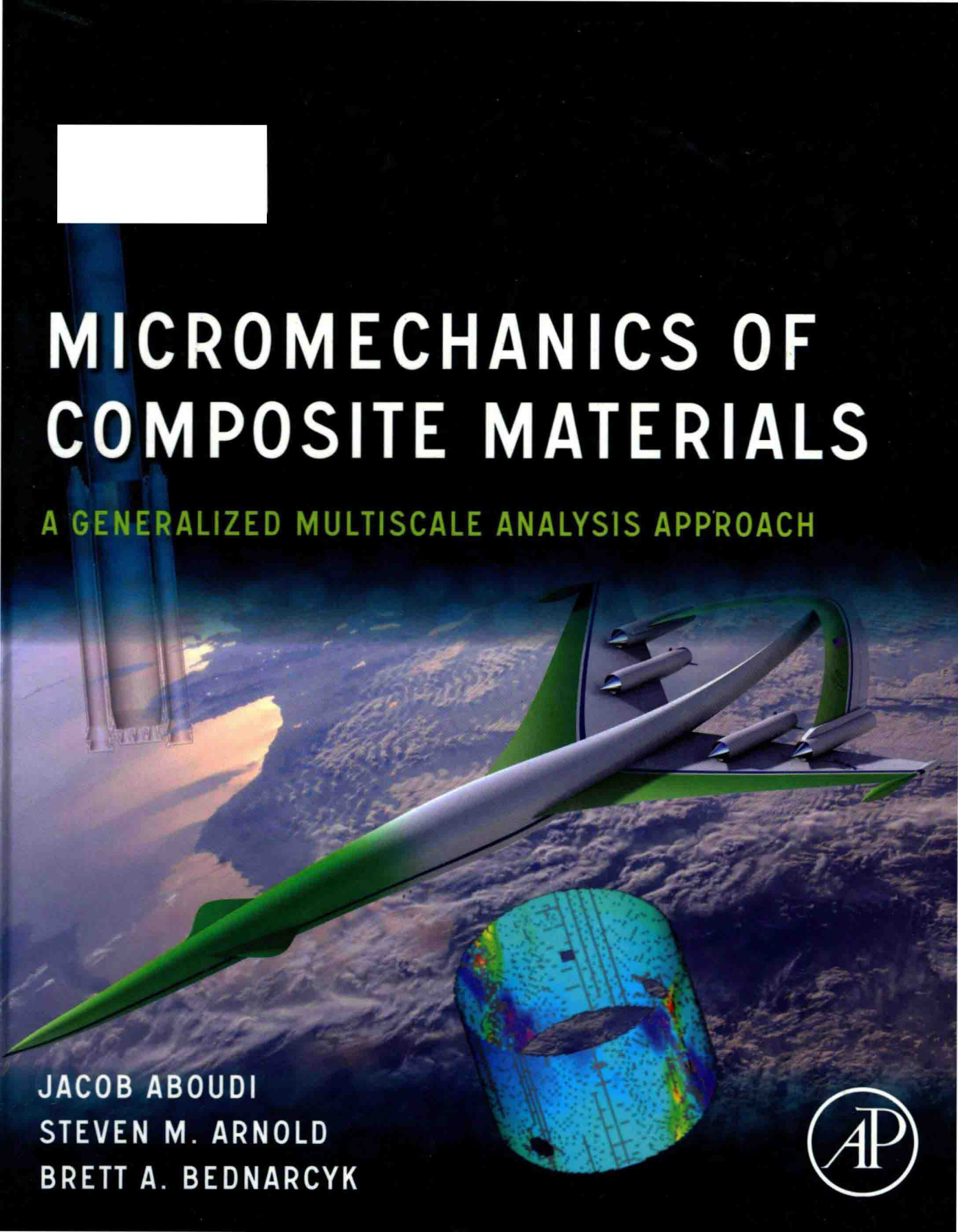


The background of the cover is a composite image. On the left, a rocket is shown launching with a large plume of fire. In the center, a green and white fighter jet is depicted in flight. On the right, a cylindrical object is shown with a colorful stress or strain analysis visualization, with red and yellow areas indicating high stress levels. The overall background is a dark, textured surface, possibly representing a composite material or a microscopic view.

MICROMECHANICS OF COMPOSITE MATERIALS

A GENERALIZED MULTISCALE ANALYSIS APPROACH

JACOB ABOUDI
STEVEN M. ARNOLD
BRETT A. BEDNARCYK



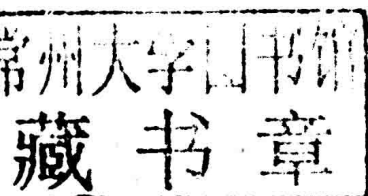
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A Generalized Multiscale Analysis Approach

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Micromechanics of Composite Materials

This book is dedicated to our families, with all of our love

*To my wife Ilana, who let me have peace of mind,
which enabled me to do what I like to do most.*

Jacob Aboudi

To my wife Debbie, and our children Graham, Leah, and Julianne.

Steven M. Arnold

To my wife Jennifer, and our daughter Adia.

Brett A. Bednarczyk

Preface

This book provides a detailed treatment of a unified family of micromechanics theories for multiphase materials developed by the authors over the past 30 years. These theories are applicable to composites with both periodic and nonperiodic (bounded) microstructures. A unique and important feature of these theories is their ability to provide not only the global effective composite properties, but also the varying local field distributions within the constituent materials. This capability enables the modeling of localized nonlinear phenomena such as damage and inelasticity, which are critical to the prediction of composite failure and life. In addition, because these theories can produce a macroscopic, nonlinear, anisotropic constitutive relation for the multiphase material, they are ideal for incorporation within multiscale analyses. Any higher scale method or model can therefore call these theories as an effective constitutive equation to obtain the local nonlinear response and to recover the local fields at any point within the composite structure. The resulting micro-macro-structural analysis capability is quite unique and is facilitated by the inherent computational efficiency of these micromechanics theories. Further, the nonperiodic versions of the micromechanics theories explicitly link the macro and micro scales, thus enabling concurrent analysis of problems where no repeating unit cell exists.

An additional unique feature of the unified micromechanics approach described herein is its ability to be readily extended to handle many technologically relevant aspects of advanced composite materials. These include composites (1) undergoing finite deformations, (2) subjected to dynamic impact conditions, (3) composed of smart (electro-magneto-thermo-elastic, electrostrictive, and shape memory alloy) constituents, and (4) exhibiting full (two-way) thermomechanical coupling. Thus, the authors believe that this book fills a void as most other books on composites emphasize the macromechanics approach and provide little treatment of nonlinearity in general and the above topics in particular.

The three of us wrote this book over the past several years, predominantly while the first author visited NASA Glenn Research Center in Cleveland, OH each year. We have attempted to highlight key lessons learned in developing and applying these theories over the past two

decades. Consequently, we hope that this unified multiscale approach will help provide materials scientists, researchers, engineers, and structural designers with a better understanding of composite mechanics at all scales, and thereby contribute to composites reaching their full potential. More related materials to this book could be found at the companion website: <http://booksite.elsevier.com/9780123970350/>. The password is “Solutions”.

Jacob Aboudi
Steve Arnold
Brett Bednarczyk
August 2012

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Without the hard work of many dedicated students and colleagues, the book you are reading would not exist. We extend a tremendous ‘Thank You’ to everyone who contributed to and helped us complete this book. At times, it certainly seemed like it would never reach this point.

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I, Jacob Aboudi, would like to thank Professor Isaac Elishakoff, who kept urging me to write a book on the micromechanical analysis of composites.

Finally, I, Steve Arnold, would like to thank my Lord and Savior, Jesus Christ, for providing me with the numerous opportunities throughout my career that have made this endeavor possible and most of all for bringing two such excellent gentlemen (Jacob and Brett) into my life, with whom I am proud to have been associated for so many years. I specifically want to thank Jacob for his mentorship and Brett for his attention to detail, but most of all for their friendship. They have made this specific journey, although intense at times, a very special and memorable time in my life, and I consider myself blessed to have been able to make it with them.

Acronyms

5HS	5-harness satin
AR	aspect ratio
BK	Benzeggagh-Kenane
BP	Bodner-Partom
CCA	concentric cylinder assemblage
CCI	constant compliant interface (model)
CDM	Continuum Damage Mechanics
CFRP	carbon fiber-reinforced polymers
CMC	ceramic matrix composite
CTE	coefficient of thermal expansion
CVI	chemical vapor infiltration
DCB	double cantilever beam
DS	differential scheme
EAM	element array model
ECI	evolving compliant interface (model)
ER	electrorheological
FCTM	fully coupled thermomicromechanical
FE	finite element
FEA	finite element analysis
FGM	functionally graded material
FI	flexible interface (model)
FS	facesheet
FSGMC	Finite Strain Generalized Method of Cells
FSHFGMC	Finite Strain High-Fidelity Generalized Method of Cells
GMC	Generalized Method of Cells
GMC-3D	triply periodic Generalized Method of Cells
gps	generalized plane strain
GSCS	generalized self-consistent scheme
GVIPS	generalized viscoplasticity with potential structure (model)
HFGMC	High-Fidelity Generalized Method of Cells
HOTCFGM	Higher-Order Theory for Cylindrical Functionally Graded Materials
HOTFGM	Higher-Order Theory for Functionally Graded Materials
HOTFGM-1D	one-directional Higher-Order Theory for Functionally Graded Materials
HOTFGM-2D	two-directional Higher-Order Theory for Functionally Graded Materials
H-S	Hashin-Shtrikman
ICME	integrated computational materials engineering
MAC/GMC	Micromechanics Analysis Code with Generalized Method of Cells (software)
MCCM	multiple concentric cylinder model

MD	molecular dynamics
MI	melt infiltrated
MKM	modified Kabelka model
MM	multiscale modeling
MMC	metal matrix composite
MMCDM	Mixed Mode Continuum Damage Mechanics
MMPM	modified mosaic parallel model
MOC	Method of Cells
MOC-TI	Method of Cells, transversely isotropic
MR	magnetorheological
MSGMC	Multiscale Generalized Method of Cells
MT	Mori-Tanaka (theory)
NDE	nondestructive evaluation
NI	Needleman Interface (model)
NLCDR	NonLinear Cumulative Damage Rule
ONERA	Office Nationale d'Études et de Recherches Aéronautiques
PLS	proportional limit stress
PMC	polymer matrix composite
PMN	lead magnesium niobate
ps	plane strain
PVDF	polyvinylidene fluoride
PZT	lead zirconium titanate
QLV	quasilinear viscoelasticity
RCS	representative cross-section
ROM	rule of mixtures
RUC	repeating unit cell
RVE	representative volume element
SAM	slice array model
SCDR	surface of constant dissipation rate
SCIP	surface of constant inelastic power
SCISR	surface of constant inelastic strain rate
SCS	self-consistent scheme
SIF	statistical interfacial failure (model)
SMA	shape memory alloy
SOM	strength of materials
SPL	sound pressure level
TBC	thermal barrier coating
TE	thermoelastic
TGVIPS	transversely isotropic GVIPS
TMC	thermomechanical coupling
TMC	titanium matrix composite
TRIP	transformation-induced plasticity
TRL	Technology Readiness Level
TVE	thermoviscoelastic
UTS	ultimate tensile strength
VCCT	virtual crack closure technique
VCM	variable constraint model
VE	viscoelastic
VFD	vanishing fiber diameter
WWFE	World-Wide Failure Exercise

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