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OCEAN WAVES

The Stochastic Approach



Michel K. Ochi

Paperback Re-issue

This book describes the stochastic method for ocean wave analysis. This method provides a route to predicting the characteristics of random ocean waves – information vital for the design and safe operation of ships and ocean structures.

Assuming a basic knowledge of probability theory, the book begins with a chapter describing the essential elements of wind-generated random seas from the stochastic point of view. The following three chapters introduce spectral analysis techniques, probabilistic predictions of wave amplitudes, wave height and periodicity. A further four chapters discuss sea severity, extreme sea state, the directional wave energy spreading in random seas and special wave events such as wave breaking and group phenomena. Finally the stochastic properties of non-Gaussian waves are presented. Useful appendices and an extensive reference list are included. Examples of practical applications of the theories presented can be found throughout the text.

This book will be suitable as a text for graduate students of naval, ocean and coastal engineering. It will also serve as a useful reference for research scientists and engineers working in this field.

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OCEAN WAVES

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PREFACE

This book is intended to provide uniform and concise information necessary to comprehend stochastic analysis and probabilistic prediction of wind-generated ocean waves.

Description and assessment of wind-generated ocean waves provide information vital for the design and operation of marine systems such as ships and ocean and coastal structures. Wind-generated seas continuously vary over a wide range of severity depending on geographical location, season, presence of tropical cyclones, etc. Furthermore, the wave profile in a given sea state is extremely irregular in time and space – any sense of regularity is totally absent, and thereby properties of waves cannot be readily defined on a wave-by-wave basis.

Characterization of the stochastic properties of ocean waves was first presented in the early 1950s; Neumann (1953), Pierson (1952, 1955), St Denis and Pierson (1953) introduced the stochastic approach for analysis of random seas, and Longuet-Higgins (1952) demonstrated the probabilistic estimation of random wave height. The four decades following the introduction of the stochastic prediction approach have seen phenomenal advances in the probabilistic analysis and prediction methodologies of random seas.

For the design of marine systems, information on the real world is required. Recent advances in technology permit the use of the probabilistic approach to estimate the responses of marine systems in a seaway, including extreme values, with reasonable accuracy. Such technology lends itself to application of the probabilistic approach as an integrated part of modern design technology in naval, ocean and coastal engineering.

In view of the growing need for more comprehensive advances in prediction methodologies and for application of the probabilistic approach in naval, ocean and coastal engineering, this book is designed as a text book at the graduate level and as a reference book for researchers and designers. The intent is to provide a thorough understanding of the modern concept of stochastic analysis and probabilistic prediction of wind-generated random seas. Specific efforts are made in this work to explain the basic principles supporting current prediction techniques and to provide practical applications of prediction methods.

Readers are expected to be familiar with basic probability theory and fundamental stochastic processes. For the readers' convenience, however, definitions, theorems and relevant formulae on probability and stochastic process theory used in the text are summarized in the appendixes without proof or derivation.

I am grateful to the College of Engineering, University of Florida, for granting me sabbatical leave to prepare this book. Significant progress was achieved toward its completion during this period of time. I would like to acknowledge the encouragement and support received from Professor Eatock Taylor of the University of Oxford during this undertaking. Thanks are also due to Professor Isobe of the Tokyo University who provided valuable suggestions on the section addressing directional wave spectra.

I am indebted to many learned scholars and researchers who directly or indirectly inspired me to study the stochastic analysis and probabilistic prediction of ocean waves. I thank those who sponsored my research which ultimately culminated in this book; in particular, Dr Silva, Office of Naval Research. Appreciation is extended to my graduate students; in particular, Drs C.H. Tsai, D.W.C. Wang, I.I. Sahinoglou, K. Ahn and Lieut. D.J. Robillard, US Navy, who through their dedicated project support had a significant influence on the final product. Finally, I would like to acknowledge the contribution of my wife, Margaret, who read the complete manuscript and provided valuable assistance with the editorial work.

CONTENTS

<i>Preface</i>	<i>xi</i>
1 Description of random seas	1
1.1 Stochastic concept as applied to ocean waves	1
1.1.1 Introduction	1
1.1.2 Ocean waves as a Gaussian random process	2
1.1.3 Random seas	4
1.2 Mathematical presentation of random waves	8
1.3 Stochastic prediction of wave characteristics	9
2 Spectral analysis	13
2.1 Spectral analysis of random waves	13
2.1.1 Fundamentals of stochastic processes	13
2.1.2 Auto-correlation function	15
2.1.3 Spectral density function (spectrum)	16
2.1.4 Wiener–Khinchine theorem	18
2.1.5 Spectral analysis of two wave records	19
2.1.6 Wave-number spectrum	22
2.1.7 Wave velocity and acceleration spectra	25
2.2 Characteristics of wave spectra	26
2.3 Wave spectral formulations	33
2.3.1 Pierson–Moskowitz spectrum	33
2.3.2 Two-parameter spectrum	35
2.3.3 Spectral formulation as a function of ω^{-5}	36
2.3.4 Six-parameter spectrum	39
2.3.5 JONSWAP spectrum	42
2.3.6 TMA spectrum	48
2.4 Modification of wave spectrum for moving systems	50
2.5 Higher-order spectral analysis	52
3 Wave amplitude and height	58
3.1 Introduction	58
3.2 Probability distribution of amplitudes with narrow-band spectrum	60
3.2.1 Derivation of probability density function	60
3.2.2 Wave envelope process	64

3.3	Probability distribution of wave maxima with non-narrow-band spectrum	66
3.4	Joint distribution of two wave amplitudes	73
3.5	Probability distribution of peak-to-trough excursions (wave height)	78
3.6	Significant wave height	81
3.7	Probability distribution of half-cycle excursions	84
3.8	Long-term wave height distribution	88
3.9	Statistical analysis of amplitude and height from wave records	91
3.9.1	Introduction	91
3.9.2	Maximum likelihood estimation	92
3.9.3	Estimation of Rayleigh distribution parameter from a small number of observations	95
3.9.4	Goodness-of-fit tests	99
4	Wave height and associated period	103
4.1	Introduction	103
4.2	Joint probability distribution of wave height and period	104
4.3	Joint probability distribution of positive maxima and time interval	110
4.4	Probability distribution of wave period	114
4.5	Joint probability distribution of wave height and direction of wave energy travel	118
5	Sea severity	123
5.1	Statistical presentation of sea severity	123
5.1.1	Probability distribution of significant wave height	123
5.1.2	Joint probability distribution of significant wave height and period	130
5.1.3	Time series analysis of sea state data	135
5.2	Hurricane-associated seas	137
5.2.1	Introduction	137
5.2.2	Sea severity measured during hurricanes	138
5.2.3	Wave spectra and wave height in hurricane-generated seas	141
6	Estimation of extreme wave height and sea state	149
6.1	Basic concept of extreme values	149
6.2	Probable and design extreme wave height	151
6.3	Estimation of extreme wave height and sea state from data	159
6.4	Extreme wave height in a non-stationary sea state	164
6.5	Asymptotic distributions of largest waves and sea states	165
6.5.1	Type I asymptotic extreme value distribution	167
6.5.2	Type III asymptotic extreme value distribution	170

7 Directional characteristics of random seas	175
7.1 Introduction	175
7.2 Principle of evaluation of directional wave spectra	176
7.2.1 Wave probe array	176
7.2.2 Floating buoys	180
7.2.3 Pressure and current meters	188
7.3 Analysis of directional energy spreading function	190
7.4 Estimation of directional energy spreading from data	196
7.4.1 Maximum likelihood method	196
7.4.2 Maximum entropy method	207
7.4.3 Application of a Bayesian method	212
7.5 Formulation of the wave energy spreading function	216
8 Special wave events	218
8.1 Breaking waves	218
8.1.1 Wave breaking criteria	218
8.1.2 Probability of occurrence of wave breaking	222
8.1.3 Energy loss resulting from wave breaking	229
8.2 Group waves	232
8.2.1 Introduction	232
8.2.2 Statistical properties through the envelope process approach	235
8.2.3 Statistical properties through the Markov chain approach	246
8.3 Freak waves	252
9 Non-Gaussian waves (waves in finite water depth)	255
9.1 Introduction	255
9.2 Probability distribution of non-Gaussian waves	257
9.2.1 Gram–Charlier series distribution	257
9.2.2 Distribution based on Stokes waves	265
9.2.3 Distribution based on the concept of nonlinear system	269
9.3 Probability distribution of peaks and troughs	275
9.4 Transformation from Gaussian to non-Gaussian waves	280
Appendix A. Fundamentals of probability theory	283
Appendix B. Fundamentals of stochastic process theory	294
Appendix C. Fourier transform and Hilbert transform	300
<i>References</i>	304
<i>Index</i>	317

I DESCRIPTION OF RANDOM SEAS

1.1 STOCHASTIC CONCEPT AS APPLIED TO OCEAN WAVES

1.1.1 Introduction

The profile of wind-generated waves observed in the ocean changes randomly with time; it is non-repeatable in time and space. In reality, both wave height (peak-to-trough excursions) and wave period vary randomly from one cycle to another. It is often observed that waves break when the wave steepness exceeds a certain limit. Furthermore, during the process of the wind-generated waves traveling from one location to another after a storm, waves of shorter length gradually lose their energy resulting in the wave profile becoming less irregular (this situation is called swell) than that observed during a storm.

A more distinct difference in the wave profile can be observed when the water depth becomes shallow. As an example, Figure 1.1 shows portions of wave profiles recorded in severe seas; one in deep water in the North Atlantic, the other in a nearshore area of water depth 2.1 m. As seen in Figure 1(a), positive and negative sides of the wave profile in deep water are, by and large, similar, while for waves in shallow water (Figure 1(b)), peaks are much sharper than troughs, and the order of magnitude of the peaks is different from that of the troughs.

As seen in the examples shown in Figure 1.1, evaluation of the properties of random waves is almost impossible on a wave-by-wave basis in the time domain. However, if we consider the randomly changing waves as a

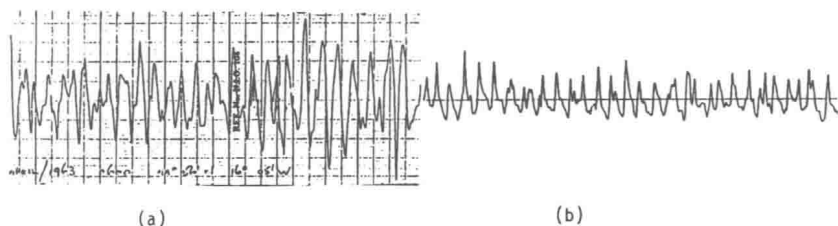


Fig. 1.1. Wave profiles in severe seas. (a) deep water; (b) shallow water.

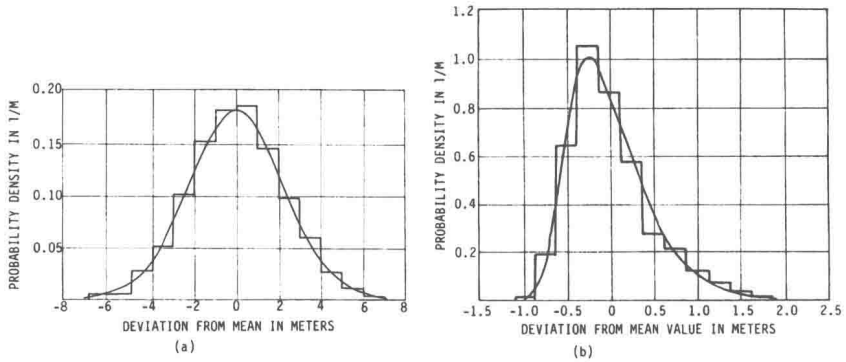


Fig. 1.2. Comparison between histogram of wave profile and theoretical probability density function: (a) deep water; (b) shallow water.

stochastic process, then it is possible to evaluate the statistical properties of waves through the frequency and probability domains.

In the stochastic process approach, waves in deep water are categorized as a Gaussian random process for which the probability distribution of displacement from the mean value (wave profile) obeys the normal probability law. On the other hand, waves in areas where the water depth affects the wave properties are categorized as a non-Gaussian random process. Examples of comparisons between histograms of the wave profile constructed from data obtained in severe seas and the theoretical probability density functions are shown in Figure 1.2. Figure 1.2(a) shows a comparison for waves in deep water in which the theoretical probability density function is a normal probability distribution with a variance evaluated from the data. In Figure 1.2(b) the comparison pertains to waves obtained in shallow water, and the theoretical probability density function given in the figure is as presented in Section 9.2.3. As seen in these examples, wave profiles in both deep and shallow water are well represented by theoretical probability distributions, and this permits us to predict various statistical properties with reasonable accuracy.

1.1.2 Ocean waves as a Gaussian random process

As stated in the preceding section, ocean waves in deep water are considered to be a Gaussian random process. This was first found by Rudnick (1951) through analysis of measured data obtained in the Pacific Ocean. In general, the Gaussian property of ocean waves depends on sea severity and water depth. It may safely be said that if the water is sufficiently deep, waves may be considered to be a Gaussian random process irrespective of sea severity, including very severe sea conditions associated with hurricanes. Waves in relatively shallow water areas may also be considered as a Gaussian random process if the sea severity is very mild. This will be discussed in

detail in Section 5.2.3 in connection with hurricane-generated seas and in Chapter 9 where wave properties in shallow water areas are presented.

The question arises as to the rationale for ocean waves being a Gaussian random process. This may be explained based on the central limit theorem in probability theory as follows.

Let η be the wave profile at a fixed time t . Here, η is a random variable defined in the sample space $(-\infty, \infty)$. We may assume that η is the sum of a large number of components X . That is,

$$\eta = X_1 + X_2 + \dots + X_n \quad (1.1)$$

where the X_i are statistically independent random variables having the same probability distribution, although the form of the probability distribution is unknown. Let the mean value of X_i be zero and its variance (the second moment) be σ^2 . Since the X_i are statistically independent, the probability distribution of η (which is unknown at this stage) has zero mean and variance $n\sigma^2$, where n is large.

We may standardize the random variable η and write the new random variable Z as follows:

$$Z = \eta / (\sqrt{n}\sigma) = \sum_{j=1}^n X_j / (\sqrt{n}\sigma) \quad (1.2)$$

Let the characteristic function of X be $\phi_x(t)$, though the form of $\phi_x(t)$ is unknown. Then, by using the properties of the characteristic function, the characteristic function of the standardized random variable $X/(\sqrt{n}\sigma)$ can be written from Eq. (1.2) as $\phi_x(t/\sqrt{n}\sigma)$. Hence, the characteristic function of Z becomes

$$\phi_z(t) = \{\phi_x(t/\sqrt{n}\sigma)\}^n \quad (1.3)$$

On the other hand, the characteristic function can be expanded in general as follows:

$$\phi_x(t) = 1 + it E[x] - \frac{t^2}{2} E[x^2] + \dots \quad (1.4)$$

Since $E[x]$ and $E[x^2]$ of the standardized random variable are 0 and 1, respectively, $\phi_x(t/\sqrt{n}\sigma)$ may be written as

$$\phi_x(t/\sqrt{n}\sigma) = 1 - (t^2/2n) + o(t^2/n) \quad (1.5)$$

and thereby we have

$$\phi_z(t) = \{1 - (t^2/2n) + o(t^2/n)\}^n \quad (1.6)$$

By letting $n \rightarrow \infty$, Eq. (1.6) yields

$$\phi_z(t) = \exp\{-t^2/2\} \quad (1.7)$$

This is the characteristic function of the standardized normal distribution. This implies that the random variable Z obeys the normal