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# OCEAN WAVES

## The Stochastic Approach

Michel K. Ochi

**Paperback Re-issue**

This book describes the stochastic method for ocean wave analysis. This method provides a route to predicting the characteristics of random ocean waves – information vital for the design and safe operation of ships and ocean structures.

Assuming a basic knowledge of probability theory, the book begins with a chapter describing the essential elements of wind-generated random seas from the stochastic point of view. The following three chapters introduce spectral analysis techniques, probabilistic predictions of wave amplitudes, wave height and periodicity. A further four chapters discuss sea severity, extreme sea state, the directional wave energy spreading in random seas and special wave events such as wave breaking and group phenomena. Finally the stochastic properties of non-Gaussian waves are presented. Useful appendices and an extensive reference list are included. Examples of practical applications of the theories presented can be found throughout the text.

This book will be suitable as a text for graduate students of naval, ocean and coastal engineering. It will also serve as a useful reference for research scientists and engineers working in this field.

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# PREFACE

This book is intended to provide uniform and concise information necessary to comprehend stochastic analyss and probabilistic prediction of wind-generated ocean waves.

Description and assessment of wind-generated ocean waves provide information vital for the design and operation of marine systems such as ships and ocean and coastal structures. Wind-generated seas continuously vary over a wide range of severity depending on geographical location, season, presence of tropical cyclones, etc. Furthermore, the wave profile in a given sea state is extremely irregular in time and space – any sense of regularity is totally absent, and thereby properties of waves cannot be readily defined on a wave-by-wave basis.

Characterization of the stochastic properties of ocean waves was first presented in the early 1950s; Neumann (1953), Pierson (1952, 1955), St Denis and Pierson (1953) introduced the stochastic approach for analysis of random seas, and Longuet-Higgins (1952) demonstrated the probabilistic estimation of random wave height. The four decades following the introduction of the stochastic prediction approach have seen phenomenal advances in the probabilistic analysis and prediction methodologies of random seas.

For the design of marine systems, information on the real world is required. Recent advances in technology permit the use of the probabilistic approach to estimate the responses of marine systems in a seaway, including extreme values, with reasonable accuracy. Such technology lends itself to application of the probabilistic approach as an integrated part of modern design technology in naval, ocean and coastal engineering.

In view of the growing need for more comprehensive advances in prediction methodologies and for application of the probabilistic approach in naval, ocean and coastal engineering, this book is designed as a text book at the graduate level and as a reference book for researchers and designers. The intent is to provide a thorough understanding of the modern concept of stochastic analysis and probabilistic prediction of wind-generated random seas. Specific efforts are made in this work to explain the basic principles supporting current prediction techniques and to provide practical applications of prediction methods.

Readers are expected to be familiar with basic probability theory and fundamental stochastic processes. For the readers' convenience, however, definitions, theorems and relevant formulae on probability and stochastic process theory used in the text are summarized in the appendixes without proof or derivation.

I am grateful to the College of Engineering, University of Florida, for granting me sabbatical leave to prepare this book. Significant progress was achieved toward its completion during this period of time. I would like to acknowledge the encouragement and support received from Professor Eatock Taylor of the University of Oxford during this undertaking. Thanks are also due to Professor Isobe of the Tokyo University who provided valuable suggestions on the section addressing directional wave spectra.

I am indebted to many learned scholars and researchers who directly or indirectly inspired me to study the stochastic analysis and probabilistic prediction of ocean waves. I thank those who sponsored my research which ultimately culminated in this book; in particular, Dr Silva, Office of Naval Research. Appreciation is extended to my graduate students; in particular, Drs C.H. Tsai, D.W.C. Wang, I.I. Sahinoglou, K. Ahn and Lieut. D.J. Robillard, US Navy, who through their dedicated project support had a significant influence on the final product. Finally, I would like to acknowledge the contribution of my wife, Margaret, who read the complete manuscript and provided valuable assistance with the editorial work.

# CONTENTS

# 1 DESCRIPTION OF RANDOM SEAS

## 1.1 STOCHASTIC CONCEPT AS APPLIED TO OCEAN WAVES

### 1.1.1 Introduction

The profile of wind-generated waves observed in the ocean changes randomly with time; it is non-repeatable in time and space. In reality, both wave height (peak-to-trough excursions) and wave period vary randomly from one cycle to another. It is often observed that waves break when the wave steepness exceeds a certain limit. Furthermore, during the process of the wind-generated waves traveling from one location to another after a storm, waves of shorter length gradually lose their energy resulting in the wave profile becoming less irregular (this situation is called swell) than that observed during a storm.

A more distinct difference in the wave profile can be observed when the water depth becomes shallow. As an example, Figure 1.1 shows portions of wave profiles recorded in severe seas; one in deep water in the North Atlantic, the other in a nearshore area of water depth 2.1 m. As seen in Figure 1 (a), positive and negative sides of the wave profile in deep water are, by and large, similar, while for waves in shallow water (Figure 1 (b)), peaks are much sharper than troughs, and the order of magnitude of the peaks is different from that of the troughs.

As seen in the examples shown in Figure 1.1, evaluation of the properties of random waves is almost impossible on a wave-by-wave basis in the time domain. However, if we consider the randomly changing waves as a

(a) (b)

Fig. 1.1. Wave profiles in severe seas. (a) deep water; (b) shallow water.

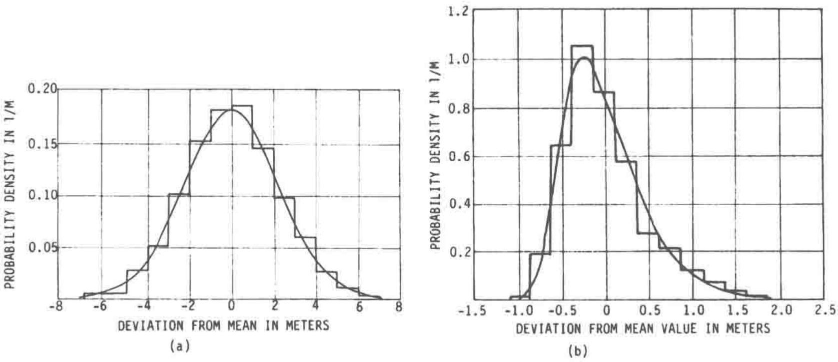


Fig. 1.2. Comparison between histogram of wave profile and theoretical probability density function: (a) deep water; (b) shallow water.

stochastic process, then it is possible to evaluate the statistical properties of waves through the frequency and probability domains.

In the stochastic process approach, waves in deep water are categorized as a Gaussian random process for which the probability distribution of displacement from the mean value (wave profile) obeys the normal probability law. On the other hand, waves in areas where the water depth affects the wave properties are categorized as a non-Gaussian random process. Examples of comparisons between histograms of the wave profile constructed from data obtained in severe seas and the theoretical probability density functions are shown in Figure 1.2. Figure 1.2(a) shows a comparison for waves in deep water in which the theoretical probability density function is a normal probability distribution with a variance evaluated from the data. In Figure 1.2(b) the comparison pertains to waves obtained in shallow water, and the theoretical probability density function given in the figure is as presented in Section 9.2.3. As seen in these examples, wave profiles in both deep and shallow water are well represented by theoretical probability distributions, and this permits us to predict various statistical properties with reasonable accuracy.

### 1.1.2 Ocean waves as a Gaussian random process

As stated in the preceding section, ocean waves in deep water are considered to be a Gaussian random process. This was first found by Rudnick (1951) through analysis of measured data obtained in the Pacific Ocean. In general, the Gaussian property of ocean waves depends on sea severity and water depth. It may safely be said that if the water is sufficiently deep, waves may be considered to be a Gaussian random process irrespective of sea severity, including very severe sea conditions associated with hurricanes. Waves in relatively shallow water areas may also be considered as a Gaussian random process if the sea severity is very mild. This will be discussed in detail in Section 5.2.3 in connection with hurricane-generated seas and in Chapter 9 where wave properties in shallow water areas are presented.

The question arises as to the rationale for ocean waves being a Gaussian random process. This may be explained based on the central limit theorem in probability theory as follows.

Let $\eta$ be the wave profile at a fixed time $t$. Here, $\eta$ is a random variable defined in the sample space $(-\infty,\infty)$. We may assume that $\eta$ is the sum of a large number of components $X$. That is,

$$\eta = X_1 + X_2 + \ldots + X_n \tag{1.1}$$

where the $X_i$ are statistically independent random variables having the same probability distribution, although the form of the probability distribution is unknown. Let the mean value of $X_i$ be zero and its variance (the second moment) be $\sigma^2$. Since the $X_i$ are statistically independent, the probability distribution of $\eta$ (which is unknown at this stage) has zero mean and variance $n\sigma^2$, where $n$ is large.

We may standardize the random variable $\eta$ and write the new random variable $Z$ as follows:

$$Z = \eta/(\sqrt{n}\sigma) = \sum_{j=1}^{n} X_j/(\sqrt{n}\sigma) \tag{1.2}$$

Let the characteristic function of $X$ be $\phi_x(t)$, though the form of $\phi_x(t)$ is unknown. Then, by using the properties of the characteristic function, the characteristic function of the standardized random variable $X/(\sqrt{n}\sigma)$ can be written from Eq. (1.2) as $\phi_x(t/\sqrt{n}\sigma)$. Hence, the characteristic function of $Z$ becomes

$$\phi_z(t) = \{\phi_x(t/\sqrt{n}\sigma)\}^n \tag{1.3}$$

On the other hand, the characteristic function can be expanded in general as follows:

$$\phi_x(t) = 1 + it\,\mathrm{E}[x] - \frac{t^2}{2}\mathrm{E}[x^2] + \ldots \tag{1.4}$$

Since $\mathrm{E}[x]$ and $\mathrm{E}[x^2]$ of the standardized random variable are 0 and 1, respectively, $\phi_x(t/\sqrt{n}\sigma)$ may be written as

$$\phi_x(t/\sqrt{n}\sigma) = 1 - (t^2/2n) + o(t^2/n) \tag{1.5}$$

and thereby we have

$$\phi_z(t) = \{1 - (t^2/2n) + o(t^2/n)\}^n \tag{1.6}$$

By letting $n \to \infty$, Eq. (1.6) yields

$$\phi_z(t) = \exp\{-t^2/2\} \tag{1.7}$$

This is the characteristic function of the standardized normal distribution. This implies that the random variable $Z$ obeys the normal