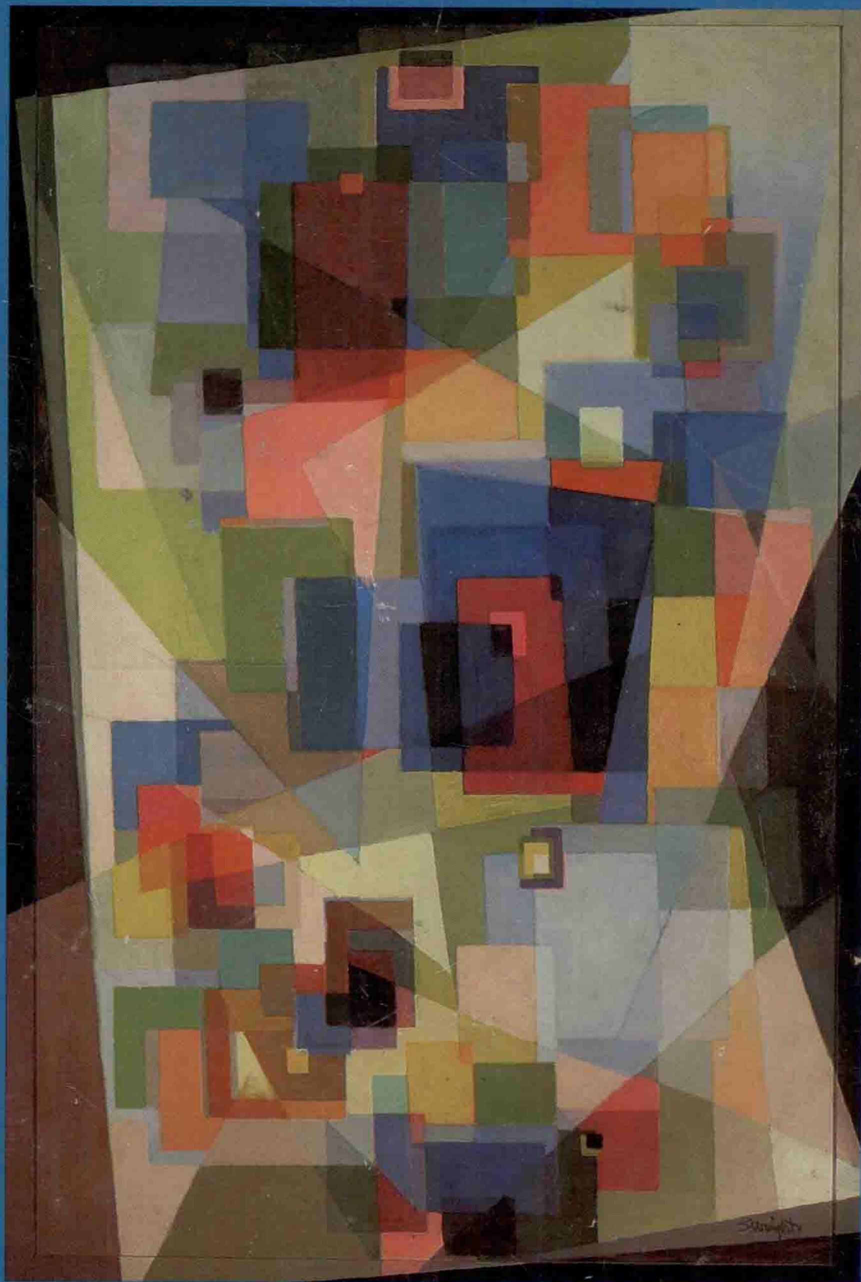
COLLEGE ALGEBRA

4th edition

Leonard I. Holder



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4th Edition

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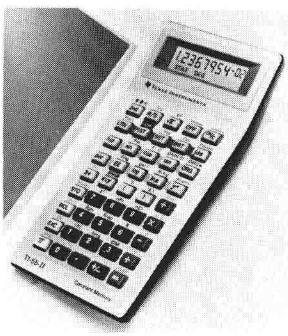
The Use of Calculators

The scientific hand calculator has made the use of tables virtually obsolete. Both in terms of accuracy and speed the calculator has a great advantage. In various places throughout this book a calculator would be useful, especially in parts of Chapters 5 and 6.

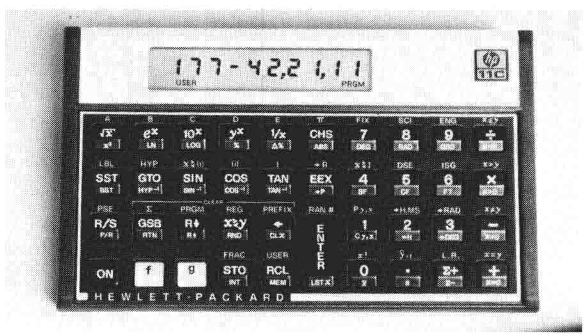
Scientific calculators include keys for the following functions: exponential, $\boxed{e^x}$; logarithmic, $\boxed{\ln x}$ and $\boxed{\log x}$; trigonometric, $\boxed{\sin x}$, $\boxed{\cos x}$, and $\boxed{\tan x}$; and inverse functions, $\boxed{\text{inv}}$ or $\boxed{\text{arc}}$. They include other functions as well, but these are the ones that distinguish the scientific calculator from the simple "four-function" and business-type calculators. For this and other courses in mathematics, science, and engineering, the scientific calculator is the appropriate one to purchase.

A great variety of makes and models of scientific calculators exist, many of which are not very expensive. Most employ either what is called an algebraic operating system (AOS) or reverse Polish notation (RPN). It is a matter of individual preference as to which type is the more convenient to use. In either case you will need to study the instructions to learn how to use your calculator correctly and effectively.

In a few selected examples in Chapters 5 and 6, we have shown sequences of calculator operations, called "keystroke" displays, for a calculator with an algebraic operating system. You may need to modify these for your own calculator.



TI-55-II Courtesy of Texas Instruments



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Preface

This new edition of *College Algebra* has the same objective as its predecessors: to present algebra as a coherent body of knowledge in an informal yet mathematically sound style that is appropriate for students at this level. The changes from the third edition include further refinements in exposition, some merging of sections, the simplification of certain proofs and derivations, and alterations to some examples and exercises. Many of these changes reflect suggestions offered by users of the third edition.

Major Changes

1. Chapters 1 and 2 have been combined into a new first chapter called "Basic Algebraic Concepts."
2. The introduction to the binomial formula has been shortened and incorporated into the section on polynomial arithmetic.
3. The section on composite functions has been rewritten so as to be more easily read and understood by students.
4. The midpoint and distance formulas now are combined with parallel and perpendicular lines in a new section called "Further Properties of Lines and Line Segments."
5. A simpler proof is given of the condition for perpendicularity of two lines.
6. New examples and exercises have been added.
7. All of the chapter introductions taken from textbooks in other fields have been changed.

The merging of Chapters 1 and 2, while retaining most of the ideas, has permitted some economy of coverage, making it possible for instructors to move more rapidly through the material. The total number of sections has been reduced from thirteen to eight. Although this is largely review material, my own experience is that most students need this review. For better prepared students, however, instructors may wish to delete the chapter, or assign it as required reading, without taking class time on it. Selective coverage is another possibility.

I wish to express my appreciation to Jim Harrison, Mathematics Editor at Wadsworth, for his helpful suggestions, and to my typist, Donna R. Cullison, who deserves much credit for deciphering my involved instructions.

Leonard I. Holder

Books that are effective teaching tools (for us) and learning tools (for our students) are not so much *written* as they are *developed* out of our classroom experience. The more experiences of colleagues an author is exposed to, the more the text is thoroughly developed into a reliable teaching tool. I am indebted to many users of the previous editions of this book for their helpful suggestions. In particular, I wish to thank the following colleagues:

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Contents

1 Basic Algebraic Concepts

To introduce each chapter we present an excerpt from a textbook in some discipline that employs mathematics, illustrating one or more of the concepts we will study in that chapter. Do not be concerned if you do not understand the material in these excerpts. You should simply observe the mathematics being used.

The first excerpt, illustrating some algebraic simplification, is from a genetics textbook.*

> By the same method we used to get the frequencies of the *M* and *N* genes in the Hardy–Weinberg equilibrium, we can obtain an equation that yields the value of a gene after a generation of selection (q_{n+1}), as follows:
>
> $$q_{n+1} = \frac{p_n q_n + q_n^2(1 - s)}{1 - sq_n^2}$$
>
> Substituting $(1 - q_n)$ for p_n and multiplying the various terms in the numerator, one gets:
>
> $$q_{n+1} = \frac{(1 - q_n)q_n + q_n^2(1 - s)}{1 - sq_n^2}$$
>
> $$q_{n+1} = \frac{q_n - q_n^2 + q_n^2 - sq_n}{1 - sq_n^2}$$
>
> $$q_{n+1} = \frac{q_n - sq_n^2}{1 - sq_n^2}$$
>
> The most interesting case is that in which $s = 1$, which means that the *aa* genotype is lethal or sterile. When this is the situation, the equation is expressed as follows:
>
> $$q_{n+1} = \frac{q_n}{1 + q_n}$$

Here, p_n and q_n represent *probabilities*, with $p_n + q_n = 1$. The various steps involve several of the concepts we will study in this chapter.

* Reproduced by permission from: Louis Levine, *Biology of the Gene*, 3d ed., St. Louis, 1980, The C. V. Mosby Co.

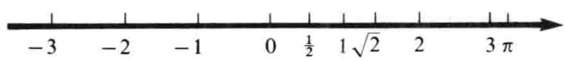
1 The Real Number System

The **real number system** is fundamental in the study of algebra, as well as in the study of many other branches of mathematics. So it is appropriate that we begin with a review of the basic properties of this most important number system. From this basic list, all other properties of real numbers can be derived. And from these properties, in turn, the so-called rules of algebra can be justified. We will not attempt such a complete deductive approach here, since to do so would constitute a course in itself, but we will indicate how the most important algebraic properties follow from those of the real numbers. Before summarizing their basic properties, we describe real numbers informally.

The simplest real numbers are the **natural numbers,** which are the ordinary counting numbers 1, 2, 3, ... Natural numbers are also called **positive integers,** and they, together with 0 and their negatives (the **negative integers,** -1, -2, $-3, \ldots$), comprise the **integers.** The **rational numbers** are numbers that can be expressed in the form of a ratio, m/n, where m and n are integers, with $n \neq 0$. Rational numbers include ordinary fractions, such as $\frac{3}{4}$, and they also include the integers themselves, since any integer m can be written $m/1$. Real numbers that are not rational are called **irrational.** Some examples are $\sqrt{2}$, π, $\sqrt[3]{7}$.

One convenient way to visualize real numbers is to use a **number line,** as shown in Figure 1. After selecting a point as the origin to correspond to 0 and another point to correspond to 1, both a scale and a direction are established, and points corresponding to both positive and negative integers can then be determined. At least theoretically, we can see how to locate a unique point for every rational number. For example, the point corresponding to $\frac{13}{25}$ could be found by dividing the unit interval from 0 to 1 into 25 equal parts and locating the right end point of the thirteenth interval, starting from 0. It is more difficult to locate irrational numbers on the line, but they are there (in abundance!). Irrationals are rather elusive, but they can always be approximated to any desired degree of accuracy by rationals. For example, the well-known number we designate by π is typically approximated by 3.1416 $\left(\frac{31,416}{10,000}\right)$, and $\sqrt{2}$ is approximately 1.414. We should keep in mind that these are approximations only; in fact, we can never write down in decimal form the exact value of any irrational number.

Figure 1



When working with a number line, we will often not distinguish between numbers and the corresponding points on the line. For example, we might say "the point 2" rather than the more precise "the point corresponding to 2." A significant feature of the real numbers is that they occupy *every* position on the number line. There are no gaps. This characteristic would not be true if we limited consideration to rational numbers only, for while the rationals are infinite in number and are dense on the line (that is, between any two points, however close together, there are rationals), they do not occupy all of the line.

In fact, in a certain sense the gaps (that is, the irrationals) are more numerous than the rational points.

The rational numbers have two possible forms when expressed as decimal expansions: (1) terminating, such as $\frac{5}{4} = 1.25$, or (2) repeating, such as $\frac{2}{3} = 0.666\ldots$. All decimal expansions that neither terminate nor repeat are irrational numbers. For example, $\sqrt{3} = 1.73205\ldots$.

We will frequently have occasion to refer to a **set** of things, by which we mean a collection. Sets are frequently designated by using braces, $\{\ \}$, and inside the braces either the **members** of the set are listed, or a rule is given that describes the set. The members of a set are called its **elements,** and we use the symbol $a \in A$ to indicate that a is an element of the set A. If every element of a set B is also an element of the set A, then B is said to be a **subset** of A, designated $B \subseteq A$. It is also convenient to be able to speak of a set that has no elements. We call such a set the **empty set** and designate it by $\varnothing$. All other sets are said to be **nonempty.** We will designate the set of all real numbers by R, the rationals by Q, the irrationals by I, the integers by J, and the natural numbers by N.

Since the notion of equality occurs throughout mathematics, it is useful at the outset to say precisely what this means. In mathematics the symbol $A = B$ means A and B are two names for the same thing. Thus, we may replace A by B or B by A in any expression involving either symbol. Following are the fundamental properties of equality as it relates to real numbers.*

Basic Properties of Equality

1. $a = a$ Reflexive property
2. If $a = b$, then $b = a$. Symmetric property
3. If $a = b$ and $b = c$, then $a = c$. Transitive property
4. If $a = b$, then $a + c = b + c$. Addition property
5. If $a = b$, then $ac = bc$. Multiplication property

By the real number *system* we mean the set R of real numbers, together with the two binary operations of addition and multiplication, that satisfy the eight basic properties listed below.

1. The set R is **closed** with respect to addition and multiplication. That is, **both $a + b$ and $a \cdot b$ are in R.**

Here, the word *closed* means that any number that results from the addition or multiplication of members of R must also be a member of R.

* The same properties hold for complex numbers, which we will study in Chapter 2, and we will use them there without further discussion.

2. Addition and multiplication in R are **commutative.** That is, $\boldsymbol{a + b = b + a}$ **and** $\boldsymbol{a \cdot b = b \cdot a}$.

Thus, the order of adding or multiplying two real numbers is immaterial.

3. Addition and multiplication in R are **associative.** That is, $\boldsymbol{a + (b + c) = (a + b) + c}$ **and** $\boldsymbol{a \cdot (b \cdot c) = (a \cdot b) \cdot c}$. Change in parenthenses

Without this property we would not know, for example, how to interpret $2 + 3 + 4$. The associative property asserts that we may add like this: $2 + (3 + 4) = 2 + 7 = 9$; or like this: $(2 + 3) + 4 = 5 + 4 = 9$. Similar comments apply to multiplication.

4. Multiplication is **distributive** over addition. That is, $\boldsymbol{a(b + c) = ab + ac}$.

For example, we may evaluate $13(12 + 8)$ either this way: $13(12 + 8) = 13(20) = 260$; or by distributing the multiplication: $13(12 + 8) = 13(12) + 13(8) = 156 + 104 = 260$.

5. The number **0 is the additive identity** in R, and the number **1 is the multiplicative identity** in R. That is, $\boldsymbol{a + 0 = a}$ **and** $\boldsymbol{a \cdot 1 = a}$.

In simpler terms, when 0 is added to any number, this leaves the number unchanged, and when a number is multiplied by 1, the number is unchanged.

6. For each element a in R there is a unique **additive inverse** in R, designated by $-a$, and for each $a \neq 0$, there is a unique **multiplicative inverse** in R, designated by a^{-1}. Thus, $\boldsymbol{a + (-a) = 0}$ **and** $\boldsymbol{a \cdot a^{-1} = 1}$.

So the additive inverse of a number is that number which must be added to it to give 0, and the multiplicative inverse of a number is that number by which it must be multiplied to give 1. For example, the additive inverse of 2 is -2, and the additive inverse of $-\frac{2}{3}$ is $\frac{2}{3}$; that is, $2 + (-2) = 0$ and $-\frac{2}{3} + \frac{2}{3} = 0$. The multiplicative inverse of 2 is $\frac{1}{2}$, and the multiplicative inverse of $\frac{2}{3}$ is $\frac{3}{2}$; that is, $2 \cdot 2^{-1} = 2 \cdot \frac{1}{2} = 1$ and $\frac{2}{3} \cdot (\frac{2}{3})^{-1} = \frac{2}{3} \cdot \frac{3}{2} = 1$. It is important to note that 0 has no multiplicative inverse. (Why?)

7. The **positive real numbers,** designated by R^+, constitute a subset of R with the following properties:
   a. **If a and b are in R^+, so are $a + b$ and $a \cdot b$.** That is, R^+ is closed with respect to addition and multiplication.
   b. Every real number falls into exactly one of three distinct categories: **It is in R^+, it is zero, or its additive inverse is in R^+.**

Property 7a says that if you add or multiply two positive real numbers, the result is positive. Property 7b divides R into three disjoint classes: (1) the positive real numbers, (2) zero, and (3) the negative real numbers (that is, numbers with additive inverses that are positive).

Before stating the final basic property, it is necessary to give some definitions. You may have wondered why the properties stated so far do not mention subtraction or division. This is because these operations are defined in terms of addition and multiplication.

DEFINITION 1 The **difference** between a and b is defined by

$$a - b = a + (-b)$$

DEFINITION 2 If $b \neq 0$, the **quotient** of a by b is defined by

$$\frac{a}{b} = a \cdot b^{-1}$$

The notation $a \div b$ is also used for the quotient of a by b. Note that $a/0$ is not defined, since 0 has no multiplicative inverse. This should be stressed. **The denominator of a fraction cannot be 0; that is, you cannot divide by 0.**

DEFINITION 3 The number a is said to be **less than *b*,** written $a < b$, provided $b - a$ is positive. Alternately, we may say b is **greater than *a*,** and write $b > a$. If a is **either less than or equal to *b*,** we write $a \leq b$. Alternately, we say b is **greater than or equal to *a*,** and write $b \geq a$.

When a number line is directed positively to the right, we can interpret $a < b$ geometrically to mean that a is to the *left* of b. So, for example, $-5 < -2$, $-1 < 0$, and $2 < 4$. In each case you can test to see that Definition 3 is satisfied.

DEFINITION 4 A subset S of R is said to be **bounded above** if there is a real number k such that every number in S is less than or equal to k. Such a number k is called an **upper bound** for S. If no upper bound of S is less than k, then k is said to be the **least upper bound** of S.

EXAMPLE 1 The set $\{1, 1\frac{1}{2}, 1\frac{3}{4}, 1\frac{7}{8}, 1\frac{15}{16}, \ldots\}$ is bounded above by 4, 5, 100, and many other numbers. But 2 is the least upper bound.

We can now state the final basic property of the real numbers:

8. The set R is **complete** in the sense that every nonempty subset of R that is bounded above has a least upper bound in R.

This property is admittedly rather difficult to understand, and in this course it will not be necessary to explore its meaning in depth. But you should be aware that this property guarantees that every point on the number line corresponds to some real number and also that every infinite decimal represents a real number.

The eight basic properties we stated characterize completely the real number system, and for this reason they can be thought of as **axioms** for R. Other number systems possess some of these properties, but the real number system is the only one having all eight.

A host of properties can be derived from the basic ones. We list some of these below for reference.

Further Properties of Real Numbers

1. The right-hand distributive property: $(a+b)c = ac + bc$.
2. The extended associative properties: The result is the same regardless of the order of adding or of multiplying any finite collection of real numbers. For example, consider

$$2 + 3 + 4 + 5$$

We may perform this addition in any of the following ways:

$$(2 + 3) + (4 + 5) = 5 + 9 = 14$$
$$[2 + (3 + 4)] + 5 = [2 + 7] + 5 = 9 + 5 = 14$$
$$2 + [3 + (4 + 5)] = 2 + [3 + 9] = 2 + 12 = 14$$

and so on.

3. When 0 is multiplied by any number, the result is 0, that is: $a \cdot 0 = 0$.
4. $-a = (-1)a$
5. $(-a)(b) = -(ab)$
6. $(-a)(-b) = ab$
7. If $ab = 0$, then either $a = 0$ or $b = 0$.

More properties will be given after introducing more definitions.

EXAMPLE 2 Show that $(-a)(b) = -(ab)$.

Note. The instruction "show that" is equivalent to "prove that."

Solution In the proof we will indicate the justification for each step to the right of the step.

$a + (-a) = 0$	Definition of additive inverse
$[a + (-a)] \cdot b = 0 \cdot b$	Equality property 5
$ab + (-a)(b) = 0 \cdot b$	Right-hand distributive property
$ab + (-a)(b) = 0$	Property of 0 (see Problem 22, Exercise Set 1)