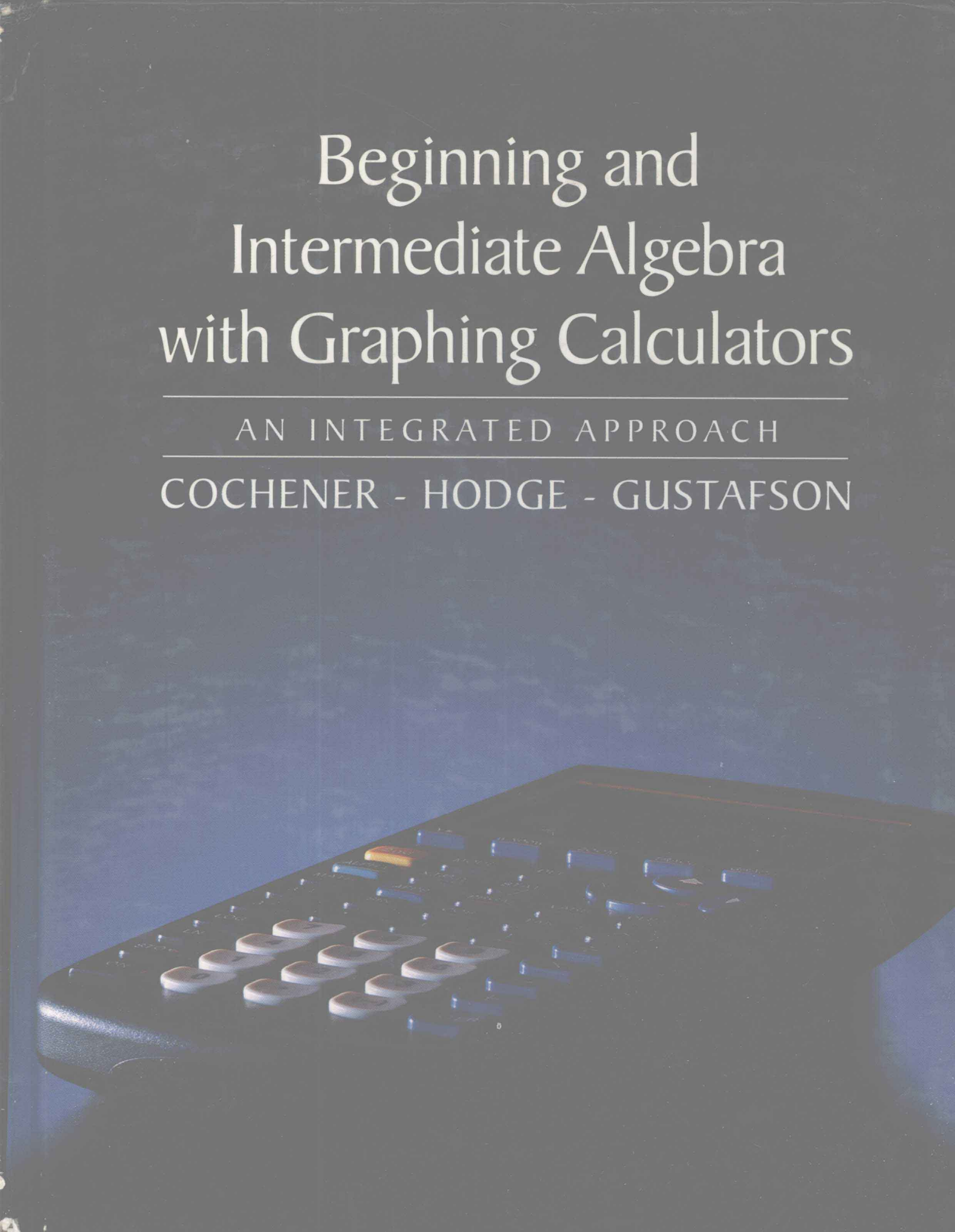
Beginning and Intermediate Algebra with Graphing Calculators

AN INTEGRATED APPROACH

COCHENER - HODGE - GUSTAFSON



Beginning and Intermediate Algebra with Graphing Calculators

An Integrated Approach

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To the Instructor and Student

Beginning and Intermediate Algebra with Graphing Calculators: An Integrated Approach combines the topics of beginning and intermediate algebra. This type of book has several advantages:

- By combining topics, much overlap and redundancy can be eliminated. The instructor will have time to teach for mastery of the material.
- For many students, the purchase of a single book will save money.
- A combination approach in one book will enable some colleges to cut back on the number of hours needed for mathematics remediation.

However, there are three concerns inherent in a combined approach:

- The first half of the book must include enough beginning algebra to ensure that students who complete the first half of the book and then transfer to another college will have the necessary prerequisite to enroll in an intermediate algebra course.
- The beginning algebra material should not get too difficult too fast.
- Intermediate algebra students beginning in the second half of the book must get some review of basic topics so that they can compete with students who are continuing from the first course.

Unlike many other texts, this book uses an *integrated approach,* which addresses each of the previous three concerns by

- including a full course in beginning algebra in the first six chapters,
- delaying the presentation of intermediate algebra topics until Chapter 7 or later, and
- providing a quick review of basic topics for those who begin in the second half of the book.

The approach throughout the text is fourfold and cyclic, using

- numerical approaches, specifically through the use of tables;
- graphical approaches, done both on graph paper and with the graphing calculator, providing students with opportunities to model data and to interpret models;
- symbolic approaches, which help students link the numerical and graphical approaches;
- verbal approaches in which students are required to express mathematical concepts in their own words; and

- continuous requests for students to demonstrate proficiency in the evaluation of expressions, the simplification of expressions, the graphing of functions, and the solving of equations.

Every effort has been made to provide the student with opportunities to become active participants in the formulation of their algebraic foundation, so that they may apply the principles of mathematics to other disciplines. The calculator is presented as a tool to be used for the confirmation of analytical processes and for the discovery of numerical and graphical patterns that lead to the formulation of fundamental algorithms.

GOALS OF THE BOOK

In addition to using a truly integrated approach, our goal has been to write a book that

- is enjoyable to read.
- is easy to understand.
- is relevant.
- will develop the necessary skills for success in future academic courses or on the job.

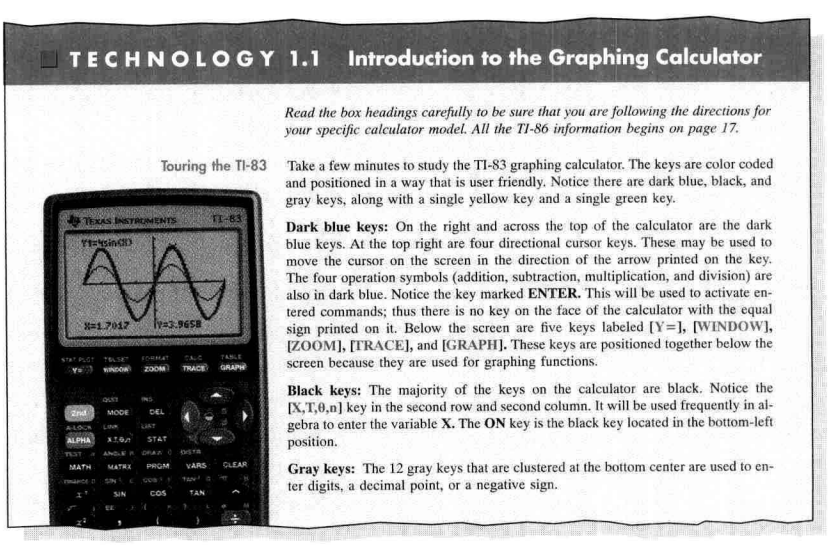
The NCTM standards, the AMATYC Crossroads, and the current trends in mathematics reform have been reinforced by emphasizing graphing and problem solving.

CALCULATORS

The use of a Texas Instruments TI-83 or TI-86 graphing calculator is assumed throughout the text. Students who use the TI-82, TI-83 Plus, or TI-85 will have no problem making the transition. We believe that students should learn calculator skills at this level of mathematics so they will be better able to use the technology in science and business classes as they progress through their individual curriculums.

FEATURES

Technology sections provide extensive keystroking information for Texas Instruments TI-83 and TI-86 graphing calculators. The information is also applicable, with slight modifications, to the TI-82 and TI-85 graphing calculators. Throughout the text the graphing calculator is used to verify results. Graphing calculator information is provided early and continues throughout the text. These sections free the instructor from having to provide keystroking information for the different calculators. ▶



TECHNOLOGY 1.1 Introduction to the Graphing Calculator

Read the box headings carefully to be sure that you are following the directions for your specific calculator model. All the TI-86 information begins on page 17.

Touring the TI-83

Take a few minutes to study the TI-83 graphing calculator. The keys are color coded and positioned in a way that is user friendly. Notice there are dark blue, black, and gray keys, along with a single yellow key and a single green key.

Dark blue keys: On the right and across the top of the calculator are the dark blue keys. At the top right are four directional cursor keys. These may be used to move the cursor on the screen in the direction of the arrow printed on the key. The four operation symbols (addition, subtraction, multiplication, and division) are also in dark blue. Notice the key marked **ENTER.** This will be used to activate entered commands; thus there is no key on the face of the calculator with the equal sign printed on it. Below the screen are five keys labeled [Y=], [WINDOW], [ZOOM], [TRACE], and [GRAPH]. These keys are positioned together below the screen because they are used for graphing functions.

Black keys: The majority of the keys on the calculator are black. Notice the [X,T,θ,n] key in the second row and second column. It will be used frequently in algebra to enter the variable **X.** The **ON** key is the black key located in the bottom-left position.

Gray keys: The 12 gray keys that are clustered at the bottom center are used to enter digits, a decimal point, or a negative sign.

Examples provide both detailed algebraic solutions as well as calculator confirmations of analytical work. ▶

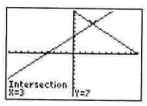
Calculator screens (using the TI-83) are provided throughout the text so that students can confirm their calculator screens. ▶

EXAMPLE 5 Solve $2(x + 2) - x = 10 - x$.

Solution *Algebraic solution:*

$$2x + 4 - x = 10 - x$$
$$x + 4 = 10 - x$$
$$2x + 4 = 10$$
$$2x = 6$$
$$x = 3$$

Graphic solution: $y1 = 2(x + 2) - x$
$y2 = 10 - x$



Notice the display at the bottom of the screen. This means that when the variable x has a value of 3, the expressions entered at the $y =$ prompts evaluate to 7.

Check: $2(3) + 4 - 3 \stackrel{?}{} 10 - 3$
$6 + 4 - 3 \stackrel{?}{} 7$
$7 = 7$ ■

Properties and procedures are boxed for easy reference. ▶

Power to a Power Rule
If m and n are natural numbers, then
$(x^m)^n = x^{mn}$

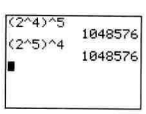
Exploring the Concept features allow students to explore the development of algorithms before they are formally presented. ▶

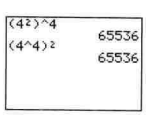
EXPLORING THE CONCEPT Use your graphing calculator to explore the relationship between each pair of numbers.

a. $(2^4)^5$; $(2^5)^4$

b. $(4^2)^4$; $(4^4)^2$

c. Which property justifies the results of each pair of exponential expressions?





Since $(2^4)^5 = 2^{4 \cdot 5}$ and $(2^5)^4 = 2^{5 \cdot 4}$, it is the commutative property of multiplication that justifies the results. Thus, $(x^m)^n = (x^n)^m$. ■

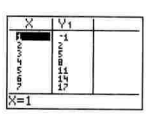
When extensive keystroking information is not necessary, Technology Tips are provided in the margin of the text to clarify the technology for the student. When more information is needed, these tips refer the student to the appropriate technology section. ▶

■ **TECHNOLOGY TIP**
Lines and curves that are graphed with the graphing calculator are actually straight lines and smooth curves. It is the resolution of the calculator screen that makes them appear "jagged."

Teaching Tips for the instructor are provided throughout the text. ▶

■ **TEACHING TIP**
Display the calculator-generated graph on the chalkboard and then plot the ordered pairs from $x = 2$ through $x = 4$ to emphasize that a line is an infinite set of collinear points.

You may now access the TABLE feature of the calculator to compare your table of values, as shown in Figure 3-9, to those of the calculator.



If you access the TRACE feature you can use the left and right cursor keys to trace along the line. You will notice that the coordinate values are cumbersome decimal values.

For this reason, we will *not* use the standard viewing window when we want to trace. The calculator has a preset window called ZInteger that yields integer values when tracing (see "Technology 3.4: Graphing Basics"). To set these values manually, enter $[-47, 47]$ by $[-31, 31]$ on your WINDOW screen now, with x and y scales equal to 10. Trace and observe the x and y coordinates.

Video icons show which examples are taught on videotape. Examples provide both analytical solutions and calculator verifications. ▶

EXAMPLE 6 If $x = 4$ and $y = 3$, evaluate $\dfrac{3x^2 - 2y}{2(x + y)}$.

Solution

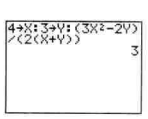
$$\frac{3x^2 - 2y}{2(x + y)} = \frac{3(4^2) - 2(3)}{2(4 + 3)}$$ Substitute 4 for x and 3 for y.

$$= \frac{3(16) - 2(3)}{2(7)}$$ Find the value of 4^2 in the numerator and do the addition in the denominator.

$$= \frac{48 - 6}{14}$$ Do the multiplications.

$$= \frac{42}{14}$$ Do the subtraction.

$$= 3$$



■

Study Tips are provided to guide the student through a comprehensive study plan. ▶

■ **STUDY TIP**
Although you are only on Section 1.2, you should begin to prepare NOW for your first major exam. When you finish tonight's homework (and as you complete each subsequent homework assignment) go back and work a few problems from each of the Exercise Sets that have been previously assigned. If you do this, you will find that much of your preparation for the test will have been accomplished by the time your instructor announces the test date.

WARNING! boxes alert students to common errors and misconceptions. ▶

NUMBER LINE

TECHNOLOGY TIP
Your graphing calculator has both a subtraction key and a negative key. The negative key is displayed as (−). Be sure you use this key to designate negative numbers.

The basic operations of addition, subtraction, multiplication, and division can be applied to all real numbers. Up to this point, it is possible that your experience has led you to deal only with the set of positive natural numbers when performing these operations. As we address these basic operations in the next two sections, the rules of operations with signed numbers will focus on the set of integers. However, these results are applicable to *all* real numbers.

We use the number line shown in Figure 1-3 to represent sets of numbers.

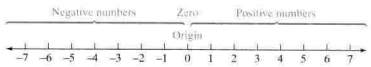


FIGURE 1-3

The number line continues forever to the left and to the right. Numbers to the left of 0 are negative, and we say that the line continues to negative infinity. Numbers to the right of 0 are positive, and we say that the line continues to positive infinity.

WARNING! The number 0 is neither positive nor negative.

Vocabulary exercises are provided at the beginning of each exercise set. ▶

Concepts exercises are provided in each exercise set. ▶

Practice problems are provided in each exercise set. ▶

EXERCISE 2.3

VOCABULARY *In Exercises 1–2, fill in the blank to make a true statement.*

1. The _____ property allows the expression $3(x + y)$ to be calculated as $3x + 3y$.
2. ____ terms have identical variable factors.

CONCEPTS *In Exercises 3–10 tell whether the terms are like or unlike terms. If they are like terms, find their sum.*

3. $3y, 4y$
4. $3x^2, 5x^2$
5. $3x, 3y$
6. $3x^2, 6x$
7. $3x^3, 4x^3, 6x^3$
8. $-2y^4, -6y^4, 10y^4$
9. $32x^5y^3, -21x^5y^3, -11x^5y^3$
10. $-x^2y, xy, 3xy^2$

In Exercises 11–18, simplify each expression, if possible. To simplify means to combine like terms.

11. $4y + 5y$
12. $-2x + 3x$
13. $-8t^2 - 4t^2$
14. $15x^2 + 10x^2$
15. $32u^3 - 16u^3$
16. $25xy^2 - 7xy^2$
17. $18x^5y^2 - 11x^5y^2$
18. $17x^6y - 22x^6y$

PRACTICE *In Exercises 19–24, simplify each expression, if possible.*

19. $3rst + 4rst + 7rst$
20. $-2ab + 7ab - 3ab$
21. $-4a^2bc + 5a^2bc - 7a^2bc$
22. $3x^2y - 5xy^2 + 7x^2y$
23. $3mn^3 - 5mn^3 + 3m^3n$
24. $6abc + 6bca + 7cba$

Applications that use real-life scenarios appear in every exercise set. ▶

Writing icons denote writing exercises. ▶

APPLICATIONS

68. **Course load** A man enrolls in college for c hours of credit, and his sister enrolls for 4 more hours than her brother. Write an expression that represents the number of hours the sister is taking. Could the sister be taking 3 hours of courses? Why or why not?
69. **Mileage** An antique Ford has 25,000 more miles on its odometer than a new car. If the new car has traveled m miles, find an expression that represents the mileage on the Ford.
70. **T-bills** Write an expression that represents the value of t T-bills, each worth $9987.

Review exercises appear at the end of exercise sets. ▶

REVIEW

27. Determine the dimensions of the viewing window that could simultaneously display the following groups of points.
    a. (2, 3), (−11, 3), (−15, −4), and (9, −20)
    b. (−18, 13.4), (16.5, −4) and (25, 30)
    c. (50, −50), (−75, 15), and (25, −80)
28. Graph the linear equation $y = x - 15$ in both the standard (ZStandard) viewing window and in the integer (ZInteger) viewing window and explain the difference in what you see.
29. Graph each of the following functions in the integer viewing window and TRACE to determine the coordinates of the x-intercepts.
    a. $y = x^2 - 14x + 45$
    b. $y = x^2 - x - 56$
    c. $y = x^2 + 11x + 24$
30. Trace to find the y-intercept of each of the functions defined in Exercise 29. Compare each of the y-values of the intercepts with the constant value of the function. What do you observe?

Chapter Summaries are provided in a format that presents the student with both the text information and representative problem exercises to maximize successful review strategies. Answers to all of these problems appear in the back of the text. ▶

CHAPTER SUMMARY

CONCEPTS	REVIEW EXERCISES
Section 3.1	**Introduction to Linear Equations in One Variable**
Any real number can be added to (or subtracted from) both sides of an equation to form another equation with the same solution(s) as the original equation. Both sides of an equation can be multiplied by (or divided by) any *nonzero* real number to form another equation with the same solution(s) as the original equation.	**1.** Tell whether the indicated number is a solution of the given equation. **a.** $3x + 7 = 1$; -2 **b.** $5 - 2x = 3$; -1 **c.** $2(x + 3) = x$; -3 **d.** $5(3 - x) = 2 - 4x$; 13 **2.** Solve each equation algebraically, showing all steps of your work. Check all results in the original equation. **a.** $3x + 7 = 1$ **b.** $7 - 9x = 8$ **c.** $\frac{x+3}{4} = 2$ **d.** $\frac{x-7}{2} = -2$ **e.** $\frac{x}{2} + \frac{7}{2} = 3$ **f.** $\frac{x}{3} - 3 = 2$

Chapter Test

1. Use exponents to rewrite $2\ x\ x\ x\ y\ y\ y$

In Problems 2–4, write each expression as an expression containing only one exponent.

2. $y^2(y\ y^3)$ **3.** $(2x^3)^5(x^2)^3$ **4.** $(2r\ r^2\ r^3)^3$

5. Simplify: $3x^0$. **6.** Identify $3x^2 + 2$ as a monomial, a binomial, or a trinomial.

7. Find the degree of the polynomial $3x^2y^3z^4 + 2x^3y^2z - 5x^2y^3z^5$. **8.** If $P(x) = x^2 + x - 2$, find $P(-2)$.

9. Simplify: $(xy)^2 + 5x^2y^2 - (3x)^2y^2$. **10.** Add: $(3x^3 + 4x^2 - x - 7) + (2x^3 - 2x^2 + 3x + 2)$.

11. Subtract: $(2x^2 - 7x + 3) - (3x^2 - 2x - 1)$. **12.** Simplify: $-6(x - y) + 2(x + y) - 3(x + 2y)$.

13. Simplify: $-2(x^2 + 3x - 1) - 3(x^2 - x + 2) + 5(x^2 + 2)$.

In Problems 14–17, find each product.

14. $(-2x^3)(2x^2y)$ **15.** $3y^2(y^2 - 2y + 3)$ **16.** $(2x - 5)(3x + 4)$ **17.** $(2x - 3)(x^2 - 2x + 4)$

18. Simplify the fraction: $\frac{8x^2y^3z^4}{16x^3y^2z^4}$. **19.** Do the division: $\frac{6a^2 - 12b^2}{24ab}$.

20. Divide: $2x + 3\overline{)2x^3 - x - 6}$.

Cumulative Review: Chapters 1 and 2

VOCABULARY / CONCEPTS

1. A number greater than one, whose only factors are one and itself is called a _____ number.

2. Explain, in a step-by-step format, how to find the least common denominator for two fractions.

Following your step-by-step directions, find the least common denominator for $\frac{5}{24}$ and $\frac{7}{18}$.

3. Explain the difference between the expressions $4x$ and x^4.

4. Explain the difference between *evaluate* and *simplify* when applied to the expression $3x + 5x$.

5. Determine the absolute value of each expression and verify with your calculator.
a. $|-3|$ **b.** $-|-3|$ **c.** $-|3|$

6. What rule supports the statement $-(-5) = 5$?

7. Explain why $\frac{x}{0}$ is undefined and $\frac{0}{x} = 0$ provided $x \neq 0$.

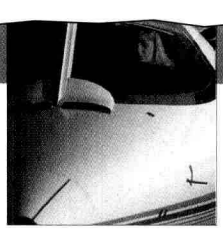
8. Which of
a. (-4) –
c. $-4(-3$

◀ Chapter Tests (with answers to all exercises) are provided at the end of each chapter.

◀ Cumulative Reviews (with answers to all exercises) appear at the end of every even-numbered chapter.

Each chapter contains a Careers & Mathematics feature that relates mathematics to career applications. ▶

CAREERS & MATHEMATICS



Pilot Pilots are highly trained professionals who fly airplanes and helicopters to carry out a wide variety of tasks. Most pilots transport passengers and cargo, but some are involved in crop dusting, spreading seed, testing aircraft, fire-fighting, tracking criminals, traffic reporting, and rescue and evacuation.

Qualifications All pilots paid to transport passengers or cargo must have a commercial pilot's license with an instrument rating. They must also have good health, vision correctable to 20/20, good hearing, and no physical handicaps that would interfere with safety. Most employers also require at least 2 years of college, and many prefer college graduates. Requirements for a private pilot's license are less restrictive.

Job Outlook Because pilots like their jobs and few change careers, competition for employment is stiff. Over the next decade, however, retirements will provide job openings for qualified applicants.

Example Application The air passing over the curved top of the wing in Illustration 1 moves faster than the air beneath, because it has a greater distance to travel. This faster-moving air exerts less pressure on the wing than the air beneath, causing *lift*.

Every chapter includes a Perspective feature that provides interesting mathematical and historical information. ▶

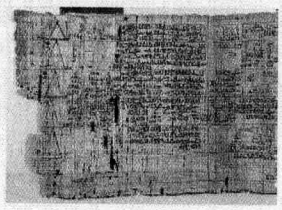
■ PERSPECTIVE

All Dark Things

The ancient Egyptians developed two systems of writing. Each symbol in one system, hieroglyphics, was a picture of an object. Because hieroglyphic writing was usually inscribed on stone, many examples still survive. For daily purposes, Egyptians used hieratic writing. Simpler than hieroglyphics, hieratic writings were done with ink on papyrus sheets. Papyrus, made from plants, is very delicate and quickly dries and crumbles. Few Egyptian papyri survive.

Those that do survive provide important clues to the content of ancient mathematics. One, the Rhind Papyrus, was discovered in 1858 by a British archeologist, Henry Rhind. Also known as the Ahmes Papyrus after its ancient author, it begins with a description of its contents: *Directions for Obtaining the Knowledge of All Dark Things.*

The Ahmes Papyrus and another, the Moscow Papyrus, together contain 110 mathematical problems and their solutions. Many of these were probably for education, beause they represented situations that the scribes, priests, and other workers in government and temple administration were expected to be able to solve.



The Ahmes Papyrus (the British Museum)

■ ANCILLARIES FOR THE INSTRUCTOR

Annotated Instructor's Edition

Free to professors when the text is adopted, the Annotated Instructor's Edition includes the complete text of the Student Edition, along with answers to all problems printed in blue next to the problems. Teaching Tips are included in the margins.

Instructor's Resource Manual

This manual contains complete, worked-out solutions for all exercises in the text, as well as printed test forms with answers and multiple-choice and free-response questions keyed to the text. Other features include:

- chapter tests for each chapter.
- final exams for both beginning and intermediate algebra courses.
- pages that can be made into transparencies for classroom use.
- suggested coverage time for each section.
- Ideas to Emphasize—main ideas to be covered in class.
- Discussion Ideas—topics for class discussion.
- Teaching Tips—alternate approaches or examples.
- Technology Notes—help with graphing calculator use.
- Points to Ponder—various types of helpful information for the instructor.

Thomson Learning Testing Tools CD

This fully integrated suite of programs allows a professor to easily post course information, office hours, lesson information, assignments, sample tests, and link to rich Web content, including review and enrichment material from Brooks/Cole. Updates are quick and easy, and customer support is available 24 hours a day, seven days a week.

Text-Specific Video Series

Free to schools when the text is adopted, this video series works through one example from each section of the text (designated by a video icon in both the Student Edition and the Annotated Instructor's Edition). Two additional examples are provided for every section.

Demo Video

The demonstration video is approximately 30 minutes long and features outtakes from the text-specific video series.

Interactive Algebra 3.0
This extremely intuitive, text-specific student tutorial is offered in student and instructor versions. It offers a mathematical guide to students of any technological proficiency and a tracking program for professors to follow their students' progress.

ANCILLARIES FOR THE STUDENT

Student Solutions Manual
This manual contains complete, worked-out solutions to all odd-numbered exercises in the text.

Greatest Hits Tutorial Video
This student video features the concepts and skills students traditionally have the most difficulty comprehending. The tape includes examples, chosen by authors, from each chapter.

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Deborah J. Cochener
Bonnie M. Hodge
R. David Gustafson

CONTENTS

Basic Arithmetic and Graphing Calculator Concepts

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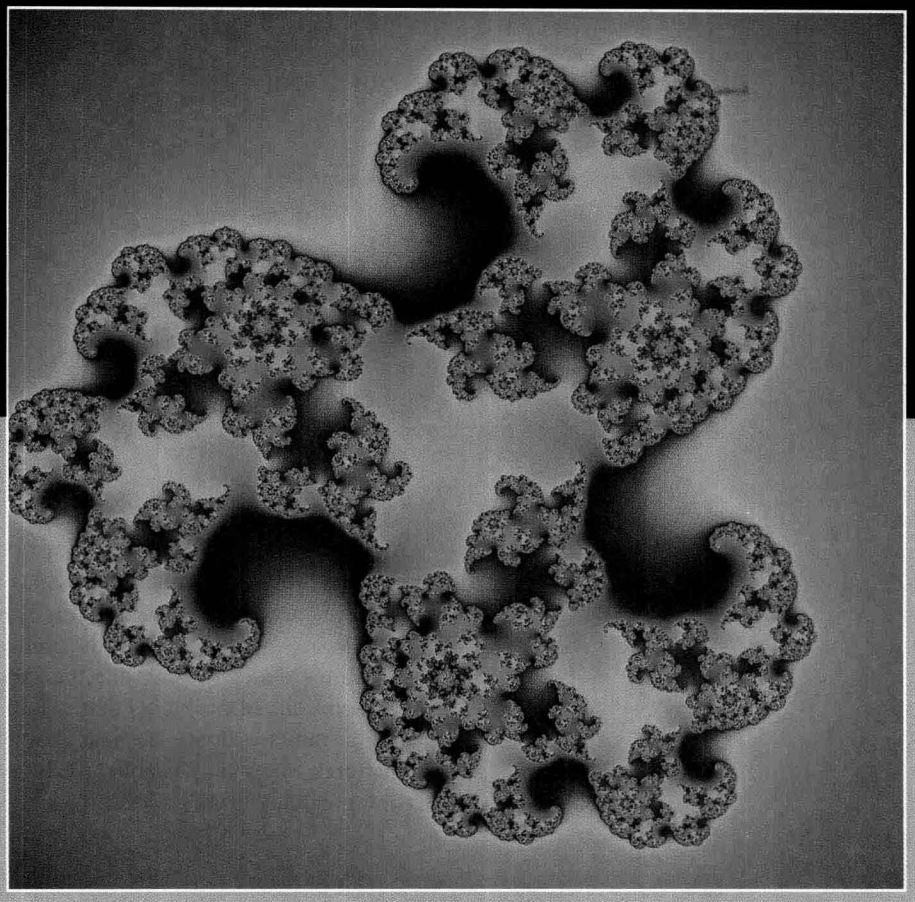

PERSPECTIVE

All Dark Things

The ancient Egyptians developed two systems of writing. Each symbol in one system, hieroglyphics, was a picture of an object. Because hieroglyphic writing was usually inscribed on stone, many examples still survive. For daily purposes, Egyptians used hieratic writing. Simpler than hieroglyphics, hieratic writings were done with ink on papyrus sheets. Papyrus, made from plants, is very delicate and quickly dries and crumbles. Few Egyptian papyri survive.

Those that do survive provide important clues to the content of ancient mathematics. One, the Rhind Papyrus, was discovered in 1858 by a British archeologist, Henry Rhind. Also known as the Ahmes Papyrus after its ancient author, it begins with a description of its contents: *Directions for Obtaining the Knowledge of All Dark Things.*

The Ahmes Papyrus and another, the Moscow Papyrus, together contain 110 mathematical problems and their solutions. Many of these were probably for education, beause they represented situations that the scribes, priests, and other workers in government and temple administration were expected to be able to solve.

The Ahmes Papyrus (the British Museum)

Algebra is an extension of arithmetic and allows us to reason from the specific to the general. Learning algebra provides us with powerful problem-solving strategies and enhances our logical thinking skills. In algebra, the operations of addition, subtraction, multiplication, and division are performed on both constants and variables, with the understanding that the variables may be replaced by constants. This leads to solutions of problems that would be difficult or impossible for us to solve by using arithmetic alone. Because humans are visual thinkers, our goal will be to draw pictures wherever possible to help our understanding. That is why we will use a graphing calculator as we develop our problem-solving skills and tools.

1.1 Review of Fractions

■ SETS OF NUMBERS ■ SIMPLIFYING FRACTIONS ■ MULTIPLYING FRACTIONS ■ DIVIDING FRACTIONS ■ ADDING FRACTIONS ■ SUBTRACTING FRACTIONS ■ MIXED NUMBERS ■ DECIMAL FRACTIONS

■ SETS OF NUMBERS

■ TECHNOLOGY TIP

Technology Tips are provided throughout this text to help you support your algebraic work with the graphing calculator. If we feel that you might need extensive help with your calculator, we will refer you to an extended explanation at the end of the section. (See "Technology 1.1: Introduction to the Graphing Calculator," page 14.)

We begin our discussion with the set of real numbers. A **set** is a well-defined collection of objects. For example, the set {1, 2, 3, 4, 5} contains the numbers 1, 2, 3, 4, and 5. The members, or elements, in a set are listed within braces { }.

Two basic sets of numbers are the **natural numbers** and the **whole numbers.**

Natural Numbers

The **natural numbers** (or the **positive integers**) are the numbers 1, 2, 3, 4, 5, ... This would be written as {1, 2, 3, 4, 5, . . .} when using set notation.

Whole Numbers

The **whole numbers** are the numbers 0, 1, 2, 3, 4, 5, ... This would be written as {0, 1, 2, 3, 4, 5, . . .} when using set notation.

The three dots in the definitions indicate that the lists continue forever.

We can use whole numbers to describe many real-life situations. For example, some cars get 30 miles per gallon of gas, some students might have paid \$1750 in tuition, and the temperature can be 0°.

Integers

The **integers** are the numbers . . . , −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, ... This would be written as {. . . , −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, . . .} when using set notation.

Numbers that show a loss or a downward direction are called **negative integers,** denoted as −1, −2, −3, and so on. For example, a debt of \$1500 can be denoted as −\$1500, and a temperature of 20° below zero can be denoted as −20°. The set of negative integers together with the set of whole numbers form the set of **integers.**

Because the set of natural numbers and the set of whole numbers are included within the set of integers, we say that these sets are **subsets** of the set of integers.

Integers cannot describe every real-life situation. For example, a student might study $3\frac{1}{2}$ hours, or a television might cost \$217.36. To describe these situations, we need fractions. A **fraction** that can be expressed as a quotient of integers, $\frac{p}{q}$, where $q \neq 0$, is called a **rational number.**

Rational Numbers

A **rational number** is any number that can be written as a fraction with an integer in its numerator and a nonzero integer in its denominator.

Fractions can also be used to indicate division. For example, the division of $8 \div 2$ can be written as a fraction, $\frac{8}{2}$. Fractions are also used to indicate parts of a whole. In Figure 1-1, a rectangle has been divided into 5 equal parts, and 3 of the parts are shaded. The fraction $\frac{3}{5}$ indicates how much of the figure is shaded. The denominator of the fraction shows the total number of equal parts into which the whole is divided, and the numerator shows the number of these equal parts that are being considered.

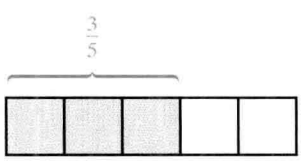

FIGURE 1-1

Some examples of rational numbers are

$$\frac{3}{2}, \frac{17}{12}, -\frac{43}{8}, 0.25, \quad \text{and} \quad -0.66666\ldots.$$

The decimals 0.25 and −0.66666 . . . are rational numbers because they can be written as fractions: 0.25 is the fraction $\frac{1}{4}$ and −0.66666 . . . is the fraction $-\frac{2}{3}$.

■ **TECHNOLOGY TIP**
Your calculator will display an error message if you attempt to divide by 0.

Because every integer can be written as a fraction with a denominator of 1, every integer is also a rational number. Because every integer is a rational number, the set of integers is a subset of the rational numbers.

WARNING! Because division by 0 is undefined, expressions such as $\frac{6}{0}$ and $\frac{0}{0}$ do not represent any number.

■ SIMPLIFYING FRACTIONS

A **prime number** is a whole number greater than 1 whose only factors are 1 and the number itself. The set of prime numbers can be represented as {2, 3, 5, 7, 11, 13, 17, 19, . . .}. A fraction is in **lowest terms** when the numerator and the denominator have no common factors (other than 1). To reduce the fraction to lowest terms (often called *simplifying*) we can factor the numerator and the denominator into **primes** and divide out common factors. The following example illustrates the **fundamental property of fractions.**