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SESSION 1

Modeling of Various Damping Systems

Nonlinear Damping Identification From Transient Data

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ABSTRACT

To study new damping augmentation methods for helicopter rotor systems, accurate and reliable nonlinear damping identification techniques are needed. For example, current studies on applications of magnetorheological (MR) dampers for rotor stability augmentation suggest that a strong Coulomb damping characteristic will be manifested as the field applied to the MR fluid is maximized. Therefore, in this work, a single degree of freedom (SDOF) system having either nonlinear Coulomb or quadratic damping is considered. This paper evaluates three analyses for identifying damping from transient test data; an FFT-based moving block analysis, an analysis based on a periodic Fourier series decomposition, and a Hilbert transform based technique. Analytical studies are used to determine the effects of block length, noise, and error in identified modal frequency on the accuracy of the identified damping level. The FFT-based moving block has unacceptable performance for systems with nonlinear damping. These problems were remedied in the Fourier series based analysis and acceptable performance is obtained for nonlinear damping identification from both this technique and the Hilbert transform based method. To more closely simulate a helicopter rotor system test, these techniques were then applied to a signal composed of two closely spaced modes. This data was developed to simulate a response containing the first lag and 1/rev modes. The primary mode of interest (simulated lag mode) had either Coulomb or quadratic damping, and the close mode (1/rev) was either undamped or had a specified viscous damping level. A comprehensive evaluation of the effects of close mode amplitude, frequency, and damping level was performed. A classifier was also developed to identify the dominant damping mechanism in a signal of 'unknown' composition. This classifier is based on the LMS error of a fit of the analytical envelope expression to the experimentally identified envelope signal. In most instances, the classifier identifies the damping mechanism correctly, erring only when the close mode significantly affects the envelope signal.

Keywords: damping identification, Hilbert transform, moving block analysis

NOMENCLATURE

$a(t)$	Analytical envelope signal
$A_1(t_k)$	Fourier cosine coefficient at time t_k
$B_1(t_k)$	Fourier sine coefficient at time t_k
$x(t)$	Transient signal of interest
ϵ	Quadratic damping ratio
μ	Coulomb damping ratio
ζ	Viscous damping ratio
C_{eq}	Equivalent viscous damping ratio
Ω	Frequency of interest in transient
ω_n	Natural frequency of transient
ω_d	Damped natural frequency of transient
j	$\sqrt{-1}$
ϕ	Phase shift of signal
$\tilde{y}(t)$	Hilbert transform of $y(t)$
y_0	Initial (maximum) displacement of transient
$q(t)$	Instantaneous phase signal
$(\cdot)_v$	Viscous damping quantities
$(\cdot)_c$	Coulomb damping quantities
$(\cdot)_q$	Quadratic damping quantities
$\hat{(\cdot)}$	Quantity estimated from experimental data

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1. INTRODUCTION

Nonlinear behavior is present in many modern mechanical systems ranging from piping networks[1] to helicopter rotors.[2,3] In addition, elastomeric bearings and ER/MR fluid filled dampers, which are used in a variety of applications such as exercise equipment and industrial vibration dampers,[4] may also contribute to nonlinear system response. In order to evaluate the damping performance of these systems, it is necessary to have an accurate and reliable method for identifying nonlinear damping mechanisms.

A particular area of interest at University of Maryland is damping augmentation in helicopter rotor systems, particularly advanced hingeless or bearingless rotors. With the advent of these new rotor systems, the mechanical complexity of the rotor system is greatly reduced. This reduction is accomplished by replacing the traditional hinges and bearings with a flexure, or flexbeam. This change results in the rotor being soft in-plane, leading to an increased susceptibility to problems of air and ground resonance. Damping is a major stabilizing influence of both air and ground resonance,[5] and methods of introducing damping into the rotor have been investigated by several researchers.[6–12] Most recently, Tracy and Chopra[9] looked at the aeromechanical stability of a hingeless rotor with coupled composite flexbeams, while Smith and Wereley[10,11] explored the addition of viscoelastic damping layers to composite flexbeams for stability augmentation. In addition, Kamath and Wereley[12] have investigated the use of magnetorheological (MR) fluid-filled dampers for increasing the damping level in a bearingless rotor.

The damping in a helicopter rotor may be nonlinear, and while these nonlinear effects are usually small, large rotor blade and flexbeam deflections can significantly increase their impact.[13] This paper addresses the accuracy and applicability of three techniques for the identification and estimation of nonlinear damping characteristics from transient data, with special consideration for the applicability to helicopter rotor test data. These techniques include an FFT-based moving block analysis, a moving block analysis exploiting the periodic Fourier series decomposition, and a Hilbert transform based damping analysis.

Moving block analyses are commonly used in the rotorcraft industry to identify damping of rotor modes such as blade flap, lag, and torsion modes. The helicopter rotor testing environment is especially prone to experimental difficulties in the characterization of damping due to high noise levels and spectrally close modes. Hammond and Doggett[14] used the moving block analysis to perform damping estimates on a rotating scale rotor system in the NASA Langley transonic dynamics tunnel, while Bousman and Winkler[15] described how the moving block analysis could identify damping in spectrally close modes. Use of the moving block analysis to identify rotor stability characteristics was addressed by Tasker and Chopra,[16] who added additional refinements to improve damping estimates, including recursive spectral analysis techniques with improved frequency resolution, and windowing to reduce leakage from closely spaced modes. They also assessed this technique for closely spaced modes (both damped and undamped), noisy data, persistent periodic vibrations from undamped modes, and lightly and heavily damped (5%) systems. In 1996, Smith and Wereley[17] applied a similar FFT based moving block analysis to analytical signals with linear viscous damping that included noisy data, spectrally close modes, and errors in the excitation frequency. This technique was then used to identify the linear viscous damping level in experimental transients obtained from both stationary and rotating composite beams with viscoelastic damping layers.

A moving block damping identification technique based on a periodic Fourier series decomposition is also considered. In this method, instead of an FFT of the data block, the periodic Fourier series decomposition is performed at the frequency of interest, attempting to spectrally isolate that mode. In addition, the use of a small block size within the moving block technique allows for more accurate envelope identification. This technique was developed to improve on the nonlinear damping identification performance of prior FFT-based moving block damping analyses.

Finally, a damping identification technique based on the Hilbert transform[18] is also considered. The determination of damping from a transient response, whether linear or nonlinear, may be reduced to determination of the decay envelope. The Hilbert transform is the standard transform used in envelope detectors, and is uniquely capable of computing the transient decay envelope. Agneni and Crema[19] first suggested this technique for time domain damping identification, while Magalas[20] refined it for determining mechanical loss spectra in metallic alloys. Simon and Tomlinson[21] used the Hilbert transform to identify the presence of nonlinear damping effects using frequency domain data, while Smith and Wereley[10] used a Hilbert transform based technique to identify the linear viscous damping level in both stationary and rotating composite beams with viscoelastic damping layers using time domain transient data. Further, an assessment was made of the effects of noisy data, spectrally close modes, and error in excitation frequency.[10]

The objectives of this paper are to evaluate three nonlinear damping identification techniques using analytically generated transients with known Coulomb and quadratic damping levels. To achieve this, both isolated modes and spectrally dense environments will be considered. For an isolated mode, the effects of block length, noise, and error in excitation frequency will be considered, while in the case of a spectrally close mode, the effects of the amplitude, frequency, and damping level of the close mode will be evaluated. The concept of a classification technique for determination of the dominant damping mechanism in a transient will also be explored.

2. DAMPING MECHANISMS

Helicopter rotor systems are complex structures that are subjected to time varying aerodynamic and structural loads. This can result in large deflections and nonlinear responses to external forces. To characterize this behavior, it is necessary to determine the type and amount of damping in the system. To accomplish this, both Coulomb and quadratic damping are considered in this study. Table 1 includes the governing equations, envelope signal expressions, and equivalent viscous damping expressions for Coulomb, viscous and quadratic damping. Viscous damping is shown for comparison purposes only.

Table 1. Coulomb, Viscous and Quadratic Damping Mechanisms.

Damping Type	Governing Equation (Homogeneous Form)	Envelope Signal	Equivalent Viscous Damping Coefficient
Coulomb	$\ddot{x}+\mu\frac{\dot{x}}{\lvert\dot{x}\rvert}+\omega_n^2 x=0$	$a_c(t)=\frac{-2\mu}{\pi\omega_n}t+y_0$	$C_{eq,c}(t)=\frac{2\hat{\mu}}{\pi\omega_n^2 a_0-2\hat{\mu}\omega_n t}$
Viscous	$\ddot{x}+2\zeta\omega_n\dot{x}+\omega_n^2 x=0$	$a_v(t)=e^{-\zeta\omega_n t}$	$C=2\zeta\omega_n$
Quadratic	$\ddot{x}+\epsilon\lvert\dot{x}\rvert\dot{x}+\omega_n^2 x=0$	$a_q(t)=\frac{3\pi a_0}{3\pi+4\epsilon\omega_n a_0 t}$	$C_{eq,q}(t)=\frac{4\hat{\epsilon}a_0}{3\pi+4\hat{\epsilon}\omega_n a_0 t}$

2.1. Coulomb Damping

Coulomb damping is most commonly associated with surface or rubbing friction, also known as dry friction. The equation of motion for a SDOF system with Coulomb damping is given by

$$\ddot{x}+\mu\frac{\dot{x}}{|\dot{x}|}+\omega_n^2 x=0 \tag{1}$$

where μ is the Coulomb damping ratio, and ω_n is the natural frequency. The initial condition response to a nonzero initial displacement was generated using an ODE solver in MATLAB, and the envelope of the response, given analytically by this expression

$$a_c(t)=\frac{-2\mu}{\pi\omega_n}t+y_0 \tag{2}$$

was determined. Once the envelope signal, $\hat{a}(t)$, has been determined from the transient test data, estimates for the Coulomb damping ratio, μ, and the equivalent viscous damping coefficient, $C_{eq,c}$, may be determined.

To obtain an estimate for the Coulomb damping ratio, μ, a cost function was defined as the LMS error between the analytical and estimated envelope signals. This expression was minimized over μ, resulting in an estimate for the Coulomb damping ratio, $\hat{\mu}$.

$$\hat{\mu}=\min J_c(\mu)=\min\sum[a_c(t_i)-\hat{a}(t_i)]^2 \tag{3}$$

The equivalent viscous damping coefficient, $C_{eq,c}$, can be calculated in one of two ways. If an estimate of the Coulomb damping ratio, $\hat{\mu}$, is available, $C_{eq,c}$ may be found analytically as follows:

$$C_{eq,c}(t) = \frac{2\hat{\mu}}{\pi\omega_n^2 a_0 - 2\hat{\mu}\omega_n t} \tag{4}$$

where a_0 is the initial amplitude of the transient/envelope signal. Otherwise, the slope between any two points of the log of the envelope signal can be defined as $-\zeta\omega_n$, and the equivalent viscous damping coefficient at that time may be easily determined. Three different Coulomb damping ratios, $\mu = 100$, 300, and 500, were used in this study to simulate light, moderate, and heavy damping. An example of a transient with Coulomb damping ($\mu = 300$) and 5% noise is shown in Fig. 1a.

2.2. Quadratic Damping

One common occurrence of quadratic damping is air damping, which occurs due to the air resistance of a moving structure. The governing equation of motion for a SDOF system with quadratic damping is given by:

$$\ddot{x} + \epsilon|\dot{x}|\dot{x} + \omega_n^2 x = 0 \tag{5}$$

The response of this system has the form:

$$y(t) = a_q(t)\sin[\omega_n(t) + \beta(t)] \tag{6}$$

where the envelope signal is:

$$a_q(t) = \frac{3\pi a_0}{3\pi + 4\epsilon\omega_n a_0 t} \tag{7}$$

Once the envelope signal, $\hat{a}(t)$, has been estimated from transient test data, estimates for ϵ and $C_{eq,q}$ may be determined.

To obtain an estimate for the quadratic damping ratio, ϵ, a cost function was defined to be the LMS error between the analytical and estimated envelope signals. This expression was minimized over ϵ, resulting in an estimate for the quadratic damping ratio, $\hat{\epsilon}$.

$$\hat{\epsilon} = \min J_q(\epsilon) = \min \sum [a_q(t_i) - \hat{a}(t_i)]^2 \tag{8}$$

As in the Coulomb damping case, the equivalent viscous damping coefficient can be calculated in one of two ways. If $\hat{\epsilon}$ is available, $C_{eq,q}$ may be determined analytically with the following relation:

$$C_{eq,q}(t) = \frac{4\hat{\epsilon}a_0}{3\pi + 4\hat{\epsilon}\omega_n a_0 t} \tag{9}$$

where a_0 is the initial amplitude of the transient/envelope signal. If $\hat{\epsilon}$ is not available, the slope between any two points of the log of the envelope signal can be defined as $-\zeta\omega_n$, and $C_{eq,q}$ at that time may be easily determined. As in the previous case, three damping ratios are considered, $\epsilon = .003$, .005, and .007, to simulate systems with light, moderate and heavy quadratic damping. An example of a transient with quadratic damping ($\epsilon = .005$) and 5% noise is shown in Fig. 1b.

3. DAMPING IDENTIFICATION TECHNIQUES

In both damping mechanisms considered here, the critical step is the identification of the envelope signal, the estimate of which is denoted by $\hat{a}(t)$. The shape of the envelope signal provides a means by which the damping mechanism can be classified (i.e. linear decay for Coulomb damping), while the duration of the transient, in conjunction with the natural frequency, provides the damping level. Three methods to estimate the envelope signal are presented: (1) an FFT-based moving block analysis, (2) a moving block analysis based on the Fourier series decomposition at the frequency of interest, and (3) a Hilbert transform based analysis. For each method, it is assumed that a transient decay response is available based on either an impulse response or a steady state sinusoidal excitation/response pair at the frequency of interest where the sinusoidal excitation is terminated at peak amplitude and the ensuing transient response acquired. In addition, the peak amplitude of each transient is determined, and when the maximum amplitude of the transient drops below a specified value, the data set is truncated for analysis. This specified value is the threshold cutoff value, and it can have a significant effect on the identified damping level. The purpose of the threshold cutoff value is to minimize the effects of noise and persistent excitation on the results of the damping identification analysis.

3.1. FFT-Based Moving Block Analysis

The moving block analysis is a digital signal processing methodology that can be implemented for estimation of modal damping using a transient decay time history. First, the mode of interest is excited sinusoidally at its modal frequency, and then the excitation is abruptly terminated, and the transient response data is acquired. The transient response amplitude at the modal frequency of interest is then computed on a block of data using either an FFT or PSD calculation. The block is then moved forward a single point in time, or a new block is formed by removing the starting point and then adding a single point to the end of the block. The computation of the transient response amplitude is then repeated. Computation of the response amplitudes continues until sufficient transient amplitude points have been computed. For viscous damping, the natural logarithm of these response amplitude points is plotted versus time to obtain the peak plot. The viscous damping ratio, ζ, is estimated from the slope of a linear LMS fit to this peak plot.

The basis of the technique is the transient behavior of a SDOF system. The FFT of a viscously damped transient response of this system is given by:

$$F(\omega, t_0) = \int_{t_0}^{T+t_0} Ae^{-\zeta\omega_n t}\sin(\omega_d t + \phi)e^{-j\omega t}dt \tag{10}$$

where ζ is the damping ratio, ω_n is the natural frequency, $\omega_d = \omega_n\sqrt{1-\zeta^2}$ is the damped natural frequency, t_0 is the initial time of the FFT, and $T + t_0$ is the final time of the FFT. Here, T is defined as the time duration of the block. Assume that (1) the system is lightly damped, or $\zeta \ll 1$, and (2) the damped natural frequency is approximately equal to the natural frequency, or $\omega_n \approx \omega_d$. Based on these assumptions, Hammond[14] showed that the magnitude of the FFT at the frequency of interest is given by:

$$|F(\omega_n, t_0)| = \frac{A}{2\omega_n}e^{-\zeta\omega_n t_0}\left[\frac{1 + f(\zeta, t_0)}{\zeta^2}\right]^{\frac{1}{2}} \tag{11}$$

and the natural logarithm of the magnitude of the FFT at the frequency of interest is given approximately by:

$$\ln|F(\omega_n, t_0)| = ln\left(\frac{A}{2\omega_n}\right) - \zeta\omega_n t_0 + \frac{1}{2}\ln\left[\frac{1 + f(\zeta, t_0)}{\zeta^2}\right] \tag{12}$$

The full expression for the function $f(\zeta, t_0)$ can be expanded as a function of the sinusoid $\sin[2(\omega_n t_0 + \phi)]$. Thus, the graph of the natural log of the magnitude of the FFT, $\ln|F(\omega_n, t)|$, versus time, t_0, is the superposition of a straight line with slope $-\zeta\omega_n$ and a component that oscillates about the line with frequency $2\omega_n$. The envelope signal is proportional to $|F(\omega_n, t)|$, and from this, the damping ratio is readily determined from the slope of the line. This moving block analysis was implemented in MATLAB for batch processing of experimental data.

An on-line damping estimate would be desirable, particularly in a rotor test as the rotor transitions to a new operating condition, as with a change of collective pitch. However, due to the heavy computational costs incurred by calculating an FFT at each time point, t_k, this particular method is not particularly well suited for real time application. In addition, a significant time period of data is required to achieve an acceptable frequency resolution for spectral isolation of rotor modes. Finally, it has been shown previously[17] that this technique is not well suited for higher damping levels (typically > 1%).

3.2. Fourier Series Based Moving Block

In order to avoid some of the limitations of the FFT based moving block mentioned previously, a different moving block analysis was also considered. Tasker and Chopra[16] used Hamming's local Fourier series solution, which is a recursive technique used to calculate the Fourier coefficients for slowly time-varying signals. This algorithm depends greatly on the concept of a slowly changing envelope signal. In cases where the damping level is high (>1%), this may not be the case, and this method may not be the most appropriate choice. In this paper, a simple compromise is made, sacrificing some computational speed in order to accurately identify envelope signals for transients associated with higher damping levels. In this case, a non-recursive periodic Fourier series based moving block (FSMB) analysis is used. It maintains the primary advantage of Goertzel's algorithm (frequency resolution) as described by Tasker and Chopra,[16] while sacrificing slightly the speed gain of the recursive method. In return, this technique does not require that the signal be slowly time varying. In other words, this method is capable of accurately identifying high damping ratios.

In this method, the signal is assumed to be of the form:

$$x(t) = A_1 \cos(\Omega t) + B_1 \sin(\Omega t) \tag{13}$$

where the Fourier coefficients are given by:

$$A_1(t_k) = \int_{t_k}^{t_k+\frac{2\pi N_c}{\Omega}} x(t)\cos(\Omega t)dt \tag{14}$$

$$B_1(t_k) = \int_{t_k}^{t_k+\frac{2\pi N_c}{\Omega}} x(t)\sin(\Omega t)dt \tag{15}$$

with N_c = block length in number of cycles of data where each cycle corresponds to a period of duration $\frac{2\pi}{\Omega}$, and t_k = time of the kth sample in the transient. These coefficients are calculated for the data window at time t_k, and then the window of data is moved forward one data point and the coefficients are recalculated. This process is repeated for the number of iterations desired, (k = 1,2,3,N_{iter}), and the envelope is then estimated using the following relation:

$$\hat{a}(t_k) = \sqrt{A_1(t_k)^2 + B_1(t_k)^2} \tag{16}$$

Once the envelope has been determined, the damping level may be identified based on the damping mechanism under consideration.

As discussed earlier, an online damping estimate would be desirable, particularly in rotor testing. Unlike the FFT-based moving block algorithm, this Fourier series based technique is much less computationally intensive. As a result, this method lends itself much better to online implementation.

3.3. Hilbert Damping Analysis

Damping estimation reduces to the determination of the amplitude of the decay envelope signal from a transient structural response. The amplitude of the envelope signal between peak points of the dispersive transient response is intuitive, however, as stated above, most damping estimation techniques do not directly address determination of the envelope signal. In contrast, the Hilbert transform of a real valued signal, $y(t)$, provides a means for calculating this envelope signal directly.[18] The Hilbert transform, $\tilde{y}(t)$, is defined as the convolution integral:

$$\tilde{y}(t) = H[y(t)] = \int_{-\infty}^{\infty} \frac{y(u)}{\pi(t-u)}du \tag{17}$$

The Hilbert transform is a linear integral transform. However, a second way to define the Hilbert transform is as a 90° phase shift system. Essentially, the Hilbert transform can be thought of as passing a signal through a filter that leaves its magnitude unchanged, but shifts the phase by 90° for positive frequencies. Thus, $\tilde{y}(t)$ is simply $y(t)$ shifted by 90°. Thus, assuming viscous damping, the decaying transient is given by:

$$y(t) = e^{-\zeta\omega_n t}\cos(\omega_d t + \phi) \tag{18}$$

The Hilbert transform is then the signal, phase shifted by 90°, as here:

$$\tilde{y}(t) = e^{-\zeta\omega_n t}\sin(\omega_d t + \phi) \tag{19}$$

The third and most useful interpretation of the Hilbert transform is as the imaginary part of an analytic signal. We define the analytic signal:

$$z(t) = y(t) + j\tilde{y}(t) \tag{20}$$

as the sum of the real signal, $y(t)$, plus an imaginary Hilbert transform signal, $j\tilde{y}(t)$. The phasor form is:

$$z(t) = a(t)e^{j\theta(t)} \tag{21}$$

where $a(t)$ is the envelope signal of $y(t)$, and $\theta(t)$ is the instantaneous phase signal of $y(t)$. Using the signal, $y(t)$, and its Hilbert transform, $\tilde{y}(t)$, the envelope signal is given by:

$$a(t) = \sqrt{y(t)^2 + \tilde{y}(t)^2} \tag{22}$$

and the instantaneous phase signal is given by:

$$\theta(t) = \tan^{-1}\left[\frac{\tilde{y}(t)}{y(t)}\right] = 2\pi f_0 t \tag{23}$$

In addition, the instantaneous frequency, f_0, is given by

$$f_0(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} \tag{24}$$

The instantaneous phase and frequency are not used extensively in this study due to the fixed nature of the frequency of interest. These quantities would, however, be of benefit in studying nonlinear damping mechanisms where the nonlinear effects cause a shift in the frequency of interest in the response. Again, once the envelope signal has been determined, the damping level may be identified based on the damping mechanism under consideration.

As in the Fourier series based moving block analysis, the Hilbert damping analysis is much less computationally intensive as compared to the FFT-based moving block technique. Therefore, the Hilbert damping analysis is also a good candidate for online implementation.

4. TYPICAL ENVELOPE SIGNALS

Accurate identification of the envelope signal from transient data is the key to all of the damping identification techniques considered here. Fig. 1 illustrates the difference between envelope signals identified by the Hilbert damping analysis and those identified by the FSMB damping analysis. It is easily seen that the upper envelope signal, identified using the Hilbert damping analysis, contains more high frequency information than the lower envelope, identified using the FSMB damping analysis. This high frequency content comes from the 5% noise present in the transients shown. The FSMB analysis exhibits a smoothing or averaging effect on the envelope, providing a cleaner, clearer looking envelope for analysis. A similar effect could be obtained by filtering the envelope signal obtained using the Hilbert damping analysis.

5. DAMPING IDENTIFICATION FOR ISOLATED MODES

In order to evaluate the performance of the damping identification techniques considered here, nonlinear Coulomb and nonlinear quadratic damping mechanisms are investigated via simple simulation studies in order to evaluate the effectiveness of each method. The effects on damping estimates by changes in block length, noise level, and error in assumed natural frequency are investigated. To better understand the effects of these factors on the accuracy of the damping identification algorithms used in this study, a single spectrally isolated mode was chosen for analysis. In this case, a natural frequency of 9 Hz was chosen, as it is nominally the first lag frequency of the Froude-scaled bearingless composite rotor in our experimental setup at the University of Maryland. In addition to the baseline cases, analytical responses were generated and damping identification results calculated which included noise, varied block lengths, and error in the assumed frequency of interest.

5.1. Effects of Block Length

In the moving block analyses, the damping estimate is very dependent on the size of the data window used. In the case of the FFT-based analysis, this is the number of points over which the FFT is calculated each time, while in the case of the Fourier series based analysis, this corresponds to the number of complete cycles for which the Fourier coefficients are calculated each time.

It has been shown previously that the moving block analysis works well for systems with low viscous damping levels.[16,17] However, damping identification in systems with nonlinear damping mechanisms such as Coulomb and quadratic damping can provide a greater challenge.

In Figs. 2, 3 and 4, the equivalent viscous damping is plotted vs. time for the Coulomb ($\mu = 300$) or quadratic ($\epsilon = 0.005$) damping mechanisms. In addition, analytical results of the equivalent viscous damping level are shown, having been calculated using Eq. 4 or Eq. 9 and the corresponding nominal damping ratio known to be present. These analytical results are compared to damping ratio estimates using the Hilbert damping analysis and moving block analyses. The Hilbert damping analysis performs well for all cases shown, typically performing as well or better than the moving block analyses. It should be noted that the Hilbert damping analysis does not rely on a moving block methodology.

Fig. 2 also shows the results of varying the block length, N_b, of the FFT-based analysis for the quadratic damping case. The damping estimates vary greatly with block length, and improve as the block length is shortened. However, even at a block length of 64 points, which corresponds to 1/16th of a second of data, this analysis has poor performance, with the error ranging from 50% early in the signal to 10% at the end of the transient. Further, at this block length, the frequency resolution of this technique has degraded to 16 Hz. If a slightly better block length is chosen, say 256 points, the frequency resolution is improved to 4 Hz, but the error grows to between 90% and 100%. It is obvious from this that the FFT-based analysis simply cannot accurately identify nonlinear damping mechanisms such as Coulomb and quadratic damping.

The results of varying the number of cycles used in the Fourier series based analysis with 10% noise are shown in Figs. 3 and 4. As in the FFT-based case, it is evident that as the number of cycles, N_c, is decreased, (i.e. as a smaller number of data points is used in each computation), the equivalent viscous damping estimate improves. Unlike the FFT-based technique, however, there is no loss of frequency resolution because the Fourier coefficients are calculated at a specific predetermined frequency, Ω. Thus, there is no penalty for reducing the number of cycles in the moving block, and this technique can accurately be used to identify nonlinear damping mechanisms such as Coulomb and quadratic damping. Therefore, for all further analyses, the FFT-based moving block will be dropped from consideration, and only the Fourier series based moving block and the Hilbert damping analyses will be considered.

5.2. Effects of Noise

Another concern when trying to identify the damping level in an experimental data set is the presence of noise. To identify the effects of noise on the accuracy of the damping identification analyses, analytical transients with varying noise levels (0%, 1%, 5%, and 10%) were generated. In this study, the addition of 10% noise corresponds to the addition of a signal with a normal distribution of random values with a mean of zero and a variance of 10% of the initial amplitude of the transient being studied. These transients were analyzed using both the Hilbert and Fourier series based moving block analyses. Equivalent viscous damping estimates for the Coulomb and quadratic damping mechanisms with 10% noise are shown in Figs. 3 and 4 respectively. Analytical results are also shown for comparison. It can be seen from Fig. 3 that for Coulomb damping with 10% noise, the Hilbert damping analysis outperforms the

Fourier series based moving block analysis. In the case of quadratic damping, however, Fig. 4 shows that the Fourier series based damping analysis actually performs slightly better than the Hilbert damping analysis at 10% noise.

To better understand the effects of noise on the identified damping level, a series of transients was generated in which the Coulomb damping ratio, μ, was varied over a range of 100 to 500 in increments of 25, and the quadratic damping ratio, ϵ, was varied over a range of 0.003 to 0.007 in increments of 0.00025. In addition, the noise level was varied from 0% to 10% in increments of 0.5%. These transients were analyzed using both damping identification techniques, and the percent error between actual and identified damping level was calculated. This matrix of error values was then used to generate the contours of constant error shown in Figs. 5 and 6.

It can be seen in Fig. 5 that for a single mode with Coulomb damping, low noise levels have very little or no effect on the identified damping level. As the noise level increases beyond this, the error in the identified damping level begins to increase as well. It may also be seen that there is a strong dependence on the threshold cutoff value used in the analysis. As the threshold value is raised, the error for a given noise level is reduced. As an example, for 8% noise and a threshold cutoff of 20%, the error can be as large as 50%. At this same noise level, a threshold cutoff of 25% will result in an error no higher than 15%.

Fig. 6 illustrates the effects of noise on a single mode with quadratic damping. In the case of the Hilbert damping analysis, the lower threshold cutoff values provide slightly better performance than the higher values, indicating the importance of including as much of the transient as possible to help identify the correct envelope behavior. The results for the FSMB show very little variation in error over the entire range of noise and damping levels. This is attributed to the averaging effect of the FSMB mentioned earlier.

In general, it should be noted that the threshold cutoff value should be set higher than any background noise or persistent excitation whenever possible. Further, the shape of the envelope signal is a critical factor in accurate damping identification. Defining this shape accurately is therefore one of the most important considerations when determining the most effective cutoff level.

5.3. Effects of Frequency Error

Most damping identification analyses require knowledge of the frequency of interest in the damped response to accurately identify the damping level. The intent of this study is to determine the effect of error in the initial identification of the frequency of interest on the accuracy of the damping estimate. In order to accomplish this, a damped analytical response was generated at 9 Hz for each of the three damping mechanisms studied, and the damping identification analyses were run with the identified frequency of interest being varied from 8 Hz to 10 Hz. In the case of Coulomb damping, the tendency is for the algorithms to underpredict m at lower frequencies and overpredict m at higher frequencies. In the case of quadratic damping, this trend is reversed, with the results for e being too high at lower frequencies and too low at higher frequencies. Also in this case, 10% noise causes much more erratic behavior in the Hilbert damping analysis. Finally, in most cases, the identified damping level was relatively insensitive to the addition of noise.

As a general rule, if the frequency of interest can be identified within a reasonable margin, say 0.5 Hz, the error associated with the identified damping level will be minimal. If the frequency of interest cannot be identified this accurately, the associated error in damping level can become unacceptable.

6. DAMPING IDENTIFICATION FOR SPECTRALLY CLOSE MODES

To more closely emulate helicopter rotor system test data, these techniques were then applied to a signal composed of two closely spaced modes. This data was developed to simulate a response containing the first lag and 1/rev modes. The primary mode of interest (simulated lag mode) had either Coulomb or quadratic damping, and the close mode (1/rev) was either undamped or had a specified amount of viscous damping. A comprehensive evaluation of the effects of close mode amplitude, frequency, and damping level was then performed. In these studies, the Coulomb damping ratio, μ, was varied over a range of 100 to 500 in increments of 25, and the quadratic damping ratio, ϵ, was varied over a range of 0.003 to 0.007 in increments of 0.00025.