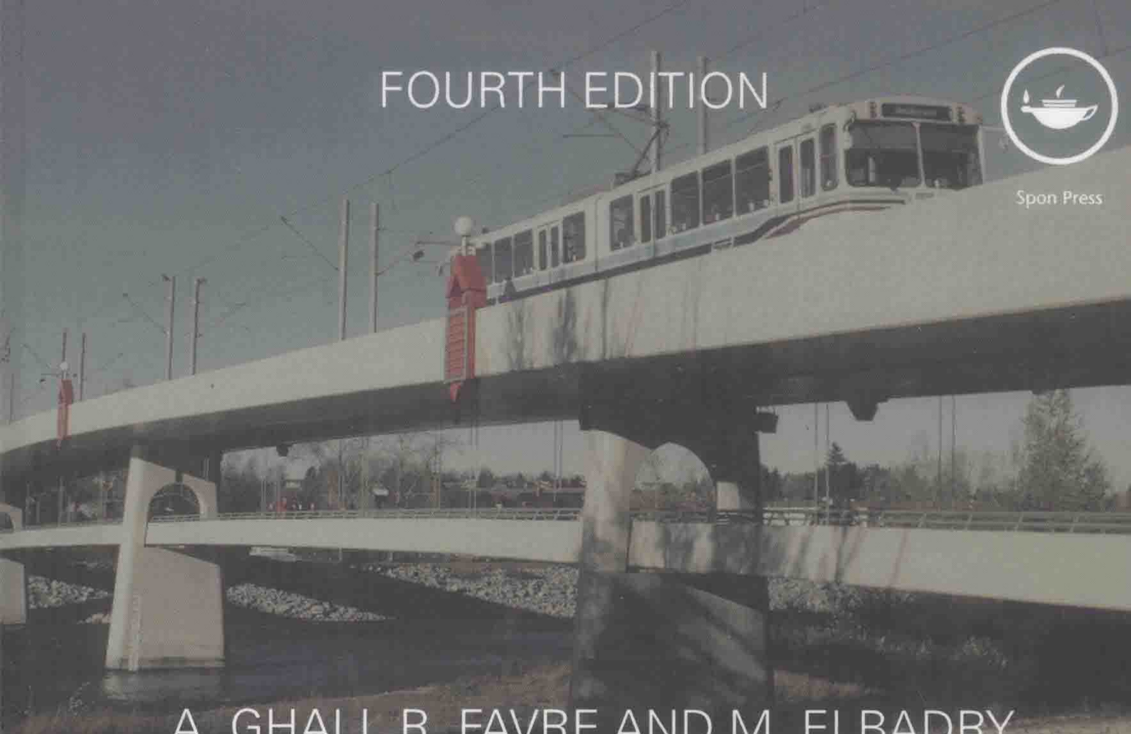

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A. GHALI, R. FAVRE AND M. ELBADRY

CONCRETE STRUCTURES

STRESSES AND DEFORMATIONS: ANALYSIS AND DESIGN FOR SERVICEABILITY

Concrete Structures

Stresses and Deformations: Analysis and Design for Sustainability

Fourth Edition

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values of these parameters for use in design. Appendix E, also based on the same sources, deals with crack width and crack spacing.

The methods of analysis of stresses and deformations presented in the chapters of the text are applicable in design of concrete structures regardless of codes. Thus, future code revisions as well as codes of other countries may be employed.

Some of the examples in the text are dimensionless. Some examples are worked out in the SI units and others in the so-called British (Imperial) units, customary to engineers in the USA; the input data and the main results are given in both SI and Imperial Units. It is hoped that the use of both systems of units will make the text equally accessible to readers in all countries. Working out different examples in the two systems of units is considered more useful than the simpler task of working each example in both units.

In the second edition, a chapter discussing control of cracking was added. Four new chapters are added in the third edition. The new Chapter 6 explains how linear computer programs, routinely used by almost all structural engineers, can be employed for analysis of the time-dependent effects of creep, shrinkage and relaxation. Chapter 12 discusses the choice of amount and distribution of prestressed and non-prestressed reinforcements to achieve best serviceability. Fibre-reinforced polymer (FRP) bars and strands are sometimes used as reinforcement of concrete in lieu of steel. Chapter 14 is concerned with serviceability of concrete structures reinforced with these materials. The effect of cracking on the reactions and the internal forces of statically indeterminate reinforced concrete structures requires non-linear analysis discussed in Chapter 13.

The analysis procedures presented in the text can in part be executed using computer programs provided on www.sponpress.com/concretestructures, for use as an optional companion to this book. The new Appendix G describes the programs on the website and how they can be used.

Mr. S. Youakim, doctoral candidate, and Mr. R. Gayed, M.Sc. student, at the University of Calgary prepared the figures and checked the revisions in the third edition; Mrs. K. Knoll-Williams typed the new material. We are grateful to them as well as to those who have helped in the earlier editions.

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January, 2002

Preface to the fourth edition

This and earlier editions of this book serve as guides on the design of structures for serviceability. Many concrete structures have deteriorated prematurely; thus, modern design pays more attention to sustainability – a structure has to offer high-quality service over a longer life span. Nowadays, specifying a life span of 100 years for infrastructures is not uncommon. For structural concrete, sustainable high-quality service requires appropriate analysis of stresses and deformations to control deflections and cracking. The material added in this fourth edition should enhance this objective.

Earlier editions of this book presented analyses of the strains and stresses in sections subjected to a normal force at a point on an axis of symmetry, combined with a bending moment about a perpendicular axis. The neutral axis is perpendicular to the axis of symmetry and generally moves, without change of direction, due to creep and shrinkage of concrete and relaxation of pre-stressing steel and cracking. This fourth edition includes analyses of sections of general shape, subjected to a normal force at any position and bending moments about two orthogonal axes. The analysis gives the position of the neutral axis and its direction and the changes in the two parameters with time or cracking. The analysis can be done using a computer program called CGS (Cracked General Sections), included on the Website of this edition (Appendix G).

Concrete of high specified strength is often used to minimize the size of heavily loaded columns. Creep and shrinkage of concrete gradually alleviate the compressive stress on the concrete, but substantially increase the compressive stress in the steel; subsequently applied bending moments due to lateral forces or live gravity loads can cause yielding of the highly compressed steel. The conditions under which this can occur, the resulting drop in stiffness and how it can be avoided are discussed.

The time functions for the modulus of elasticity, the creep coefficient, the shrinkage and the aging coefficient of concrete are expected to change in a coming issue of the fib CEB-FIP Model Code 2010. Those time functions that are known at the time of writing are used in preparing graphs for creep and aging coefficients in Appendix A and in a computer program called CRP

(Creep and Shrinkage) available on the Website (Appendix G). CRP also gives the drying and the autogenous shrinkage, the latter being particularly significant in high-performance concrete.

Problems and answers are added in an appendix. SI units of measurements are used in some of the problems and Imperial units, customary in the USA, in different problems. This was considered better use of the space available than presenting the same problems in the two systems.

Mr C. Peiris, a doctoral student at the University of Calgary, and his wife, Mrs D. Perera, typed the text and prepared the figures for the fourth edition. We are grateful to them.

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Calgary, Canada
January, 2011

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The authors would like to thank those who helped in the preparation of the first edition of the book. In Lausanne, Dr M. Koprna, Research Associate, reviewed parts of the text and collaborated in writing Chapter 8 and Appendix A; Mr J. Trevino, Research Assistant, made a considerable contribution by providing solutions or checking the numerical examples and preparing the manuscript for the publisher; Mr B.-F. Gardel prepared the figures. In Calgary, Mr M. Elbadry and Mr A. Mokhtar, graduate students, checked parts of the text, Mr B. Unterberger prepared by computer the graphs of Appendix F; Miss C. Larkin produced an excellent typescript.

The authors deeply appreciate the work of Dr S. El-Gabalawy of the Department of English at the University of Calgary, who revised the manuscript.

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Note

It has been assumed that the design and assessment of structures are entrusted to experienced civil engineers, and that calculations are carried out under the direction of appropriately experienced and qualified supervisors. Users of this book are expected to draw upon other works on the subject including national and international codes of practice, and are expected to verify the appropriateness and content of information they draw from this book.

The SI system of units and Imperial equivalents

Length	
metre (m)	1 m = 39.37 in 1 m = 3.281 ft
Area	
square metre (m^2)	1 m^2 = 1550 in^2 1 m^2 = 10.76 ft^2
Volume	
cubic metre (m^3)	1 m^3 = 35.32 ft^3
Moment of inertia	
metre to the power four (m^4)	1 m^4 = 2403 × 10^3 in^4
Force	
newton (N)	1 N = 0.2248 lb
Load intensity	
newton per metre (N/m)	1 N/m = 0.06852 lb/ft
newton per square metre (N/m^2)	1 N/m^2 = 20.88 × 10^{-3} lb/ft^2
Moment	
newton metre (N-m)	1 N-m = 8.851 lb-in 1 N-m = 0.7376 × 10^{-3} kip-ft 1 kN-m = 8.851 kip-in
Stress	
newton per square metre (pascal)	1 Pa = 145.0 × 10^{-6} lb/in^2 1 MPa = 0.1450 ksi
Curvature	
(metre)$^{-1}$	1 m^{-1} = 0.0254 in^{-1}

Temperature change

degree Celsius (°C)	1 °C = (5/9) °Fahrenheit

Energy and power

joule (J) = 1 N-m	1 J = 0.7376 lb-ft
watt (W) = 1 J/s	1 W = 0.7376 lb-ft/s
	1 W = 3.416 Btu/h

Nomenclature for decimal multiples in the SI system

10^9 giga (G)
10^6 mega (M)
10^3 kilo (k)
10^{-3} milli (m)

Notation

The following is a list of symbols which are common in various chapters of the book. All symbols are defined in the text when they first appear and again when they are used in equations which are expected to be frequently applied. The sign convention adopted throughout the text is also indicated where applicable.

A	Cross-sectional area
$\{A\}$	Vector of actions (internal forces or reactions)
$\bar{A}$, $\bar{B}$ and $\bar{I}$	Area, first moment of area and moment of inertia of the age-adjusted transformed section, composed of area of concrete plus $\bar{\alpha}$ times area of reinforcement
B	First moment of area. For $\bar{B}$, see $\bar{A}$
b	Breadth of a rectangular section, or width of the flange of a T-section
c	Depth of compression zone in a fully cracked section
D	Displacement
d	Distance between extreme compressive fibre to the bottom reinforcement layer
E	Modulus of elasticity
$\bar{E}_c$	$= E_c(t_0)/[1 + \chi\varphi(t, t_0)]$ = age-adjusted elasticity modulus of concrete
e	Eccentricity
F	Force
f	Stress related to strength of concrete or steel
$[f]$	Flexibility matrix
f_{ct}	Tensile strength of concrete
$[H]$	Matrix of area or transformed area of a cross-section

$$[H] = \begin{bmatrix} A & & \text{symmetrical} \\ B_x & I_x & \\ B_y & I_{xy} & I_y \end{bmatrix}$$

h	Height of a cross-section
I_x, I_y, I_{xy}	Moments of inertia and product of inertia about axes x and y
i, j, m, n	Integers
l	Length of a member
M	Bending moment. In a horizontal beam, a positive moment produces tension at the bottom fibre
M_{r} and/or N_{r}	Values of the bending moment and/or the axial force which are just sufficient to produce cracking
N	Normal force, positive when tensile
P	Force
$\{R\}$	Vector composed of a normal force and moments about axes x and y $\{R\} = \{N, M_x, M_y\}$
RH (percent)	Relative humidity
r	Radius of gyration
$r(t, t_0)$	Relaxation function = concrete stress at time t due to a unit strain imposed at time t_0 and sustained to time t
$[S]$	Stiffness matrix
s_{r}	Spacing between cracks
T	Temperature
t	Time or age (generally in days)
VS (mm)	Volume / Surface
W	Section modulus (length^3)
y	Coordinate defining location of a fibre or a reinforcement layer; y is measured in the downward direction from a specified reference point
α	$= E_{\mathrm{s}}/E_{\mathrm{c}}(t_0)$ = ratio of elasticity modulus of steel to elasticity modulus of concrete at age t_0
$\bar{\alpha}$	$= \alpha[1 + \chi\varphi(t, t_0)] = E_{\mathrm{s}}/\bar{E}_{\mathrm{c}}$ = ratio of elasticity modulus of steel to the age-adjusted elasticity modulus of concrete
α_{t}	Coefficient of thermal expansion (degree^{-1})
$\Delta\sigma_{\mathrm{pr}}$ and $\Delta\bar{\sigma}_{\mathrm{pr}}$	Intrinsic and reduced relaxation of prestressed steel
ε	Normal strain, positive for elongation
ζ	Coefficient of interpolation between strain, curvature and deflection values for non-cracked and fully cracked conditions (states 1 and 2, respectively)
η	Dimensionless multiplier for calculation of time-dependent change in axial strain
$\mathrm{\kappa}$	Dimensionless multiplier for calculation time-dependent change of curvature
ν	Poisson's ratio
ξ	Dimensionless shape function

ρ, ρ'	Ratio of tension and of compression reinforcement to the area (bd); $\rho = A_s/bd$; $\rho' = A'_s/bd$
σ	Normal stress, positive when tensile
τ	Instant of time
$\varphi(t, t_0)$	Creep coefficient of concrete = ratio of creep to the instantaneous strain due to a stress applied at time t_0 and sustained to time t
$\chi(t, t_0)$	Aging coefficient of concrete (generally between 0.6 and 0.9; see Section 1.7 and Figs A.6–45)
$\chi\varphi(t, t_0)$	$= \chi(t, t_0)\, \varphi(t, t_0)$ = aging coefficient × creep coefficient
χ_r	Relaxation reduction coefficient for prestressed steel
ψ	Curvature (length^{-1}). Positive curvature corresponds to positive bending moment
{ }	Braces indicate a vector; i.e. a matrix of one column
[ ]	A rectangular or a square matrix
→, ↷	Single-headed arrows indicate a displacement (translation or rotation) or a force (a concentrated load or a couple)
↠	Double-headed arrow indicates a couple or a rotation; its direction is that of the rotation of a right-hand screw progressing in the direction of the arrow

Subscripts

c	Concrete
cs	Shrinkage
m	Mean
ns	Non-prestressed steel
O	Reference point
0	Initial or instantaneous
pr	Relaxation in prestressed steel
ps	Prestressed steel
s	Steel
st	Total steel, prestressed and non-prestressed
u	Unit force effect, unit displacement effect
φ	Creep effect
1,2	Uncracked or cracked state

Contents