INTERMEDIATE ALGEBRA FOR COLLEGE

HARRY LEWIS

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Preface

In developing the concepts in this text, an attempt has been made to strike a middle ground between the "cookbook" style of mathematics texts of pre-Sputnik days and the esoteric explanations of the "new" mathematics. The former leads to rote learning that, in turn, leads to very little comprehension or retention. For an understanding of the latter, the level of mathematical maturity required is well beyond that of the college student in a developmental course in mathematics.

To develop this subject area without giving the student some glimpse into the nature of mathematics would raise grave questions. The text, therefore, introduces the eleven field properties and shows how each of the principles found in this course is an outgrowth of these properties. There are relatively few proofs, and the groundwork for each of them is well laid to assure understanding.

The term "principle" in this text would normally be called a "theorem" in a somewhat more rigorous development. In addition, the language used in the statement of either a principle or a definition is frequently a compromise between "mathematical symbolism" and the far less exacting "English" interpretation. The objective is to ease students into college mathematics without drowning them in a sea of symbolic logic!

Even those few proofs that appear in the text may be deleted from the course, if the instructor prefers. The explanatory material preceding the statement of a principle and the illustrative examples that follow it are enough to give the students confidence in applying the principle.

The style of writing is "light" in order to encourage student reading. To simplify the explanatory material further, extensive use is made of mapping diagrams. Students who will take the time to read should have no difficulty understanding what they have read.

A deliberate effort has been made to treat the elementary aspects of algebra more extensively than is usually done in a text in intermediate algebra, for several reasons. Perhaps the most important of these is the fact that this single text can be used during a full year's program to cover both elementary and intermediate algebra, leading to a smooth, uninterrupted flow in the learning of these subjects. In addition, for those students who failed to master the concepts in elementary algebra, the coverage of the fundamental operations with polynomials is more extensive than that usually found in an intermediate text. Finally, instructors who feel that their students need a review of elementary algebra will find sufficient material for their needs; instructors whose classes do not require this review can skim the material rapidly.

As an aid in determining the extent of the review that may be necessary with any particular class or individual student, there are four diagnostic tests in the *Instructor's Manual*. Each of these tests covers the material in each of the first four chapters of the text. Student achievement on these tests will indicate whether there is a need for review and, if so, what direction that review should take.

These are a few of the distinguishing features of this textbook:

1. The text is divided into chapters that, in turn, are divided into sections. Wherever necessary, the sections are further separated into subsections. This design lends itself to the introduction of only one or two concepts per learning segment. There are sufficient exercises at the end of each subsection to provide the student with ample practice to consolidate an understanding of that concept.
2. Each chapter closes with a review test of every topic covered in that chapter.
3. The style of writing and the readability of the expository material permit the use of the text for self-instruction.
4. The style of writing gives students the feeling that the instructor is standing at their side making certain that each point is fully understood before moving along to the next one.
5. The exercises are abundant and carefully graded.
6. The examples that appear prior to each set of exercises cover the background needed for successful completion of the exercises. The instructor can make assignments without fear of including exercises that are in no way related to the explanatory material.
7. Every example is accompanied by an explanation that shows the need for each step of the solution that follows. The solution itself provides a guide by which students can pattern their work when doing the exercises.

Answers to the odd-numbered exercises appear at the end of the textbook. Answers to the even-numbered exercises appear in the *Instructor's Manual*, which also contains a separate test for each chapter, a mid-course examination, a final examination, and the four diagnostic tests.

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1

An Introduction to Sets

Within recent years it has been felt that the unifying thread tying together the various subjects found under the umbrella called mathematics is the concept of a "set." The formal definition of the word is,

A set is a clearly defined collection of elements.

1.1 IDENTIFYING A SET

There are two ways by which a set can be identified. In one of these we merely *list* those things that are to be in the set. We then enclose this listing by drawing braces, { } , before the first item and after the last item. An example of this is the set below.

$$\{ a, e, i, o, u \}$$

In this situation, each of the letters a, e, i, o, and u is called an *element* or *member* of the set.

The procedure for identifying a set by listing its members is called *the roster method.*

Another manner by which a set can be identified is to give a description of the elements of that set; for example,

"The set of vowels"

This procedure is called the *description method* or *rule method.*

Example 1

Change the identification of the following set from the description method to the roster method.

"The set of odd numbers greater than five and less than twenty"

Solution $\{7, 9, 11, 13, 15, 17, 19\}$

Example 2

Change the identification of the following set from the roster method to the description method.

$$\{5, 7, 11, 13, 17\}$$

Solution "The set of prime numbers greater than 3 and less than 19."

Explanation A prime number is a number that has *exactly two* divisors, the number 1 and the number itself.

There are times when it is impossible to list all the elements of a set. This occurs in a situation where the description of the set is an expression such as

"The set of counting numbers."

Were we to try to list the elements, the process would continue without end. To indicate that this is so, we write the first few elements and indicate that this pattern continues endlessly by writing three dots, called *elipsis*, after the last element shown. Thus for the set of counting numbers this will be

$$\{1, 2, 3, 4, 5, \ldots\}.$$

A set that contains more elements than can possibly be listed is called an *infinite set.* On the other hand, if it is possible to list all the elements of a set, then that set is called a *finite set.*

Although we can list all the elements of a finite set, there are times when we would prefer not to do so. Consider the following description:

"The set of whole numbers less than one million"

In designating the members of this set we write the first few elements, the last few elements, and elipsis between them.

$$\{0, 1, 2, 3, \ldots, 999{,}997, 999{,}998, 999{,}999\}$$

Notice that the *counting numbers*, also called *natural numbers*, begin with the number 1; the *whole number* include the counting numbers, however they also include the number 0.

Example 3

Change the identification of the following set from the roster method to the description method.

$$\{6, 8, 10, 12, \ldots\}$$

Solution The set of even numbers greater than 5

Example 4

Change the identification of the following set from the description method to the roster method.

"The set of natural numbers less than 100 that are divisible by 3"

Solution $\{3, 6, 9, \ldots, 93, 96, 99\}$

EXERCISES 1.1

A

Change the identification of each of the following sets from the description method to the roster method.

1. The set of even numbers greater than 1 and less than 10.
2. The set of odd numbers greater than 4 and less than 12.
3. The set of even numbers greater than 21 and less than 35.
4. The set of odd numbers greater than 30 and less than 50.
5. The set of whole numbers less than 6.
6. The set of counting numbers less than 6.
7. The set of counting numbers greater than 7 and less than 14.
8. The set of natural numbers less than 19 that are divisible by 5.
9. The set of natural numbers less than 12 that are divisible by 8.
10. The set of whole numbers greater than 10 and less than 30 that are divisible by 6.
11. The set of prime numbers less than 10.
12. The set of prime numbers greater than 20 and less than 30.
13. The set of the names of the days of the week that begin with the letter S.
14. The set of the names of the days of the week that begin with the letter M.
15. The set of fractions whose numerators are elements of $\{5\}$ and whose denominators are elements of $\{8, 9\}$.
16. The set of fractions whose numerators are elements of $\{2, 3\}$ and whose denominators are elements of $\{7, 17\}$.

B

Change the identification of each of the following sets from the roster method to the description method.

1. $\{2, 4, 6\}$
2. $\{1, 3, 5, 7, 9, 11\}$
3. $\{12, 14, 16, 18\}$
4. $\{17, 19, 21, 23\}$
5. $\{5, 10, 15, 20\}$
6. $\{12, 16, 20, 24, 28\}$
7. $\{1, 2, 3, 4, 5, 6, 7\}$
8. $\{0, 1, 2, 3, 4, 5, 6, 7\}$
9. $\{2, 3, 5, 7, 11, 13, 17\}$
10. $\{23, 29, 31, 37\}$
11. $\{12, 15, 18, 21, 24, 27\}$
12. $\{10, 20, 30, 40, 50\}$
13. {March, May}
14. {Tuesday, Thursday}

C

Change the identification of each of the following sets from the description method to the roster method.

1. The set of natural numbers
2. The set of whole numbers
3. The set of natural numbers greater than 5
4. The set of whole numbers greater than 5
5. The set of even numbers
6. The set of odd numbers greater than 10
7. The set of whole numbers less than 150
8. The set of counting numbers less than 500
9. The set of counting numbers divisible by 5
10. The set of counting numbers less than 1,000 that are divisible by 5
11. The set of even numbers greater than 10 and less than 100
12. The set of fractions whose numerators are 1 and whose denominators are the counting numbers
13. The set of fractions whose numerators are 1 and whose denominators are the natural numbers less than 1,000
14. The set of fractions whose numerators are 1 and whose denominators are the natural numbers greater than 10 and less than 100

D

Change the identification of each of the following sets from the roster method to the description method.

1. $\{4, 5, 6, \ldots\}$
2. $\{6, 9, 12, \ldots\}$
3. $\{22, 24, 26, \ldots\}$
4. $\{11, 12, 13, \ldots, 78, 79\}$
5. $\{101, 103, 105, \ldots, 997, 999\}$

1.2 THE OPERATIONS WITH SETS

PART 1

Our objective at this time is to examine the operations that are performed with sets. They are called binary operations, for each of them assigns a single set to a *pair* of sets.

The first of these operations is called *intersection.* The single set assigned to two sets under this operation is the set consisting of their *common* elements. Thus, consider the following two sets:

$$\{ a, b, c \} \quad \text{and} \quad \{ b, c, e, f \} .$$

The *common* elements of these two sets are b and c. In view of this the intersection of these two sets is

$$\{ b \quad c \} .$$

We express the above with symbols by writing

Symbol for intersection

$$\{ a, b, c \} \cap \{ b, c, e, f \} = \{ b, c \} .$$

This is read as

"The intersection of set a, b, c and set b, c, e, f is the set b, c."

The formal definition of *intersection* is—

The intersection of two sets is the set of their common elements.

If two sets have no common elements, then the set assigned to them as their intersection is called the *null set*, or *empty set*, and is signified by either of these symbols:

$$\{ \quad \} \text{ or } \phi .$$

The symbol ϕ is the Greek letter "phi."

Hence, the intersection of

$$\{ a, b, c\} \quad \text{and} \quad \{ d, e \}$$

is the null set. This can be expressed with symbols in either of two ways:

$$\{ a, b, c \} \cap \{ d, e\} = \{ \quad \}$$

or

$$\{ a, b, c \} \cap \{ d, e\} = \phi .$$

In order to simplify our work, a single capital letter is often used to represent an entire set. Thus, the letter A can be used to represent the set of even natural numbers less than 10:

$$A = \{2, 4, 6, 8\}.$$

Similarly, the letter B can be used to represent the natural numbers divisible by 4 that are less than 21:

$$B = \{4, 8, 12, 16, 20\}.$$

In view of this, the expression

$$A \cap B$$

calls for the common elements of set A and set B. Thus,

$$A \cap B = \{4, 8\}.$$

Example 1

If $A = \{1, 3, 5, 7, 9\}$ and $B = \{1, 2, 3, 4, 5\}$, then find $A \cap B$.

Solution $A \cap B = \{1, 3, 5\}$

Example 2

If $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 4, 6, 8\}$, find $A \cap B$.

Solution $A \cap B = \phi$

EXERCISES 1.2 (Part 1)

A

Determine the intersection of A and B in each of the following exercises.

1. $A = \{1, 3, 5\}$, $B = \{1, 2, 3, 4, 5\}$
2. $A = \{3, 6, 9, 12, 18\}$, $B = \{6, 12, 18, 24\}$
3. $A = \{a, c, e, g, i\}$, $B = \{a, e, i, o, u\}$
4. $A = \{$Monday, Tuesday, Wednesday, Thursday$\}$,
   $B = \{$Tuesday, Thursday$\}$
5. $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8\}$
6. $A = \{2, 4, 6, 8\}$, $B = \{2, 4, 6, 8\}$
7. $A = \{3, 6, 9, 12\}$, $B = \{3, 9, 12\}$
8. $A = \{2, 4, 6, \ldots, 16, 18, 20\}$, $B = \{10, 12, 14, \ldots\}$
9. $A = \{2, 4, 6, \ldots\}$, $B = \{1, 3, 5, \ldots\}$
10. $A = \{3, 6, 9, \ldots, 27, 30\}$, $B = \{5, 10, 15, \ldots\}$

B

Determine the intersection of A and B in each of the following exercises.

1. A is the set of the names of the days of the week.
   B is the set of the names of the days cf the week beginning with the letter S.
2. A is the set of the letters in the alphabet.
   B is the set of vowels in the alphabet.
3. A is the set of vowels.
   B is the set of consonants.
4. A is the set of even numbers greater than 10 and less than 20.
   B is the set of numbers divisible by 4 that are greater than 10 and less than 20.
5. A is the set of even numbers.
   B is the set of odd numbers.
6. A is the set of numbers greater than 10 that are divisible by 3.
   B is the set of numbers less than 20 that are divisible by 3.
7. A is the set of natural numbers that are divisible by 4.
   B is the set of natural numbers less than 30 that are divisible by 6.
8. A is the set of counting numbers.
   B is the set of whole numbers.

C

Perform the operation indicated in each of the following exercises.

1. $\{2, 3, 4, 5\} \cap \{2, 4, 6, 8\}$
2. $\{a, b, c, d, e\} \cap \{d, a, c, y\}$
3. $\{b\} \cap \{a, b, c\}$
4. $\{10, 11, 12\} \cap \{14, 15, 16\}$
5. $\{3, 6, 9\} \cap \{\ \}$
6. $\{1, 3, 5, 7\} \cap \{1, 3, 5, 7\}$
7. $\{5, 10, 15, \ldots, 95, 100\} \cap \{80, 85, 90, \ldots\}$

PART 2

The second operation performed with sets is called *union.* This operation also consists of a process by which a single set is assigned to two sets. Thus, if A and B are two sets, then the set assigned to them by this operation has as its elements all the elements of set A and of set B.

As an example, consider the following two sets:

$$\{2, 4, 6, 8\} \quad \text{and} \quad \{4, 8, 12, 16, 20\}.$$