

EIGHTH EDITION

AIRCRAFT BASIC SCIENCE

Michael J. Kroes • James R. Rardon • Michael S. Nolan



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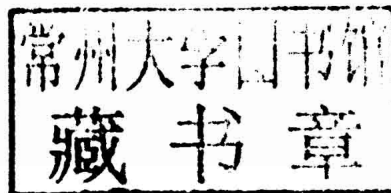
Basic Science

Eighth Edition

Michael J. Kroes

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Michael S. Nolan



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Aircraft

Basic Science

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Preface to the Eighth Edition

The purpose of *Aircraft Basic Science* is to provide the general technical information needed as a foundation for work as a technician in the field of aviation maintenance. The subjects covered in this text are those that are usually applicable to both airframes and powerplants and their associated systems. The material is of a general nature since, in addition to applying to the maintenance technician, it is also relevant to many related fields of study in aviation.

This edition has been updated to reflect changes in the aerospace industry as well as in regulatory information, and meets the requirements for general subject material specified in FAR Part 147. Electricity and electronics are not covered but are treated in detail elsewhere.

In particular, the text has been updated in the areas of high-speed aerodynamics and properties of advanced materials and composites, as well as to reflect the many FAA and regulatory changes that might be expected in this highly regulated industry. This book, when used along with complementary books and in conjunction with classroom and laboratory instruction, will provide the student with the technical information needed to qualify for certification as an airframe and powerplant technician.

This book is intended as a text and general reference book for individuals involved in aircraft maintenance and operation. The material presented is for use in understanding aircraft materials, processes, practices, and operations. Technical information contained in this book should not substitute for that provided by either the FAA or aerospace manufacturers.

Michael J. Kroes
Michael S. Nolan

Preface to the Seventh Edition

Aircraft Basic Science is one of five textbooks in the Aviation Technology Series. Its purpose is to provide the general technical information needed as a foundation for work as a technician in the field of aviation maintenance. The subjects covered in this text are those that are usually applicable to both airframes and powerplants and their associated systems. The material is of a general nature since, in addition to applying to the maintenance technician, it is also applicable to many related fields of study in aviation.

This edition has been updated to include many recent advances in aviation technology and, with the exception of electricity, has been revised to meet the requirements for general subject material specified in FAR Part 147. Electricity and electronics are not covered but are treated in detail in another text in this series.

The current “aging-aircraft” situation requires that the maintenance technician have more knowledge of materials, processes, and inspection procedures. The chapter involving fabrication processes has been expanded to provide more information in this area.

Throughout the text an increased emphasis has been placed on relating the material to both general-aviation and transport-category aircraft. In addition, the chapters on aerodynamics have been reorganized and expanded to provide a solid foundation in the theory of flight.

Due to the increased emphasis on understanding the Federal Aviation Regulations and on the ability to use technical maintenance publications, this material has been expanded and divided into two new chapters.

This book, when used along with the other books in the series and in conjunction with classroom and shop instruction, will provide the student with the technical information needed to qualify for certification as an airframe and powerplant technician.

This book is intended as a text and general reference book for those individuals involved in aircraft maintenance and operation. The material presented is for use in understanding aircraft materials, processes, practices, and operations. Technical information contained in this book should not substitute for that provided by manufacturers.

Michael J. Kroes
James R. Rardon

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We may now observe that the total number consists of 3 millions, 2 hundred thousands, 3 ten thousands, 5 thousands, 4 hundreds, 6 tens, and 7 units. The total number is read “three million, two hundred thirty-five thousand, four hundred sixty-seven.”

There are several classes of numbers. **Whole numbers**, also called **integers**, are those which contain no fractions. Examples of such numbers are 3, 10, 250, and 435. A **fraction** is a part of a unit. A **mixed number** contains a whole number and a fraction. An **even number** is one which is divisible by 2. The numbers 2, 4, 6, 8, 10, 48, and 62 are even. **Odd numbers** are those which are not divisible by 2. The numbers 3, 5, 11, 13, 53, and 61 are odd.

Addition and Subtraction

Addition and subtraction may be considered the simplest of mathematical operations; however, these operations require practice to do quickly and accurately.

Addition

Addition is the process of combining the values of two or more numbers into a single value. The combined value is called the **sum** of the values (numbers). The sign for addition is the **plus sign** (+). This sign placed between numbers indicates that they are to be added. Numbers to be added may be arranged horizontally or vertically in columns, as shown here:

324

25

+ 78

427

324 + 25 + 78 = 427

Numbers to be added are usually arranged in columns for more speed and convenience in performing the addition.

7

6

3

8

+5

29

32

420

8

19

26

+ 248

753

4382

276

1820

2753

47

+ 238

9516

Practice is one of the surest ways to learn to add accurately and rapidly. If you want to attain proficiency, you should take time to make up problems or find problems already prepared and then practice solving the problems until you feel comfortable.

It is recommended that you practice adding by sight. It is quite easy to learn to add by sight when the numbers to be added contain only one digit. With a little practice, the sight of any two digits will immediately bring the sum to mind. Thus when seeing the digits 6 and 5, for example, you should immediately think *11*, or upon seeing 9 and 7, you should instantly think *16*.

When we want to add two-digit numbers by sight, it is merely necessary to add the units and then the tens.

Suppose that the numbers 45 and 23 are presented for addition. The units are 5 and 3, so we immediately think *8 units*. The tens are 4 and 2, so we think *6 tens*. The sum of 6 tens and 8 units is 68. If the units in an addition add to a sum greater than 9, we must remember to add the ten or tens to the sum of the tens. If we wish to add 36 and 57, for example, we see that the units add to 13, or 1 ten and 3 units. We record the 3 units and **carry** the ten, adding it to the 3 tens and 5 tens. The result is 9 tens and 3 units, or 93.

Subtraction

Subtraction is the reverse of addition. The sign for subtraction is the minus sign (–). In ordinary arithmetic a smaller number is always subtracted from a larger number.

In subtraction the number from which another is to be subtracted is called the **minuend**, the number being subtracted from the other is called the **subtrahend**, and the result is called the **difference**.

675

–342

333

minuend

subtrahend

difference

In subtraction it is important to remember the components of a number, that is, the units, tens, hundreds, and so on. This will make it easier to perform the necessary operations. In the preceding example, the numbers in the subtrahend are smaller than the corresponding numbers in the minuend, and the operation is simple. If a number in the minuend is smaller than the corresponding number in the subtrahend, it is necessary to **borrow** from the next column. For example,

853

–675

178

In the first column we find the 3 smaller than the 5, and therefore we must borrow 1 ten from the next column. We then subtract 5 from 13 to obtain 8. We must remember that there are only 4 tens left in the second column, and we have to borrow 1 hundred from the next column to make 140. We subtract 70 from 140 and obtain 70, and so we place a 7 in the tens column of the answer. Since we have borrowed 1 hundred from the 8 hundreds of the third column, only 7 hundreds are left. We subtract 6 hundreds from 7 hundreds, thus leaving 1 hundred. We therefore place a 1 in the hundreds column of the answer.

Multiplication

The act of **multiplication** may be considered multiple addition. If we add 2 + 2 to obtain 4, we have multiplied 2 by 2, because we have taken 2 two times. Likewise, if we add 2 + 2 + 2 + 2 to obtain 8, we have multiplied 2 by 4, because we have taken 2 four times.

In multiplication the number to be multiplied is called the **multiplicand**, and the number of times the multiplicand is to be taken is called the **multiplier**. The answer obtained

from a multiplication is the **product**. The following example illustrates these terms:

$$\begin{array}{r} 425 \text{ multiplicand} \\ \times 62 \text{ multiplier} \\ \hline 850 \\ 2550 \\ \hline 26350 \text{ product} \end{array}$$

Note that the terms *multiplicand* and *multiplier* may be interchanged. For example, 2×4 is the same as 4×2 .

When we use multiplication to solve a specific problem, the names of the terms have more significance. For example, if we wish to find the total weight of 12 bags of apples and each bag weighs 25 pounds (lb), then the multiplicand is 25 and the multiplier is 12. We then say 12 times 25 lb is 300 lb, or $12 \times 25 = 300$.

We can understand the multiplication process by analyzing a typical but simple problem, such as multiplying 328 by 6.

$$\begin{array}{r} 328 \\ \times 6 \\ \hline 48 \\ 120 \\ 1800 \\ \hline 1968 \end{array} \quad \begin{array}{l} = 6 \times 8 \\ = 6 \times 20 \\ = 6 \times 300 \end{array}$$

In actual practice we do not write down each separate operation of the multiplication as shown in the foregoing problem, but we shorten the process by carrying figures to the next column. In the problem shown we can see that 6×8 is 48 and that the 4 goes into the tens column. Therefore, when we multiply, we merely carry the 4 over and add it to the next multiplication, which is in the tens column. When we use this method, the operation is as follows:

$$\begin{array}{r} 14 \\ 328 \\ \times 6 \\ \hline 1968 \end{array}$$

The first step in this operation is to multiply 8 by 6.

$$6 \times 8 = 48$$

Record the 8 (units) and carry the 4 (tens), then multiply 2 by 6.

$$6 \times 2 = 12$$

Add the 4 to obtain 16. Record the 6 (tens) and carry the 1 (hundreds). Then multiply 3 by 6.

$$6 \times 3 = 18$$

Add the 1 and obtain 19. Record the 19.

When there is more than one digit in the multiplier, we repeat the process for each digit, but we must shift one column to the left for each digit. This is because the right-hand digit of the multiplier is units, the next digit to the left is tens,

the next is hundreds, and so on. If we multiply 328 by 246, we proceed as follows:

$$\begin{array}{r} 328 \\ \times 246 \\ \hline 1968 \\ 13120 \\ 65600 \\ \hline 80688 \end{array} \quad \begin{array}{r} 328 \\ \times 246 \\ \hline 1968 \\ 1312 \\ 656 \\ \hline 80688 \end{array}$$

Zeros were placed at the end of the second and third multiplications in the first example to show that we were multiplying by 40 and 200, respectively. In actual practice the zeros are not usually recorded. In the preceding multiplication we multiplied 328 first by 6, then by 40, and finally by 200. When we added these products, we obtained the answer, 80 688.

Accurate multiplication requires great care. First, it is important to know the multiplication tables. Second, care must be taken to record products in the correct column. Third, the addition must be made carefully and accurately. In order to acquire proficiency in multiplication, practice is essential.

In any mathematical problem it is smart to check the answer for accuracy. There are a number of methods for checking multiplication, and the most obvious is to divide the product by either the multiplicand or the multiplier. If the product is divided by the multiplicand, the quotient (answer) should be the multiplier.

Another method for checking multiplication is to repeat the problem, reversing the multiplicand and multiplier. If the product is the same in each case, the answer is probably correct.

Division

The act of **division** may be considered the reverse of multiplication; that is, division is the separating or dividing of a number into a certain number of equal parts. The symbol for division is the division sign (\div), and it is read "divided by." For example, $98 \div 4$ is read "98 divided by 4." In arithmetic there are two commonly used methods for the division of whole numbers. These are **short division** and **long division**. The terms used to describe the elements of a division problem are **dividend**, which is the number to be divided; **divisor**, the number of times the dividend is to be divided; and **quotient**, the number of times the divisor goes into the dividend. In the problem $235 \div 5 = 47$, the number 235 is the dividend, 5 is the divisor, and 47 is the quotient.

The process of short division is often used to divide a number by a divisor having only one digit. This is accomplished as follows:

$$\begin{array}{r} 3 \\ 7 \overline{)3857} \\ \underline{551} \end{array}$$

The first step is to divide 38 by 7. Since $7 \times 5 = 35$, it is obvious that after the division of 38 by 7 there will be a remainder of 3. This 3 is held over in the hundreds column and becomes the first digit of the next number to be divided. This number is 35, and 7 goes into 35 five times without leaving a remainder. The only number left to divide is the 7, into which the divisor goes once. The quotient is thus 551.

The process of division as just explained may be understood more thoroughly if we analyze the numbers involved. The dividend 3857 may be expressed as $3500 + 350 + 7$. These numbers divided separately by 7 produce the quotients 500, 50, and 1. Adding these together gives 551, which is the quotient obtained from the short division.

Long division is employed most often when the dividend and the divisor both contain more than one digit. The process is somewhat more complex than that of short division, but with a little practice, long division may be accomplished easily and accurately.

To solve the problem $18\,116 \div 28$, we arrange the terms of the problem as shown here:

$$\begin{array}{r} 647 \\ 28 \overline{)18116} \\ \underline{168} \\ 131 \\ \underline{112} \\ 196 \\ \underline{196} \\ 0 \end{array}$$

The first step in solving the problem is to divide 181 by 28, because 181 is the smallest part of the dividend into which 28 can go. It is found that 28 will go into 181 six times, with a remainder of 13. The number 168 (6×28) is placed under the digits 181 and is subtracted. The number 13, which is the difference between 168 and 181, is placed directly below the 6 and 8 as shown, and then the number 1 is brought down from the dividend to make the number 131. The divisor 28 will go into 131 four times, with a remainder of 19. The final digit 6 of the dividend is brought down to make the number 196. The divisor 28 will go into 196 exactly seven times. The quotient of the entire division is thus 647.

If we study the division shown in the foregoing example, we will find that the dividend is composed of $28 \times 600 = 16\,800$, $28 \times 40 = 1120$, and $28 \times 7 = 196$. Then by adding $16\,800 + 1120 + 196$, we find the sum, which is 18 116, the original dividend. We could divide each part of the dividend by 28 separately to obtain 600, 40, and 7 and then add these quotients together; however, it is usually quicker and simpler to perform the divisions as shown.

If a divisor does not go into a dividend an even number of times, there will be a remainder. This remainder may be expressed as a whole number, a fraction, or a decimal. Fractions and decimals are discussed later in this chapter.

In the following example the divisor will not go into the dividend an even number of times, so it is necessary to indicate a remainder:

$$\begin{array}{r} 223\frac{10}{16} \\ 16 \overline{)3578} \\ \underline{32} \\ 37 \\ \underline{32} \\ 58 \\ \underline{48} \\ 10 \end{array}$$

Fractions

A **fraction** may be defined as a part of a quantity, unit, or object. For example, if a number is divided into four equal parts, each part is one-fourth ($\frac{1}{4}$) of the whole number. The parts of a fraction are the **numerator** and the **denominator**, separated by a line indicating division.

In Fig. 1-1 a rectangular block is cut into four equal parts; each single part is $\frac{1}{4}$ of the total. Two of the parts make $\frac{1}{2}$ the total, and three of the parts make the fraction $\frac{3}{4}$ of the total.

A fraction may be considered an indication of a division. For example, the fraction $\frac{3}{4}$ indicates that the numerator 3 is to be divided by the denominator 4. One may wonder how a smaller number, such as 3, can be divided by a larger number, such as 4. It is actually a relatively simple matter to accomplish such a division when we apply it to a practical problem. Suppose we wish to divide 3 gallons (gal) of water into four equal parts. Since there are 4 quarts (qt) in a gallon, we know that there are 12 qt in 3 gal. We can then divide the 12 qt into four equal parts of 3 qt each. Three quarts is $\frac{3}{4}$ gal; thus we see that 3 divided by 4 is equal to $\frac{3}{4}$. The principal fact to remember concerning fractions is that a fraction indicates a division. The fraction $\frac{1}{2}$ means that 1 is to be divided by 2, or that the whole is to be cut in half.

A fraction whose numerator is less than its denominator is called a **proper fraction**. Its value is less than 1. If the numerator is greater than the denominator, the fraction is called an **improper fraction**.

A **mixed number** is a combination of a whole number and a fraction, such as $32\frac{2}{5}$ and $325\frac{23}{35}$, which mean $32 + \frac{2}{5}$ and $325 + \frac{23}{35}$.

Fractions may be changed in form without changing their values. If the numerator and the denominator of a fraction are both multiplied by the same number, the value of the fraction remains unchanged, as shown in the following example:

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

The value of $\frac{9}{12}$ is the same as $\frac{3}{4}$. In a similar manner, the value of a fraction is not changed if both the numerator and the denominator are divided by the same number.

$$\frac{24 \div 12}{36 \div 12} = \frac{2}{3}$$

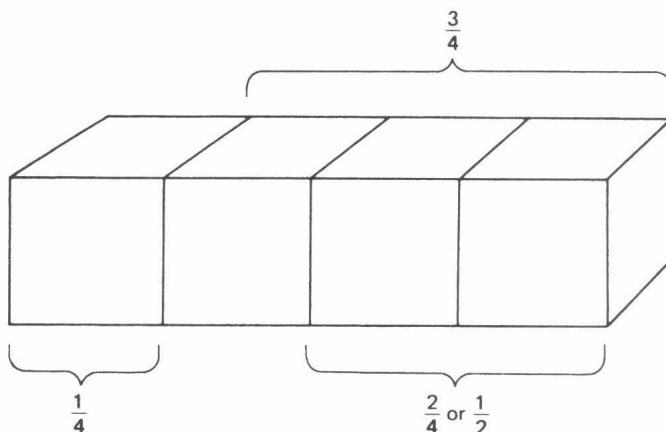


FIGURE 1-1 Fractions of a whole.

Thus we see that a large fraction may be simplified in some cases. This process is called **reducing the fraction**. To reduce a fraction to its lowest terms, we divide both the numerator and the denominator by the largest number that will go into each without leaving a remainder. This is accomplished as follows:

$$\frac{36 \div 4}{40 \div 4} = \frac{9}{10} \quad \text{and} \quad \frac{525 \div 25}{650 \div 25} = \frac{21}{26}$$

Addition and Subtraction of Fractions

In order to add or subtract fractions, the denominators of the fractions must have equal values. For example, it is not possible to add $\frac{1}{3}$ to $\frac{2}{5}$ until the denominators of the fractions have been changed to equal values. Since 3 and 5 will both go evenly into 15, we can change $\frac{1}{3}$ to $\frac{5}{15}$, and $\frac{2}{5}$ to $\frac{6}{15}$. In this case, 15 is called the **lowest common denominator (LCD)** of the fractions being considered. It is now a simple matter to add the fractions.

$$\frac{5}{15} + \frac{6}{15} = \frac{11}{15}$$

We can see that this addition makes sense because $5 + 6 = 11$. Since both the 5 and the 6 denote a specific number of fifteenths, we add them to obtain the total number of fifteenths.

The foregoing principle may be understood more easily if we apply it to a practical problem. Suppose we wish to add 3 gal and 5 qt and 1 pint (pt) of gasoline. The most logical method is to convert all quantities to pints. In 3 gal of gasoline there are 3×8 or 24 pt; in 5 qt there are 5×2 or 10 pt. Then we add 24 pt + 10 pt + 1 pt. The answer is 35 pt. If we wish to convert this quantity to gallons, we must divide the 35 by 8. We find that we have 4 gal and 3 pt, or $4\frac{3}{8}$ gal.

To prepare fractions for adding or subtracting we proceed as follows:

1. Find the LCD.
2. Divide the LCD by each denominator.
3. Multiply the numerator and denominator of each fraction by the quotient obtained when the LCD was divided by the denominator.

To practice these steps, perform the following addition:

$$\frac{3}{4} + \frac{7}{8} + \frac{5}{6}$$

The LCD is 24. Divide the LCD by the first denominator, and then multiply the fraction by this quotient:

$$24 \div 4 = 6$$

$$\frac{3 \times 6}{4 \times 6} = \frac{18}{24}$$

Do the same for the second fraction:

$$24 \div 8 = 3$$

$$\frac{7 \times 3}{8 \times 3} = \frac{21}{24}$$

And for the third fraction:

$$24 \div 6 = 4$$

$$\frac{5 \times 4}{6 \times 4} = \frac{20}{24}$$

Then add all the fractions:

$$\frac{18}{24} + \frac{21}{24} + \frac{20}{24} = \frac{59}{24} = 2\frac{11}{24}$$

Adding and Subtracting Mixed Numbers

When adding and subtracting mixed numbers, we must consider both the whole numbers and the fractions. To add $5\frac{3}{8} + 7\frac{2}{3}$, we should first add 5 and 7 to obtain 12, and then we must add the fractions. We find that $\frac{3}{8} = \frac{9}{24}$, $\frac{2}{3} = \frac{16}{24}$, and thus $\frac{9}{24} + \frac{16}{24} = \frac{25}{24}$, or $1\frac{1}{24}$. Then $12 + 1\frac{1}{24} = 13\frac{1}{24}$, the total sum of the mixed numbers.

Subtraction of mixed numbers is accomplished by subtracting the whole numbers and then the fractions. For example, subtract $8\frac{2}{3}$ from $12\frac{3}{4}$.

$$\begin{array}{r} 12\frac{3}{4} = 12\frac{9}{12} \\ - 8\frac{2}{3} = -8\frac{8}{12} \\ \hline 4\frac{1}{12} \end{array}$$

If the fraction of the subtrahend is greater than the fraction of the minuend, it is necessary to borrow 1 from the whole number in the minuend to increase the fraction of the minuend. If we wish to subtract $5\frac{7}{8}$ from $9\frac{1}{3}$, we must increase the $\frac{1}{3}$ to a value greater than $\frac{7}{8}$. The LCD of the fractions is 24, and so $5\frac{7}{8}$ becomes $5\frac{21}{24}$ and $9\frac{1}{3}$ becomes $9\frac{8}{24}$. We must then borrow 1 from 9 and add the 1 to $\frac{8}{24}$. The minuend then becomes $8\frac{32}{24}$. The final form of the problem is then

$$\begin{array}{r} 8\frac{32}{24} \\ - 5\frac{21}{24} \\ \hline 3\frac{11}{24} \end{array}$$

Multiplication of Fractions

Multiplication of fractions is accomplished by placing the product of the numerators over the product of the denominators. This result is then reduced to lowest terms. For example,

$$\frac{2}{5} \times \frac{1}{2} \times \frac{3}{4} = \frac{6}{40} = \frac{3}{20}$$

Where possible in the multiplication of fractions, cancellation is employed to simplify the fractions before final multiplication takes place.

$$\begin{array}{c} 3 \\ \cancel{8} \times \cancel{2} \times \cancel{3} \times \cancel{4} = \frac{3}{10} \\ \cancel{4} \end{array}$$

In the preceding problem we have canceled all values except the 3 in the numerator and the 10 in the denominator. First we canceled the 5s, and next we divided the 8 in the denominator by the 2 in the numerator. The 4 that was left in the denominator was then canceled by the 4 in the numerator. The 9 in the numerator was divided by the 3 in the denominator to leave a 3 in the numerator. The product, $\frac{3}{10}$, contains the product of the numerators and the product of the denominators reduced to their lowest terms. This may be proved by multiplying the numerators and denominators without canceling any values, as follows:

$$5 \times 2 \times 9 \times 4 = 360$$

$$8 \times 3 \times 10 \times 5 = 1200$$

Then,

$$\frac{360}{1200} = \frac{3}{10}$$

The reduction of the fraction was accomplished by dividing both the numerator and the denominator by 120.

A problem involving a few more operations than that given previously is

$$\frac{5}{\cancel{6}} \times \frac{\cancel{3}^3}{\cancel{7}} \times \frac{\cancel{11}}{\cancel{20}_2} \times \frac{9}{\cancel{44}_4} = \frac{135}{56} = 2\frac{23}{56}$$

In this problem note that the 25 in the numerator and the 20 in the denominator were both divided by 5 to obtain a 5 in the numerator and a 4 in the denominator.

Dividing Fractions

The division of fractions is simply accomplished by inverting the divisor and multiplying. Inverting a fraction means to turn it over; for example, if we invert $\frac{3}{4}$, it becomes $\frac{4}{3}$. It is also of interest to note that $\frac{4}{3}$ is the reciprocal of $\frac{3}{4}$. If we invert a whole number, we merely place a 1 above it. Hence, 3 becomes $\frac{1}{3}$ when it is inverted. To practice, divide $\frac{5}{8}$ by $\frac{7}{15}$.

$$\frac{5}{8} \div \frac{7}{15} = \frac{5}{8} \times \frac{15}{7} = \frac{75}{56} = 1\frac{19}{56}$$

Multiplying and Dividing Mixed Numbers

Mixed numbers may be multiplied or divided by changing the mixed numbers to improper fractions and then proceeding as with fractions. For example, try multiplying $5\frac{7}{8}$ by $3\frac{2}{3}$.

$$5\frac{7}{8} = \frac{47}{8} \text{ and } 3\frac{2}{3} = \frac{11}{3}$$

$$\frac{47}{8} \times \frac{11}{3} = \frac{517}{24} = 21\frac{13}{24}$$

For another example, divide $9\frac{3}{4}$ by $4\frac{2}{3}$.

$$9\frac{3}{4} = \frac{39}{4} \text{ and } 4\frac{2}{3} = \frac{14}{3}$$

$$\frac{39}{4} \div \frac{14}{3} = \frac{39}{4} \times \frac{3}{14} = \frac{117}{56} = 2\frac{5}{56}$$

The procedures explained in the preceding sections are not the only possible methods. Other methods will provide the same results, but those given here are commonly used.

Decimals

Decimal fractions, or **decimals**, provide a means of performing mathematical operations without using the time-consuming and complex methods of common fractions. A decimal fraction is a common fraction converted to tenths, hundredths, thousandths, or other power of ten. For example, if we convert the common fraction $\frac{3}{4}$ to a decimal, we find that it becomes 0.75. This is accomplished by dividing the numerator by the denominator:

$$\begin{array}{r} 0.75 \\ 4 \overline{)3.00} \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Any fraction may be converted to a decimal by this same process. Let us assume that we wish to convert the fraction $\frac{28}{35}$ to a decimal.

$$\begin{array}{r} 0.80 \\ 35 \overline{)28.00} \\ \underline{28.00} \\ 0 \end{array}$$

The decimal 0.80 is the same as 0.8 and may be read "eighty hundredths" or "eight tenths."

If we want to convert a fraction to a decimal when the denominator will not go evenly into the numerator, the decimal will be carried to the nearest tenth, hundredth, thousandth, or ten-thousandth according to the degree of accuracy required for the problem. For example, we may wish to convert the fraction $\frac{25}{33}$ to a decimal carried to the nearest ten-thousandth. We proceed as follows:

$$\begin{array}{r} 0.7576 \\ 33 \overline{)25.0000} \\ \underline{231} \\ 190 \\ \underline{165} \\ 250 \\ \underline{231} \\ 190 \\ \underline{198} \\ 20 \end{array}$$

Rounding Off Decimals

In the preceding problem, the answer would be alternately 7 and 5 indefinitely if we continued to carry the division onward.