Physics

FOR SCIENTISTS AND ENGINEERS

Serway

Fourth Edition

Chapters 16 - 22

2

PHYSICS

For Scientists & Engineers

Fourth Edition

Solar System Data

Body	Mass (kg)	Mean Radius (m)	Period (s)	Distance from Sun (m)
Mercury	3.18×10^{23}	2.43×10^{6}	7.60×10^{6}	5.79×10^{10}
Venus	4.88×10^{24}	6.06×10^{6}	1.94×10^{7}	1.08×10^{11}
Earth	5.98×10^{24}	6.37×10^{6}	3.156×10^{7}	1.496×10^{11}
Mars	6.42×10^{23}	3.37×10^{6}	5.94×10^{7}	2.28×10^{11}
Jupiter	1.90×10^{27}	6.99×10^{7}	3.74×10^{8}	7.78×10^{11}
Saturn	5.68×10^{26}	5.85×10^{7}	9.35×10^{8}	1.43×10^{12}
Uranus	8.68×10^{25}	2.33×10^{7}	2.64×10^{9}	2.87×10^{12}
Neptune	1.03×10^{26}	2.21×10^{7}	5.22×10^{9}	4.50×10^{12}
Pluto	$\approx 1.4 \times 10^{22}$	$\approx 1.5 \times 10^{6}$	7.82×10^{9}	5.91×10^{12}
Moon	7.36×10^{22}	1.74×10^{6}	—	—
Sun	1.991×10^{30}	6.96×10^{8}	—	—

Physical Data Often Used[a]

Average Earth-Moon distance	3.84×10^{8} m
Average Earth-Sun distance	1.496×10^{11} m
Average radius of the Earth	6.37×10^{6} m
Density of air (20°C and 1 atm)	1.20 kg/m^3
Density of water (20°C and 1 atm)	1.00×10^{3} kg/m^3
Free-fall acceleration	9.80 m/s^2
Mass of the Earth	5.98×10^{24} kg
Mass of the Moon	7.36×10^{22} kg
Mass of the Sun	1.99×10^{30} kg
Standard atmospheric pressure	1.013×10^{5} Pa

[a] These are the values of the constants as used in the text.

Some Prefixes for Powers of Ten

Power	Prefix	Abbreviation	Power	Prefix	Abbreviation
10^{-18}	atto	a	10^{1}	deka	da
10^{-15}	femto	f	10^{2}	hecto	h
10^{-12}	pico	p	10^{3}	kilo	k
10^{-9}	nano	n	10^{6}	mega	M
10^{-6}	micro	μ	10^{9}	giga	G
10^{-3}	milli	m	10^{12}	tera	T
10^{-2}	centi	c	10^{15}	peta	P
10^{-1}	deci	d	10^{18}	exa	E

Standard Abbreviations and Symbols of Units

Abbreviation	Unit	Abbreviation	Unit
A	ampere	in.	inch
Å	angstrom	J	joule
u	atomic mass unit	K	kelvin
atm	atmosphere	kcal	kilocalorie
Btu	British thermal unit	kg	kilogram
C	coulomb	kmol	kilomole
°C	degree Celsius	lb	pound
cal	calorie	m	meter
deg	degree (angle)	min	minute
eV	electron volt	N	newton
°F	degree Fahrenheit	Pa	pascal
F	farad	rev	revolution
ft	foot	s	second
G	gauss	T	tesla
g	gram	V	volt
H	henry	W	watt
h	hour	Wb	weber
hp	horsepower	μm	micrometer
Hz	hertz	Ω	ohm

Mathematical Symbols Used in the Text and Their Meaning

Symbol	Meaning
$=$	is equal to
$\equiv$	is defined as
$\neq$	is not equal to
$\propto$	is proportional to
$>$	is greater than
$<$	is less than
$\gg (\ll)$	is much greater (less) than
$\approx$	is approximately equal to
Δx	the change in x
$\sum_{i=1}^{N} x_i$	the sum of all quantities x_i from $i = 1$ to $i = N$
$\lvert x \rvert$	the magnitude of x (always a nonnegative quantity)
$\Delta x \rightarrow 0$	Δx approaches zero
$\frac{dx}{dt}$	the derivative of x with respect to t
$\frac{\partial x}{\partial t}$	the partial derivative of x with respect to t
$\int$	integral

Aerial view of windsurfer riding on the crest of a wave. This dramatic photograph illustrates many physical principles that are described in the text. For example, the water wave carries energy and momentum as it travels from one location to another. The surfer and the surfboard move in a complex path under the action of several forces, including gravity, wind resistance, and the force of water on the surfboard. *(© Darrell Wong/Tony Stone Images)*

PART II

Mechanical Waves

> The impetus is much quicker than the water, for it often happens that the wave flees the place of its creation, while the water does not; like the waves made in a field of grain by the wind, where we see the waves running across the field while the grain remains in place.
>
> **LEONARDO DA VINCI**

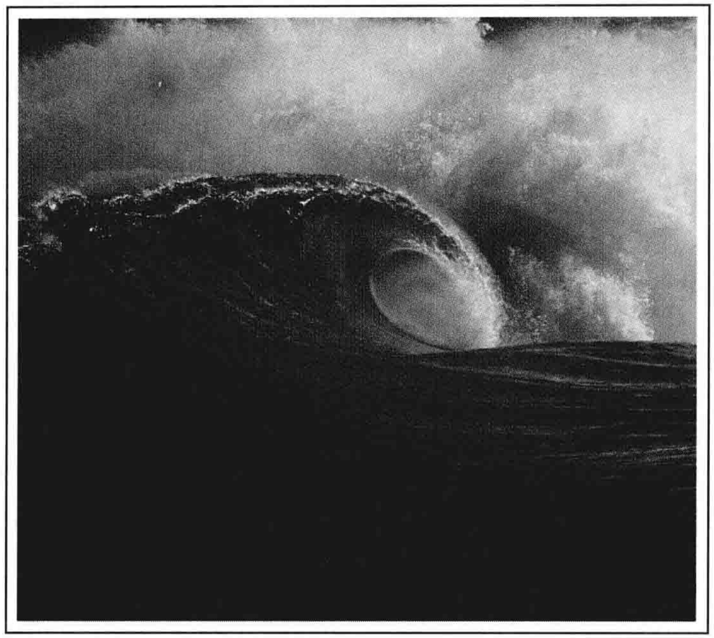
As we look around us, we find many examples of objects that vibrate: a pendulum, the strings of a guitar, an object suspended on a spring, the piston of an engine, the head of a drum, the reed of a saxophone. Most elastic objects vibrate when an impulse is applied to them. That is, once they are distorted, their shape tends to be restored to some equilibrium configuration. Even at the atomic level, the atoms in a solid vibrate about some position as if they were connected to their neighbors by imaginary springs.

Wave motion is closely related to the phenomenon of vibration. Sound waves, earthquake waves, waves on stretched strings, and water waves are all produced by some source of vibration. As a sound wave travels through some medium, such as air, the molecules of the medium vibrate back and forth; as a water wave travels across a pond, the water molecules vibrate up and down and backward and forward. As waves travel through a medium, the particles of the medium move in repetitive cycles. Therefore, the motion of the particles bears a strong resemblance to the periodic motion of a vibrating pendulum or a mass attached to a spring.

There are many other phenomena in nature whose explanation requires us to understand the concepts of vibrations and waves. For instance, although many large structures, such as skyscrapers and bridges, appear to be rigid, they actually vibrate, a fact that must be taken into account by the architects and engineers who design and build them. To understand how radio and television work, we must understand the origin and nature of electromagnetic waves and how they propagate through space. Finally, much of what scientists have learned about atomic structure has come from information carried by waves. Therefore, we must first study waves and vibrations in order to understand the concepts and theories of atomic physics.

CHAPTER 16

Wave Motion



Large waves sometimes travel great distances over the surface of the ocean, yet the water does not flow with the wave. The crests and troughs of the wave often form repetitive patterns. *(Superstock)*

Most of us experienced waves as children when we dropped a pebble into a pond. At the point where the pebble hits the water surface, waves are created by the impact. These waves move outward from the creation point in expanding circles until they finally reach the shore. If you were to examine carefully the motion of a leaf floating on the disturbance, you would see that the leaf moves up, down, and sideways about its original position but does not undergo any net displacement away from or toward the point where the pebble hits the water. The water molecules just beneath the leaf, as well as all the other water molecules on the pond surface, behave in the same way. That is, the water wave moves from one place to another, and yet the water is not carried with it.

An excerpt from a book by Einstein and Infeld gives the following remarks concerning wave phenomena.[1]

[1] A. Einstein and L. Infeld, *The Evolution of Physics,* New York, Simon and Schuster, 1961. Excerpt from "What is a Wave?".

A bit of gossip starting in Washington reaches New York very quickly, even though not a single individual who takes part in spreading it travels between these two cities. There are two quite different motions involved, that of the rumor, Washington to New York, and that of the persons who spread the rumor. The wind, passing over a field of grain, sets up a wave which spreads out across the whole field. Here again we must distinguish between the motion of the wave and the motion of the separate plants, which undergo only small oscillations. . . . The particles constituting the medium perform only small vibrations, but the whole motion is that of a progressive wave. The essentially new thing here is that for the first time we consider the motion of something which is not matter, but energy propagated through matter.

Water waves and the waves across a grainfield are only two examples of physical phenomena that have wavelike characteristics. The world is full of waves, the two main types of which are mechanical waves and electromagnetic waves. We have already mentioned examples of mechanical waves: sound waves, water waves, and "grain waves." In each case, there is some physical medium being disturbed—air molecules, water molecules, and stalks of grain in our three particular examples. Electromagnetic waves are a special class of waves that do not require a medium in order to propagate, some examples being visible light, radio waves, television signals, and x-rays. Here in Part II of this book, we shall study only mechanical waves.

The wave concept is abstract. When we observe what we call a water wave, what we see is a rearrangement of the water's surface. Without the water, there would be no wave. A wave traveling on a string would not exist without the string. Sound waves travel through air as a result of pressure variations from point to point. In such cases involving mechanical waves, what we interpret as a wave corresponds to the disturbance of a body or medium. Therefore, we can consider a wave to be the *motion of a disturbance.*

The mathematics used to describe wave phenomena is common to all waves. In general, we shall find that mechanical wave motion is described by specifying the positions of all points of the disturbed medium as a function of time.

16.1 INTRODUCTION

The mechanical waves discussed in this chapter require (1) some source of disturbance, (2) a medium that can be disturbed, and (3) some physical connection through which adjacent portions of the medium can influence each other. We shall find that all waves carry energy. The amount of energy transmitted through a medium and the mechanism responsible for that transport of energy differ from case to case. For instance, the power of ocean waves during a storm is much greater than the power of sound waves generated by a single human voice.

Three physical characteristics are important in characterizing waves: wavelength, frequency, and wave speed. One **wavelength** is the *minimum distance between any two points on a wave that behave identically,* as shown in Figure 16.1.

Most waves are periodic, and the **frequency** of such periodic waves is *the time rate at which the disturbance repeats itself.*

Waves travel with a specific speed, which depends on the properties of the medium being disturbed. For instance, sound waves travel through air at 20°C with a speed of about 344 m/s (781 mi/h), whereas the speed of sound in most solids is higher than 344 m/s. Electromagnetic waves travel very swiftly through a vacuum with a speed of approximately 3.00×10^8 m/s (186 000 mi/s).

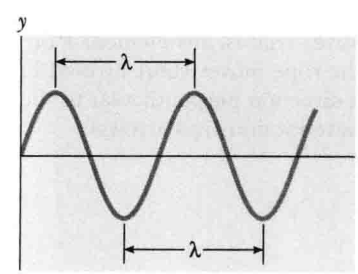


FIGURE 16.1 The wavelength λ of a wave is the distance between adjacent crests or adjacent troughs.

16.2 TYPES OF WAVES

One way to demonstrate wave motion is to flick one end of a long rope that is under tension and has its opposite end fixed, as in Figure 16.2. In this manner, a single wave bump (called a wave pulse) is formed and travels (to the right in Fig. 16.2) with a definite speed. This type of disturbance is called a **traveling wave,** and Figure 16.2 represents four consecutive "snapshots" of the traveling wave. As we shall see later, the speed of the wave depends on the tension in the rope and on the properties of the rope. The rope is the medium through which the wave travels. The shape of the wave pulse changes very little as it travels along the rope.[2]

As the wave pulse travels, *each segment of the rope that is disturbed moves in a direction perpendicular to the wave motion.* Figure 16.3 illustrates this point for one particular segment, labeled *P*. Note that no part of the rope ever moves in the direction of the wave.

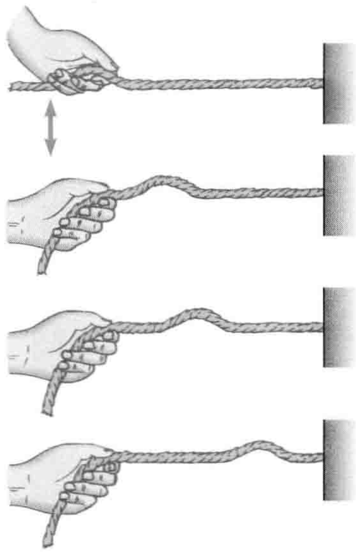


FIGURE 16.2 A wave pulse traveling down a stretched rope. The shape of the pulse is approximately unchanged as it travels along the rope.

A traveling wave that causes the particles of the disturbed medium to move perpendicular to the wave motion is called a **transverse wave.**

A traveling wave that causes the particles of the medium to move parallel to the direction of wave motion is called a **longitudinal wave.**

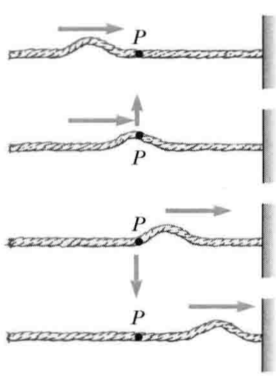


FIGURE 16.3 A pulse traveling on a stretched rope is a transverse wave. That is, any element *P* on the rope moves (blue arrows) in a direction perpendicular to the wave motion (red arrows).

Sound waves, which we discuss in Chapter 17, are one example of longitudinal waves. Sound waves in air are a series of high- and low-pressure regions, or disturbances, traveling in the same direction as the displacements. A longitudinal pulse can be easily produced in a stretched spring, as in Figure 16.4. The left end of the spring is given a sudden movement (consisting of a brief push to the right and equally brief pull to the left) along the length of the spring; this movement creates a sudden compression of the coils. The compressed region *C* (pulse) travels along the spring, and so we see that the disturbance is parallel to the wave motion. The compressed region is followed by a region where the coils are stretched.

Some waves in nature are neither transverse nor longitudinal, but a combination of the two. Surface water waves are a good example. When a water wave travels on the surface of deep water, water molecules at the surface move in nearly circular paths, as shown in Figure 16.5, where the water surface is drawn as a series of crests and troughs. Note that the disturbance has both transverse and longitudinal components. As the wave passes, water molecules at the crests move in the direction of

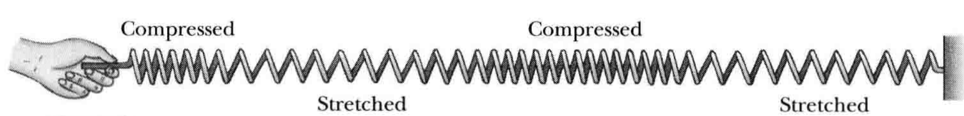


FIGURE 16.4 A longitudinal pulse along a stretched spring. The displacement of the coils is in the direction of the wave motion. For the starting motion described in the text, the compressed region is followed by a stretched region.

[2] Strictly speaking, the pulse will change its shape and gradually spread out during the motion. This effect is called *dispersion* and is common to many mechanical waves.

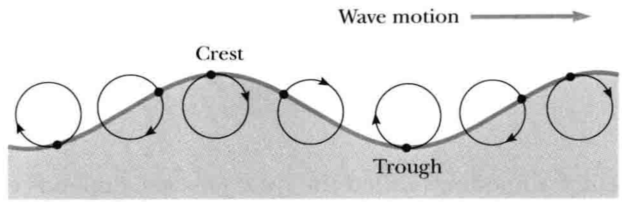


FIGURE 16.5 Wave motion on the surface of water. The molecules at the water's surface move in nearly circular paths. Each molecule is displaced horizontally and vertically from its equilibrium position, represented by circles.

the wave, and molecules at the troughs move in the opposite direction. Since the molecule at the crest in Figure 16.5 will soon be at a trough, its movement in the direction of the wave will soon be canceled by its movement in the opposite direction. Since this argument holds for every disturbed water molecule, we conclude that there is no net displacement of any water molecule.

16.3 ONE-DIMENSIONAL TRAVELING WAVES

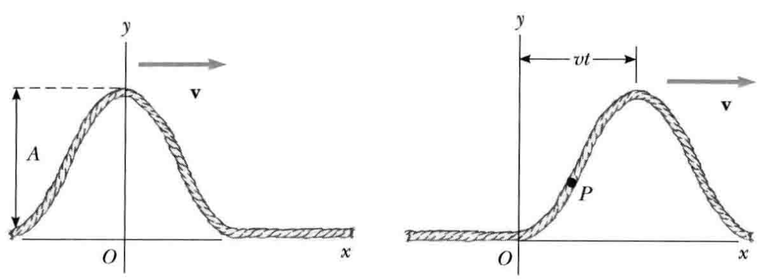
Let us now give a mathematical description of a one-dimensional traveling wave. Consider again a wave pulse traveling to the right with constant speed v on a long taut string, as in Figure 16.6. The pulse moves along the x axis (the axis of the string), and the transverse displacement of the string (the medium) is measured with the coordinate y.

Figure 16.6a represents the shape and position of the pulse at time $t = 0$. At this time, the shape of the pulse, whatever it may be, can be represented as $y = f(x)$. That is, y is some definite function of x. The *maximum displacement of the string, A*, is called the **amplitude** of the wave. Since the speed of the wave pulse is v, it travels to the right a distance vt in a time t (Fig. 16.6b).

If the shape of the wave pulse doesn't change with time, we can represent the string displacement y for all later times measured in a stationary frame having the origin at 0 as

$$y = f(x - vt) \tag{16.1}$$

Wave traveling to the right



(a) Pulse at $t = 0$ (b) Pulse at time t

FIGURE 16.6 A one-dimensional wave pulse traveling to the right with a speed v. (a) At $t = 0$, the shape of the pulse is given by $y = f(x)$. (b) At some later time t, the shape remains unchanged and the vertical displacement of any point P of the medium is given by $y = f(x - vt)$.

If the wave pulse travels to the *left*, the string displacement is

Wave traveling to the left

$$y = f(x + vt) \tag{16.2}$$

The displacement y, sometimes called the *wave function*, depends on the two variables x and t. For this reason, it is often written $y(x, t)$, which is read "y as a function of x and t."

It is important to understand the meaning of y. Consider a particular point P on the string, identified by a particular value of its coordinates. As the wave passes P, the y coordinate of this point increases, reaches a maximum, and then decreases to zero. Therefore, the **wave function** *y represents the y coordinate of any medium point P at any time t.* Furthermore, if t is fixed, then the wave function y as a function of x *defines a curve representing the shape of the pulse at this time.* This curve is equivalent to a "snapshot" of the wave at this time.

For a pulse that moves without changing shape, the speed of the pulse is the same as that of any feature along the pulse, such as the crest in Figure 16.6b. To find the speed of the pulse, we can calculate how far the crest moves in a short time and then divide this distance by the time interval. In order to follow the motion of the crest, some particular value, say x_0, must be substituted in Equation 16.1 for $x - vt$. Regardless of how x and t change individually, we must require that $x - vt = x_0$ in order to stay with the crest. This expression, therefore, represents the equation of motion of the crest. At $t = 0$, the crest is at $x = x_0$; at a time dt later, the crest is at $x = x_0 + v\,dt$. Therefore, in a time dt, the crest has moved a distance $dx = (x_0 + v\,dt) - x_0 = v\,dt$. Hence, the wave speed is

Wave speed

$$v = \frac{dx}{dt} \tag{16.3}$$

As noted above, the wave velocity must not be confused with the transverse velocity (which is in the y direction) of a particle in the medium (nor with the longitudinal velocity for a longitudinal wave).

EXAMPLE 16.1 A Pulse Moving to the Right

A wave pulse moving to the right along the x axis is represented by the wave function

$$y(x, t) = \frac{2}{(x - 3.0t)^2 + 1}$$

where x and y are measured in centimeters and t is in seconds. Let us plot the waveform at $t = 0$, $t = 1.0$ s, and $t = 2.0$ s.

Solution First, note that this function is of the form $y = f(x - vt)$. By inspection, we see that the speed of the wave is $v = 3.0$ cm/s. Furthermore, the wave amplitude (the maximum value of y) is given by $A = 2.0$ cm. At times $t = 0$, $t = 1.0$ s, and $t = 2.0$ s, the wave function expressions are

$$y(x, 0) = \frac{2}{x^2 + 1} \qquad \text{at } t = 0$$

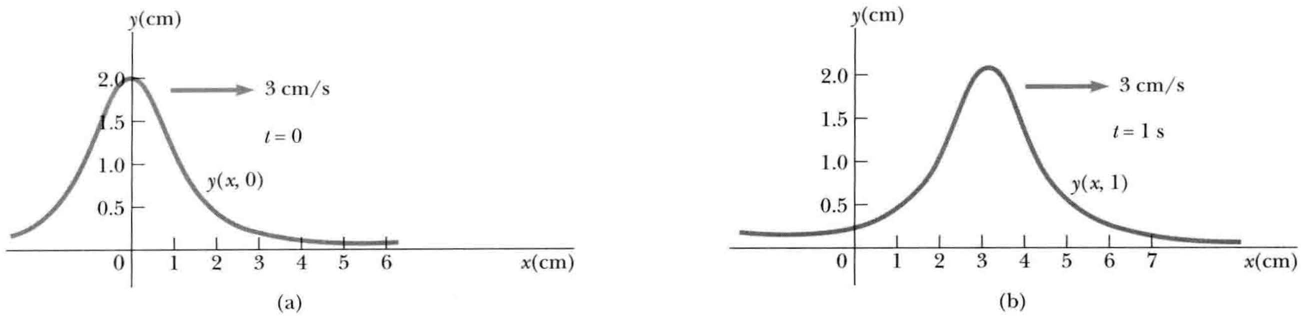
$$y(x, 1.0) = \frac{2}{(x - 3.0)^2 + 1} \qquad \text{at } t = 1.0 \text{ s}$$

$$y(x, 2.0) = \frac{2}{(x - 6.0)^2 + 1} \qquad \text{at } t = 2.0 \text{ s}$$

We can now use these expressions to plot the wave function versus x at these times. For example, let us evaluate $y(x, 0)$ at $x = 0.50$ cm:

$$y(0.50, 0) = \frac{2}{(0.50)^2 + 1} = 1.6 \text{ cm}$$

Likewise, $y(1.0, 0) = 1.0$ cm, $y(2.0, 0) = 0.40$ cm, and so on. A continuation of this procedure for other values of x yields the waveform shown in Figure 16.7a. In a similar manner, one obtains the graphs of $y(x, 1.0)$ and $y(x, 2.0)$, shown in Figures 16.7b and 16.7c, respectively. These snapshots show that the wave pulse moves to the right without changing its shape and has a constant speed of 3.0 cm/s.



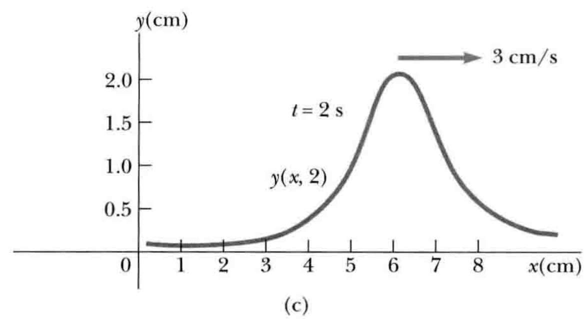


FIGURE 16.7 (Example 16.1) Graphs of the function $y(x, t) = 2/[(x - 3t)^2 + 1]$. (a) $t = 0$, (b) $t = 1$ s, and (c) $t = 2$ s.

16.4 SUPERPOSITION AND INTERFERENCE OF WAVES

Many interesting wave phenomena in nature cannot be described by a single moving pulse. Instead, one must analyze complex waveforms in terms of a combination of many traveling waves. To analyze such wave combinations, one can make use of the **superposition principle:**

If two or more traveling waves are moving through a medium, the resultant wave function at any point is the algebraic sum of the wave functions of the individual waves.

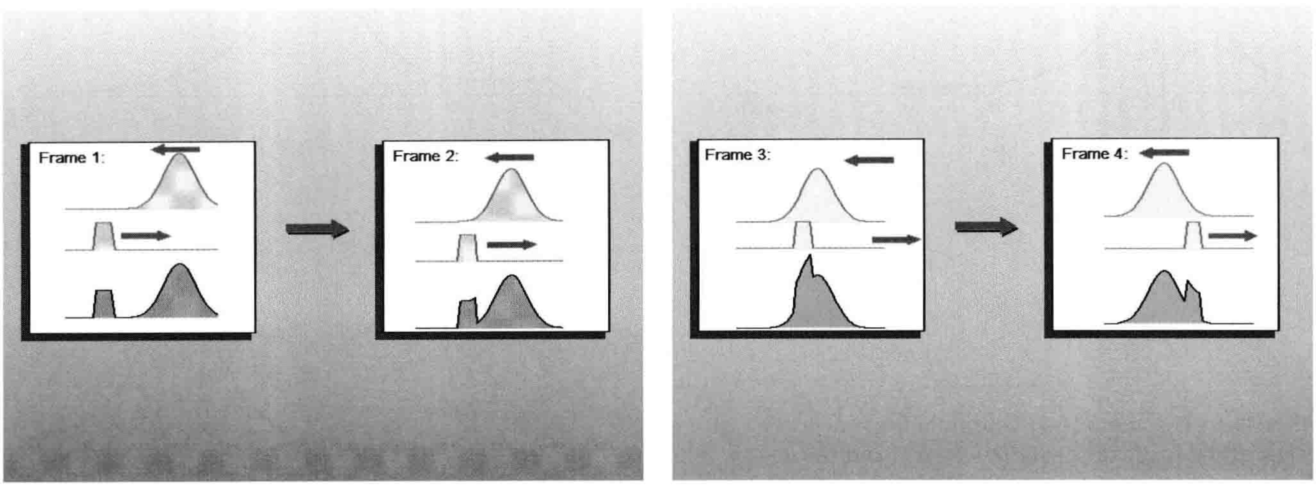
Linear waves obey the superposition principle

Waves that obey this principle are called *linear waves,* and they are generally characterized by small wave amplitudes. Waves that violate the superposition principle are called *nonlinear waves* and are often characterized by large amplitudes. In this book, we deal only with linear waves.

One consequence of the superposition principle is that *two traveling waves can pass through each other without being destroyed or even altered.* For instance, when two pebbles are thrown into a pond and hit the surface at two places, the expanding circular surface waves do not destroy each other. In fact, they pass right through each other. The complex pattern that is observed can be viewed as two independent sets of expanding circles. Likewise, when sound waves from two sources move

Wave Motion

This simulator allows you to model wave motion involving one or two traveling waves. For the case of a single wave, you will be able to specify the velocity and shape of the wave by selecting values from a given list, or define your own wave shape and velocity. For a wave traveling on a string, you can specify whether an end of the string is fixed or free and examine how the wave is reflected at this end in each case. When studying two waves traveling on a string, you can investigate their superposition as they move through each other.



through air, they also pass through each other. The resulting sound one hears at a given point is the resultant of both disturbances.

A simple pictorial representation of the superposition principle is obtained by considering two pulses traveling in opposite directions on a taut string, as in Figure 16.8. The wave function for the pulse moving to the right is y_1, and the wave function for the pulse moving to the left is y_2. The pulses have the same speed but different shapes. Each pulse is assumed to be symmetric, and the displacement of the medium is in the positive y direction for both pulses. (Note that the superposition principle applies even if the two pulses are not symmetric and even when they travel at different speeds.) When the waves begin to overlap (Fig. 16.8b), the resulting complex waveform is given by $y_1 + y_2$. When the crests of the pulses coincide (Fig. 16.8c), the resulting waveform $y_1 + y_2$ is symmetric. The two pulses finally separate and continue moving in their original directions (Fig. 16.8d). Note that the final waveforms remain unchanged, as if the two pulses had never met!

The combination of separate waves in the same region of space to produce a resultant wave is called **interference.** For the two pulses shown in Figure 16.8, the displacement of the medium is in the positive y direction for both pulses, and the resultant waveform (when the pulses overlap) exhibits a displacement greater than those of the individual pulses. Since the displacements caused by the two pulses are in the same direction, we refer to their superposition as **constructive interference.**

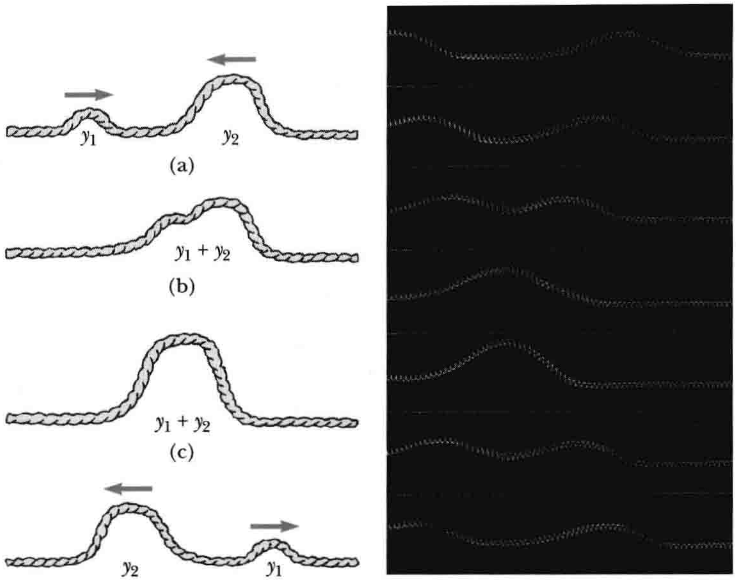


FIGURE 16.8 *(Left)* Two wave pulses traveling on a stretched string in opposite directions pass through each other. When the pulses overlap, as in (b) and (c), the net displacement of the string equals the sum of the displacements produced by each pulse. Since each pulse produces positive displacements of the string, we refer to their superposition as *constructive interference. (Right)* Photograph of superposition of two equal and symmetric pulses traveling in opposite directions on a stretched spring. *(Photo, Education Development Center, Newton, Mass.)*

Now consider two pulses traveling in opposite directions on a taut string, where now one is inverted relative to the other, as in Figure 16.9. In this case, when the pulses begin to overlap, the resultant waveform is given by $y_1 - y_2$. Again the two pulses pass through each other as indicated. Since the displacements caused by the two pulses are in opposite directions, we refer to their superposition as **destructive interference.**

Interference patterns produced by outward spreading waves from several drops of water falling into a pond. *(Martin Dohrn/SPL/Photo Researchers)*

Interference of water waves produced in a ripple tank. The sources of the waves are two objects that vibrate perpendicularly to the surface of the tank. *(Courtesy of Central Scientific CO.)*

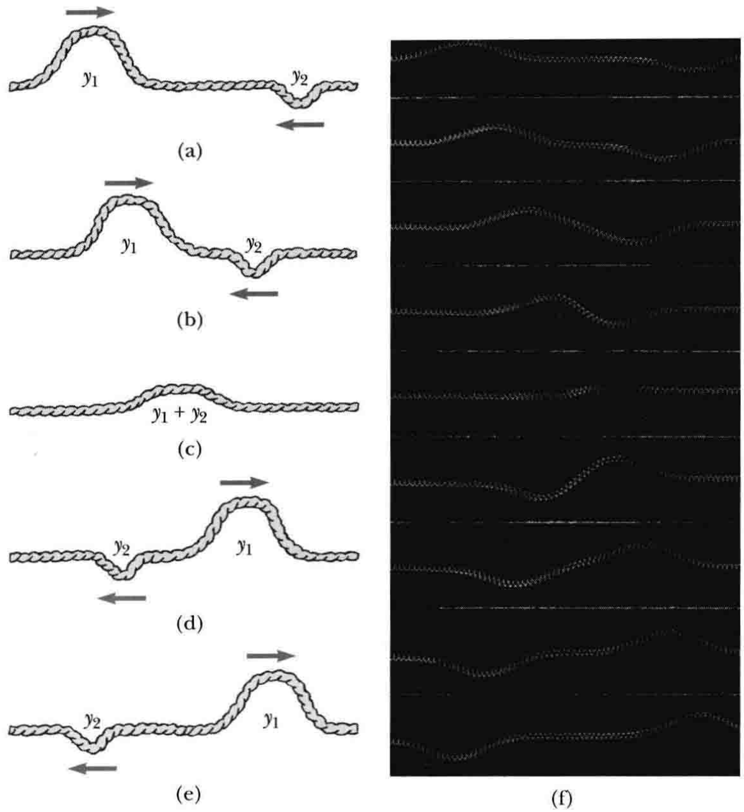


FIGURE 16.9 *(Left)* Two wave pulses traveling in opposite directions with displacements that are inverted relative to each other. When the two overlap as in (c), their displacements subtract from each other. *(Right)* Photograph of superposition of two symmetric pulses traveling in opposite directions, where one is inverted relative to the other. *(Photo, Education Development Center, Newton, Mass.)*

16.5 THE SPEED OF WAVES ON STRINGS

The speed of linear mechanical waves depends only on the properties of the medium through which the wave travels. In this section, we focus on determining the speed of a transverse pulse traveling on a taut string. If the tension in the string is F and its mass per unit length is μ, then as we shall show, the wave speed is

Speed of a wave on a stretched string

$$v = \sqrt{\frac{F}{\mu}} \tag{16.4}$$

First, let us verify that this expression is dimensionally correct. The dimensions of F are MLT^{-2}, and the dimensions of μ are ML^{-1}. Therefore, the dimensions of F/μ are L^2/T^2; hence the dimensions of $\sqrt{F/\mu}$ are L/T, which are indeed the dimensions of speed. No other combination of F and μ is dimensionally correct if we assume that they are the only variables relevant to the situation.

Now let us use a mechanical analysis to derive the above expression. Consider a pulse moving to the right with a uniform speed v, measured relative to a stationary frame of reference. Instead of staying in this frame, it is more convenient to choose as our reference frame one that moves along with the pulse with the same speed, so that the pulse is at rest in this frame, as in Figure 16.10a. This change of reference frame is permitted because Newton's laws are valid in either a stationary frame or one that moves with constant velocity.

A small segment of the string of length Δs forms an approximate arc of a circle of radius R, as shown in Figure 16.10a and magnified in Figure 16.10b. In the pulse's frame of reference (which is moving to the right along with the pulse), the shaded segment is moving down with a speed v. This small segment has a centripetal acceleration equal to v^2/R, which is supplied by the force of tension **F** in the string. The force **F** acts on each side of the segment, tangent to the arc, as in Figure 16.10b. The horizontal components of **F** cancel, and each vertical component $F \sin \theta$ acts radially inward toward the center of the arc. Hence, the total radial force is $2F \sin \theta$. Since the segment is small, θ is small and we can use the familiar small-angle approximation $\sin \theta \approx \theta$. Therefore, the total radial force can be expressed as

$$F_r = 2F \sin \theta \approx 2F\theta$$

The small segment has a mass $m = \mu\, \Delta s$. Since the segment forms part of a circle and subtends an angle 2θ at the center, $\Delta s = R(2\theta)$, and hence

$$m = \mu\, \Delta s = 2\mu R\theta$$

If we apply Newton's second law to this segment, the radial component of motion gives

$$F_r = \frac{mv^2}{R} \quad \text{or} \quad 2F\theta = \frac{2\mu R\theta v^2}{R}$$

where F_r is the force that supplies the centripetal acceleration of the segment and maintains the curvature at this point. Solving for v gives Equation 16.4. Notice that this derivation is based on the assumption that the pulse height is small relative to the length of the string. Using this assumption, we were able to use the approximation that $\sin \theta \approx \theta$. Furthermore, the model assumes that the tension F is not affected by the presence of the pulse, so that F is the same at all points on the string. Finally, this proof does *not* assume any particular shape for the pulse. Therefore, we conclude that a pulse of *any shape* will travel on the string with speed $v = \sqrt{F/\mu}$ without any change in pulse shape.

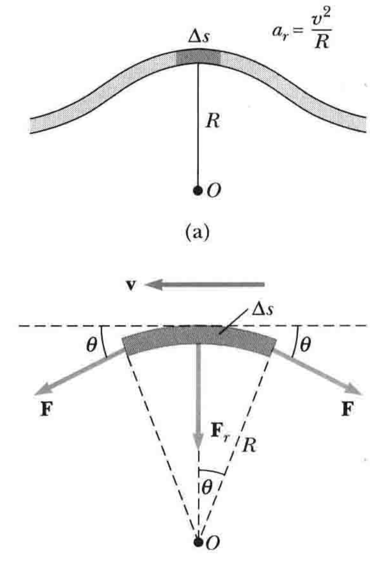


FIGURE 16.10 (a) To obtain the speed v of a wave on a stretched string, it is convenient to describe the motion of a small segment of the string in a moving frame of reference. (b) The net force on a small segment of length Δs is in the radial direction. The horizontal components of the tension force cancel.

EXAMPLE 16.2 The Speed of a Pulse on a Cord

A uniform cord has a mass of 0.300 kg and a length of 6.00 m (Fig. 16.11). Find the speed of a pulse on this cord.

Solution The tension F in the cord is equal to the weight of the suspended 2.00-kg mass:

$$F = mg = (2.00 \text{ kg})(9.80 \text{ m/s}^2) = 19.6 \text{ N}$$

(This calculation of the tension neglects the small mass of the cord. Strictly speaking, the cord can never be exactly horizontal, and therefore the tension is not uniform.)

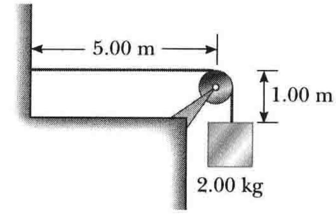


FIGURE 16.11 (Example 16.2) The tension F in the cord is maintained by the suspended mass. The wave speed is given by the expression $v = \sqrt{F/\mu}$.

The mass per unit length μ is

$$\mu = \frac{m}{\ell} = \frac{0.300 \text{ kg}}{6.00 \text{ m}} = 0.0500 \text{ kg/m}$$

Therefore, the wave speed is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{19.6 \text{ N}}{0.0500 \text{ kg/m}}} = 19.8 \text{ m/s}$$

Exercise Find the time it takes the pulse to travel from the wall to the pulley.

Answer 0.253 s.

16.6 REFLECTION AND TRANSMISSION OF WAVES

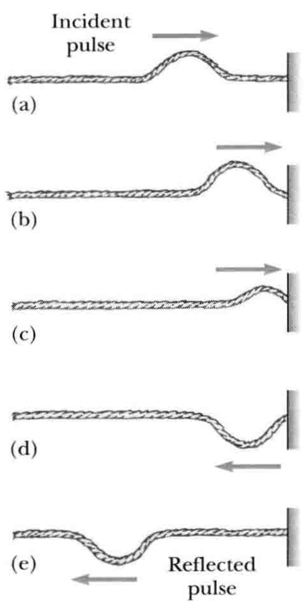


FIGURE 16.12 The reflection of a traveling wave pulse at the fixed end of a stretched string. The reflected pulse is inverted, but its shape remains the same.

Whenever a traveling wave reaches a boundary, part or all of the wave is reflected. For example, consider a pulse traveling on a string fixed at one end (Fig. 16.12). When the pulse reaches the wall, it is reflected. Because the support attaching the string to the wall is rigid, the pulse does not transmit any part of the disturbance to the wall and its amplitude does not change.

Note that the reflected pulse is inverted. This can be explained as follows. When the pulse reaches the fixed end of the string, the string produces an upward force on the support. By Newton's third law, the support must then exert an equal and opposite (downward) reaction force on the string. This downward force causes the pulse to invert upon reflection.

Now consider another case, where this time the pulse arrives at the end of a string that is free to move vertically, as in Figure 16.13. The tension at the free end is maintained by tying the string to a ring of negligible mass that is free to slide vertically on a smooth post. Again the pulse is reflected, but this time it is not inverted. As the pulse reaches the post, it exerts a force on the free end of the string, causing the ring to accelerate upward. In the process, the ring would overshoot the height of the incoming pulse except that it is pulled back by the downward component of the tension force. This ring movement produces a reflected pulse that is not inverted, and whose amplitude is the same as that of the incoming pulse.

Finally, we may have a situation in which the boundary is intermediate between these two extreme cases, that is, one in which the boundary is neither rigid nor free. In this case, part of the incident pulse is transmitted and part is reflected. For instance, suppose a light string is attached to a heavier string as in Figure 16.14. When a pulse traveling on the light string reaches the boundary between the two,

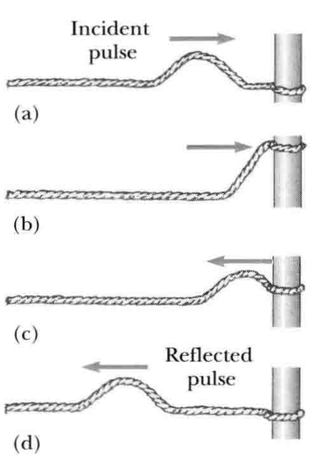


FIGURE 16.13 The reflection of a traveling wave pulse at the free end of a stretched string. The reflected pulse is not inverted.

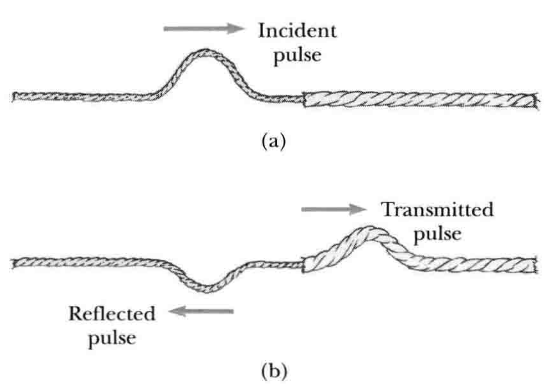


FIGURE 16.14 (a) A pulse traveling to the right on a light string attached to a heavier string. (b) Part of the incident pulse is reflected (and inverted), and part is transmitted to the heavier string. (Note that the change in pulse width is not shown.)