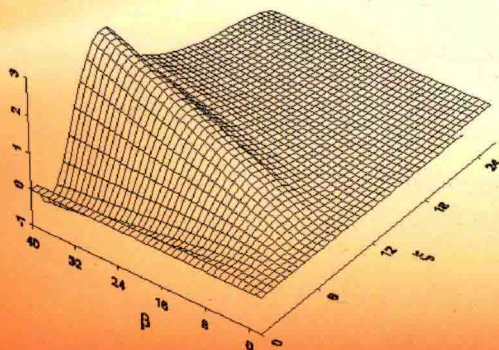
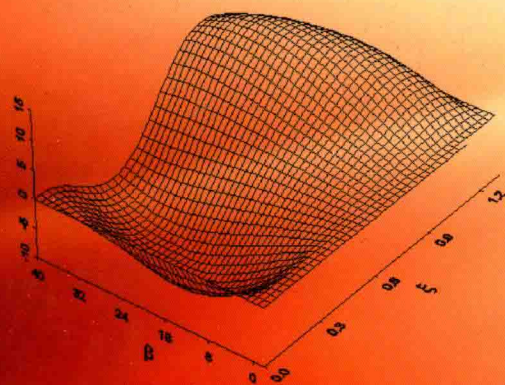
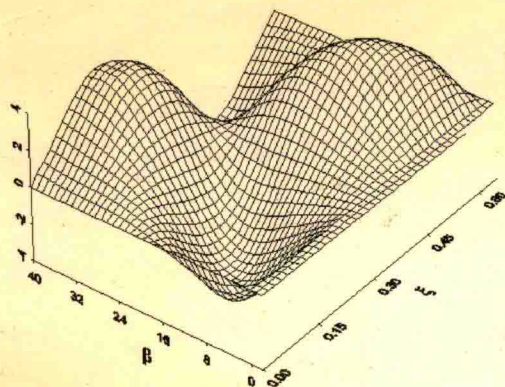
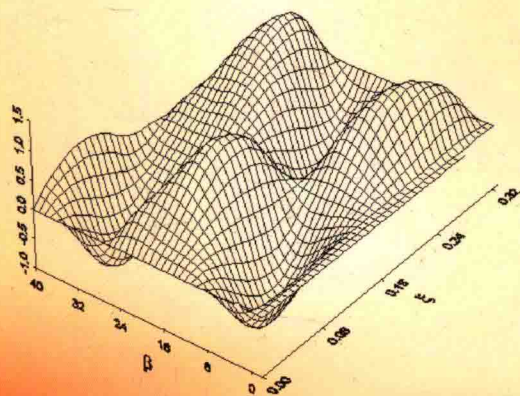


D. Y. Tzou



Macro- to Microscale Heat Transfer

The Lagging Behavior

Second Edition

WILEY

MACRO- TO MICROSCALE HEAT TRANSFER THE LAGGING BEHAVIOR

Second Edition

D. Y. Tzou

University of Missouri, USA

WILEY

This edition first published 2015
© 2015 John Wiley & Sons, Ltd.

Registered Office

John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, United Kingdom

For details of our global editorial offices, for customer services and for information about how to apply for permission to reuse the copyright material in this book please see our website at www.wiley.com.

The right of the author to be identified as the author of this work has been asserted in accordance with the Copyright, Designs and Patents Act 1988.

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, except as permitted by the UK Copyright, Designs and Patents Act 1988, without the prior permission of the publisher.

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print may not be available in electronic books.

Designations used by companies to distinguish their products are often claimed as trademarks. All brand names and product names used in this book are trade names, service marks, trademarks or registered trademarks of their respective owners. The publisher is not associated with any product or vendor mentioned in this book.

Limit of Liability/Disclaimer of Warranty: While the publisher and author have used their best efforts in preparing this book, they make no representations or warranties with respect to the accuracy or completeness of the contents of this book and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose. It is sold on the understanding that the publisher is not engaged in rendering professional services and neither the publisher nor the author shall be liable for damages arising herefrom. If professional advice or other expert assistance is required, the services of a competent professional should be sought.

Mathematica[®] is a registered trademark of Wolfram Research, Inc. and is used with permission. Wolfram Research, Inc. does not warrant the accuracy of the text or exercises in this book. The book's use or discussion of Mathematica[®] or related products does not constitute endorsement or sponsorship by Wolfram Research, Inc. nor is Wolfram Research, Inc. directly involved in this book's development or creation.

Library of Congress Cataloging-in-Publication Data

D. Y. Tzou

Macro- to microscale heat transfer : the lagging behavior / D. Y. Tzou. – Second edition.

pages cm

Includes bibliographical references and index.

ISBN 978-1-118-81822-0 (cloth)

1. Heat-Transmission. I. Title.

QC320.T96 2015

536'.2-dc23

2014021372

Set in 10/12pt Times by SPi Publisher Services, Pondicherry, India

Printed in Singapore by C.O.S. Printers Pte Ltd

MACRO- TO MICROSCALE HEAT TRANSFER

Preface

Second Edition

Rapid development of the dual-phase-lag model over the past 16 years has necessitated the publication of the second edition of *Macro- to Microscale Heat Transfer: The Lagging Behavior*. Not only has the dual-phase-lag model been applied to a wide variety of heat-transfer problems from micro- to nanoscale, but the phase-lag concept has been extended to mass transport during the ultrafast transient. Meanwhile, the theoretical foundation of the dual-phase-lag model has been continuously deepened, now including the compatibility within the framework of the Boltzmann transport equation. The nonlocal behavior posted in the first edition, which bears the same concept of thermal lagging in time but is applied to space, has now become confirmed as another salient feature in nanoscale heat transfer. Combined, it has now been clear that while the two phase lags in thermal lagging enable us to capture the ultrafast response in the femtosecond domain, the intrinsic lengths characterizing the nonlocal response enable us to describe the physical mechanisms in nanoscale.

The second edition integrates some of the milestones developed over the past 16 years. The perfect correlations to existing heat-transfer models in micro/nanoscale continue to expand, now including eleven models placed in the framework of thermal lagging/nonlocal response. New chapters and sections are added to extend the lagging behavior from heat to mass transport, which includes experimental support of the time evolution of the intermetallic layers and consequently identifications of new sources for the delayed response. The lagging/nonlocal behaviors are unveiled in coupling with other fields. The ultrafast deformation induced by the rapidly heated electrons in metals, and hence the hot electron blast responsible for the clean cut furnished by femtosecond lasers, are resolved in the picosecond transient. When coupling with the electric field, in thermoelectricity, the lagging behavior is extracted from the rapid energy exchange between the thermoelectric couple and the interstitial gas in the mushy zone of PN junctions. To support the expanding efforts in exploring the lagging behavior in biological materials, in addition, effects of multiple energy/mass carriers as well as the multistage mass diffusion across biological membranes are included. Heat and mass transport has evolved rapidly as the physical scale of observation shrinks from macro-, micro- to nanoscale. In view

of the lagging/nonlocal response, regardless of the number of carriers involved, it seems conclusive that the lagging/nonlocal response is characterized by the two phase lags (lagging response) and two nonlocal lengths during the ultrafast transient, with additional effects appearing as their high-order terms. The response regime, in time, is posted to weigh the relative importance of the two phase lags as the physical scale shrinks from micro- to nanoscale. The second edition pays even more attention to illustrate the lagging and nonlocal behavior from fundamental problems in engineering.

Examples include recovery of Newton's law from a special case in the lagging response, as well as heat transfer into the ambient from an extended surface. It is the author's hope to bring close relevance and raise attention to the lagging and nonlocal behavior from these well-known examples on the undergraduate level. Continuing the faith of the first edition of the book, tremendous effort has been put into interpreting the lagging behavior in time and nonlocal response in space, in ways that are already familiar to engineers. A new chapter is added to tackle nonlinear problems in thermal lagging/nonlocal response, where Mathematica codes are exemplified to illustrate the basic setup in solving a wide class of problems. The original FORTRAN code in the first edition is kept due to its close resemblance to language C/Matlab, should a modern computational platform be intended. The method of Laplace transform with Riemann sum approximation (for linear problems) and the finite-difference differential method (for nonlinear problems) are focused in resolving the lagging/nonlocal response in micro/nanoscale heat transfer without much distraction from other methods that often require different skill sets. Based on the fundamental understating of the lagging/nonlocal behavior thus developed, more sophisticated numerical methods could be further pursued to ensure an efficient and robust treatment in untangling the space and time tradeoffs as the response domain of heat and mass transfer continuously moves into micro/nanoscale.

In revising the book for the second edition, there are indeed materials that have become relatively obsolete due to the advancement of the dual-phase-lag model over the past 15 years. I have, however, decided to keep them, along with the new materials, since they reflect the footprints of the dual-phase-lag model since its inauguration in 1995. It is my hope that such footprints retain the original thoughts through which the dual-phase-lag model has evolved into what it is now. The second edition is dedicated to my wife, Li Na, for her decades-long, unconditional support and patience during the composition of the book.

Nomenclature

A	<ul style="list-style-type: none"> • dimensionless coefficient (2, 4, 7, 8)¹ • positive coefficient, $\text{m W}^{-1} \text{K}^{-1}$ (3) • parameter in the Laplace transform solution, m^{-1} (6) • amplitude of the near-tip temperature, K (8) • dimensionless radius (12)
A_i	<ul style="list-style-type: none"> • $i = e$ and l. Electron and lattice component in the lattice heat capacity, $[A_e] = \text{J W}^{-3} \text{K}^{-2}$; $[A_l] = \text{J W}^{-3} \text{K}^{-4}$ • $i = 1$ to 6. Positive coefficients; $[A_1, A_2, A_4] = \text{W m}^{-1} \text{K}^{-3}$; $[A_3] = \text{m W}^{-1} \text{K}^{-1}$; $[A_5] = \text{Pa}^{-1} \text{K}^{-1} \text{s}^{-1}$; $[A_6] = \text{W m}^{-3} \text{K}^{-1}$ (3) • $i = 1, 2, 3$. Coefficients in Laplace transform solutions, K s (5), dimensionless coefficients (4, 11); A_e, A_l: ratio of thermal diffusivity (5)
	$i = L$. Volumetric effective area of the vasculature, m^{-1} (10)
	$i = c, R$. Cross section area (c) or annular area surrounding the fin (R), m^2 (10)
A_{ij}	$i, j = 1, 2, 3$. Coefficients in Laplace transform solutions, K s (5)
a	<ul style="list-style-type: none"> • acceleration, m s^{-2}; discrepancy factor between conductive and thermodynamic temperatures, m^2 (3) • parameter in the normalized autocorrelation function, dimensionless (5, 11) • radius of the circular or spherical cavity, m (8) • interfacial area per unit volume, m^{-1} (9, 10) • maximum cellular uptake rate, $\text{g m}^{-3} \text{s}^{-1}$ (10)
a_i	$i = 1, 2, 3$. Generalized coefficients in the boundary conditions, dimensionless (8)
B	• $\tau_T/(2\tau_q)$ (2); τ_T/τ_q (4, 8, 12)

¹ Numbers in parentheses refer to the chapters where the corresponding symbols appear.

- positive coefficient, $\text{Pa}^{-1} \text{K}^{-1} \text{s}^{-1}$ (3)
- coefficient in Laplace transform solutions, m^{-1} (5)
- Coefficients of eigenfunctions, dimensionless (8)
- B_i • $i = 1$ to 4. Positive coefficients. $[B_1, B_2] = \text{J s m}^{-3} \text{K}^{-1}$; $[B_3] = \text{K}^{-2}$; $[B_4] = \text{K}^{-1} \text{s}^{-1}$ (3)
- $i = 1, 2$. Coefficients in Laplace transform solutions, m^{-1} (5); $i = 1, 2, 3$, dimensionless (11)
- b parameter in the Laplace transform solution, dimensionless (11, 12)
- b_i $i = 1, 2, 3$. Generalized coefficients in the boundary conditions, dimensionless (8)
- C • thermal wave speed, m s^{-1}
- volumetric heat capacity $\text{J m}^{-3} \text{K}^{-1}$
- configuration factor in slip conditions, dimensionless (1, 12)
- dimensionless volumetric heat capacity (13)
- $C_{(e,l)}$ volumetric heat capacity of electron gas (e) and metal lattice (l), $\text{J m}^{-3} \text{K}^{-1}$ (1, 5, 8, 12, 13)
- $C_{(S,g)}$ volumetric heat capacity of solid (S) and gaseous (g) phases, $\text{J m}^{-3} \text{K}^{-1}$ (9)
- C_i • $i = 1, 2, 3$. Coefficients in heat flux, W m^{-2} (1); Dimensionless coefficients in Fourier transform solutions (2); coefficients of Poisson ratio, dimensionless (11); volumetric heat capacity of carrier i , $\text{J m}^{-3} \text{K}^{-1}$ (12)
- coefficients in Laplace transform solutions, dimensionless (4, 5, 12); $[C_1] = \text{m K W}^{-1}$, $[C_2] = \text{m K S W}^{-1}$ when used with dimensions (5); $i = 1-5$, dimensionless coefficients of space and time grids (13)
- $i = 1$ to 4. $[C_1] = \text{m}^4 \text{s kg}^{-1} \text{W K}$; $[C_2] = \text{J kg}^{-1} \text{K}^{-1} \text{Pa}^{-2}$; $[C_3] = \text{m}^2 \text{J kg}^{-1} \text{K}^{-3}$; $[C_4] = \text{J kg}^{-1} \text{K}^{-1}$ (3)
- $i = E$. Dilatational wave speed, m s^{-1} (11)
- $i \equiv p, v, \kappa$, volumetric heat capacity, $\text{J m}^{-3} \text{K}^{-1}$; $i = 1, 2, 3$, coefficients involving Poisson ratio, dimensionless (11)
- C_{ij} $i, j = 1, 2$. Coefficients in Laplace transform solutions, $[C_{11}, C_{21}] = \text{m K W}^{-1}$; $[C_{12}, C_{22}] = \text{m s K W}^{-1}$ (5)
- C_p volumetric heat capacity, $\text{J m}^{-3} \text{K}^{-1}$
- $C_{L,T}$ speed of longitudinal (L) or T wave, m s^{-1} (12)
- C_v speed of CV wave, m s^{-1} (12)
- $C^{(i)}$ $i = 1, 2$. The i th wave speed in thermomechanical coupling, m s^{-1} (11)
- c • mean phonon speed, m s^{-1} (1, 2)
- $v/2\alpha$, m^{-1} (8)
- damping coefficient (spring), N s m^{-1} (10)
- parameter in the Laplace transform solution, dimensionless (11, 12)
- speed of light, m s^{-1} (12)
- $c_{E,L,I}$ concentration of free drug (E), liposome (L), and intracellular (I), g m^{-3} (10)
- c_p specific heat capacity, $\text{J kg}^{-1} \text{K}^{-1}$
- c_V liposome concentration in plasma, g m^{-3} (10)
- c_q λ_q/τ_q (12)
- D • mean diameter of grains, μm (1)
- dimensionless coefficients in Laplace transform (2)
- density of states, J^{-1} (3)
- dimensionless radius (4)

- coefficient in Laplace transform solutions, K s (6)
- fractal and fracton dimensions, dimensionless (7)
- effective diffusion coefficient, $\text{m}^2 \text{s}^{-1}$ (9, 10)
- dimensionless number (12, 13)
- $D_{D,L}$ effective diffusivity of free drug (D) or liposome drug (L), $\text{m}^2 \text{s}^{-1}$ (10)
- D_i • $i = 1, 2, 3$, dimensionless coefficients (5, 8, 9, 11). $i = 1$, coefficient, s m^{-2} ; $i = 2$, coefficient, $\text{s}^2 \text{m}^{-2}$; $i = 3$, coefficient, $\text{s}^3 \text{m}^{-2}$; (12)
- $i = 1$ to ∞ . Fourier coefficients, dimensionless (12)
- D_{21} coefficient, s (12)
- D_{22} coefficient, s^2 (12)
- d • film thickness (1)
- dimensionality of heat source or conducting media, dimensionless (7)
- optical depth of penetration, m (13)
- d_i • $i = 1, 2, 3$. Distance traveled by phonons or electrons, nm (1)
- $i = 1$ to 4. Coefficients in the asymptotic expansion, dimensionless (11)
- E • phonon/electron energy, J (1, 12)
- conjugate tensor to the Cauchy strain tensor, $\text{W m}^{-3} \text{K}^{-1}$ (3)
- averaged error threshold, dimensionless (6)
- electric field, volt m^{-1} (9)
- Young's modulus in elasticity, Pa (11, 12, 13)
- E_i $i = 1, 2$. Dimensionless coefficients in Laplace transform solutions (5)
- e • Cauchy strain tensor, mm/mm (3, 11, 12)
- volumetric or one-dimensional strain, dimensionless (11, 12)
- internal energy, J kg^{-1} (12)
- e_m mean strain, mm/mm (11)
- F • dimensionless Lamé potential (4)
- dimensionless numbers (5, 8, 9, 13)
- kernel in the memory function, $\text{W m}^{-3} \text{K}^{-1}$ (12)
- F_i $i = 1$ to 5. Dimensionless coefficients (11, 12)
- f • distribution function/probability, dimensionless (3)
- temperature rise relative to its maximum value, dimensionless (6)
- time function of diffusive temperature, $\text{s}^{-1/2}$ (7)
- transformation function or eigenfunction, dimensionless (8)
- force, N (10)
- phonon resistive force, Pa (12)
- f_i $i = 1, 2$. Nonhomogeneous functions, dimensionless (11, 13)
- $f_{x, t}$ space (x) and time (t) factors in the distribution of laser pulse, dimensionless (13)
- G • electron–phonon coupling factor, $\text{W m}^{-3} \text{K}^{-1}$ (1, 2, 3, 5, 8, 12, 13); dimensionless function (12)
- solid–gas energy coupling factor, $\text{W m}^{-3} \text{K}^{-1}$ (6, 9)
- dimensionless heat intensity (7)
- G_{ij} energy coupling factor between carriers i and j , $\text{W m}^{-3} \text{K}^{-1}$ (12)
- g • heat intensity per unit area, J m^{-2} (7)
- reciprocal of the laser penetration depth, m^{-1} (11)
- spatial distribution of the oscillating heat source (12)

- g_0 dimensionless g (11)
- g_i $i = 1, 2, 3$. Transformation function, dimensionless (8)
- G_i conjugate vector to the temperature gradient, $W m^{-2} K^{-2}$ (3)
- H
 - dimensionless number (1, 5, 9, 13)
 - angular distribution of the near-tip temperature, dimensionless (8)
 - unit step function (11, 12)
 - complex amplitude of the temperature wave, K (12)
- H_i $i = 1, 2$. Coefficient in Laplace transform solutions, s (5)
- h
 - Planck constant, $J s$ (1, 5, 12)
 - unit step function (2, 4, 5)
 - film heat transfer coefficient, $W m^{-2} K^{-1}$ (9, 10)
 - power of energy exchange per unit volume per degree, $W m^{-3} K^{-1}$ (10)
 - specific enthalpy per unit mass, $J kg^{-1}$ (11)
- \hbar Planck constant, $J s$ (13)
- I
 - identity matrix, dimensionless (3)
 - power intensity of laser beam, $W m^{-2}$ (5)
- I_n modified Bessel function of the first kind of order n (4, 8, 11)
- i number of terms in a series (2)
- J
 - entropy flux, $W m^{-2} K^{-1}$ (3)
 - electric current density, $A m^{-2}$ (9)
 - energy intensity of laser pulse, $J m^{-2}$ (5, 11, 13)
- J_n Bessel function of the first kind of order n (4)
- j
 - number of terms in a series (2)
 - mass flux density, $kg m^{-2} s^{-1}$ (4, 9)
- K
 - dimensionless number (9, 13)
 - bulk modulus in elasticity, Pa (11)
 - thermal conductivity of the electron gas; effective conductivity in phonon flow, $W m^{-1} K^{-1}$ (12)
- k
 - thermal conductivity, $W m^{-1} K^{-1}$
 - spring constant, $N m^{-1}$ (10)
 - wave number in error propagation, dimensionless (13)
- k_B Boltzmann constant, $J K^{-1}$ (13)
- k_i thermal conductivity of carrier i , $W m^{-1} K^{-1}$ (12)
- k_{fs} cross conductivity along solid/fluid interface, $W m^{-1} K^{-1}$ (9)
- $k_{E,I}$ pharmacodynamical parameters, $g m^{-3}$ (10)
- L
 - any linear operators (1)
 - thin-film thickness, μm (1, 5)
 - thickness of interfacial layer, m (7)
 - length of the one-dimensional solid, μm (2, 4)
 - dimensionless nonlocal length of the heat flux vector (12)
 - dimensionless thickness (13)
- L_i $i = 1, 2$, thickness of the contact layer i , m (13)
- l
 - effective mean free path in phonon collision, μm
 - nonlocal length, m (3)
 - dimensionless length of the one-dimensional solid (2, 4, 12)
 - half-length of the sand container, m (6)

- dimensionless interfacial thickness (9)
- intrinsic length in the thermomass model, m (12)
- thickness of acoustically thin layers, m (13)
- l_b mean free path of backscattered phonons, m (12)
- M
 - number of terms in the Riemann-sum approximation or Taylor series expansion (2)
 - number of data points in the experiment (6)
 - v/C_v , thermal Mach number (8, 10); v/C_{TM} (12)
 - figure of merit, dimensionless (9)
 - atomic mass, kg (12)
 - dimensionless number (13)
- m
 - time exponent of surface temperature; slope in the logarithmic temperature-versus-time curve, dimensionless
 - mass, kg (1, 3, 12)
- m_0 phonon mass at rest, kg (12)
- N
 - general nonhomogeneous terms; total number of atoms/particles (1, 3)
 - number of terms in the series truncation (2, 6, 7)
 - number density of electrons, m^{-3} (11)
 - number of carriers (12)
- n
 - number density per unit volume, m^{-3} (1, 3, 5, 11, 12)
 - unit normal of the differential surface area, dimensionless (11)
- n_a number density of atoms, m^{-3} (1)
- n_c critical model number for the occurrence of the thermal resonance (12)
- n_i $i = C, k, T, q$. Exponents describing the temperature dependence of volumetric heat capacity (C), thermal conductivity (k), phase lag of the temperature gradient (τ_T), and phase lag of the heat flux vector (τ_q), dimensionless (13)
- P
 - perimeter, m (10)
 - pressure, Pa (11)
 - transient matrix element (12)
- P_L apparent permeability of the vasculature, $m\ s^{-1}$ (10)
- p
 - specular reflection parameter, dimensionless (1)
 - momentum, $kg\ m\ s^{-1}$ (3)
 - Laplace transform parameter, dimensionless (2, 4, 5, 7, 8, 11, 12); s^{-1} when used with dimensions (5, 6)
 - transformation function (8)
 - phonon pressure, Pa (12)
- Q
 - axial heat flow, W (1)
 - volumetric heat source, $W\ m^{-3}$ (2, 12)
 - angular distribution of the heat flux vector (8)
 - dimensionless laser absorption rate (11)
 - dimensionless heat flux, $q/(C_p T_0 C_L)$; kernel in the memory function, $W\ m^{-1}\ K^{-1}\ s^{-1}$ (12)
- Q_i conjugate vector to the heat flux vector, $m^{-1}\ K^{-1}$ (3)
- q heat flux, $W\ m^{-2}$
- q_i $i = 1, 2$, heat fluxes in the contact region, K (13)
- q_m the metabolic heat generation, $W\ m^{-3}$ (10)
- R
 - radius of nanowires, nm (1)

	<ul style="list-style-type: none"> • reflectivity, dimensionless (1, 5, 11, 13) • mean distance traveled by random walkers (7) • dimensionless density (9) • rigidity propagator in heat transport, W m^{-3} (12)
R_c	ratio C_e/C_l (12)
Re	real part of a function
r	position, $\mu\text{m}/\text{nm}$
r_i	$i = 1, 2$. Dimensionless coefficients in Fourier transform (2)
S	<ul style="list-style-type: none"> • energy absorption rate, W m^{-3} (5, 11) • surface area, m^2 (11) • volumetric heat source, W m^{-3} (12) • dimensionless laser absorption rate (13)
S_{ij}	$i, j = 1, 2, 3$. Conjugate tensor to the Cauchy stress tensor, $\text{K}^{-1} \text{s}^{-1}$ (3)
s	<ul style="list-style-type: none"> • entropy per unit mass, $\text{J kg}^{-1} \text{K}^{-1}$ (3, 4, 11) • eigenvalues (r dependency) of the near-tip heat flux vector, dimensionless (8) • time variable in memory functions, s (12)
T	absolute temperature, K
T_i	$i = 1, 2$, temperatures in the contact region, K (13)
T_j^n	nodal temperature at spatial node j and time node n , K (13)
t	physical time, s
t_0	decaying time constant, s (10)
t_i	$i = 1 - 5$. Travel times of phonons or electrons in successive collisions; $i = 1, 2, \dots, N$. Characteristic times on the time axis, s (1, 12)
t_p	full-width-at-half-maximum pulse (5, 11, 13)
U	<ul style="list-style-type: none"> • dimensionless number (9, 13) • dimensionless displacement (11, 13)
U_i	$i = 1, 2$. Coefficients in Laplace transform solutions, $\text{W m}^{-1} \text{K}^{-1}$ (5)
u	<ul style="list-style-type: none"> • velocity, m s^{-1} (3, 10) • displacement, m (9, 11, 12, 13); velocity of phonon flow, m s^{-1} (12)
u_i	<ul style="list-style-type: none"> • $i = 1, 2$. General unknowns (1) • $i = 1, 2$. First and second sound speeds in liquid helium (4)
V	<ul style="list-style-type: none"> • electric potential, volt (9) • volume, m^3 (11)
V_i	$i = 1, 2$. Coefficients in Laplace transform solutions, K s (5)
v	<ul style="list-style-type: none"> • specific volume, $\text{m}^3 \text{kg}^{-1}$ (3, 11) • velocity, m s^{-1} (3, 4) • crack velocity, m s^{-1} (8) • mean velocity of sound in the contact region, m s^{-1} (13)
v_i	$i = 1, 2, 3$. Velocity components in the x , y , and z directions, m s^{-1} (3)
$v_{s,x}$	phonon velocity or speed of sound (v_s ; 1, 5, 12); phonon/particle speed in the x direction (v_x ; 1, 3), m s^{-1}
W_i	$i = 1, 2$. Coefficients in Laplace transform solutions, K s m^{-1} (5)
w	<ul style="list-style-type: none"> • displacement vector, m (3) • perfusion rate of blood per unit volume, s^{-1} (10) • transformation variable, dimensionless (11)
x	<ul style="list-style-type: none"> • space variable, m • displacement from equilibrium position, m (10)

x_i	$i = 1, 2, 3$. Cartesian coordinates, m (8)
Y	dimensionless Helmholtz potential (4); nondimensional elastic modulus (13)
Y_i	$i = 1, 2$. Dimensionless elastic modulus (13)
y	space variable, m (3); transformed or integral variables, dimensionless (2, 7, 11)
Z	τ_T/τ_q (2, 7, 12); τ_p/τ_j (9)
z	<ul style="list-style-type: none"> • space variable, m (3) • transformed or integral variable, dimensionless (2, 4, 8) • ratio of phase lag (τ) to diffusion time (l^2/α), dimensionless (6) • τ_C/τ_j (10) • dimensionless phase lags (11, 12)
z_i	$i = C, k, T, q$; $(T/T_0)^{n_i}$, temperature dependence of volumetric heat capacity (C), thermal conductivity (k), phase lag of the temperature gradient (τ_T), and phase lag of the heat flux vector (τ_q), dimensionless (13)

Greek Symbols

α	<ul style="list-style-type: none"> • thermal diffusivity, $\text{m}^2 \text{s}^{-1}$ • coefficient in the size effect of thermal conductivity, dimensionless (1)
α_S	Seebeck coefficient, V K^{-1} (9, 13)
β	<ul style="list-style-type: none"> • dimensionless time • diameter/thickness to mean-free-path ratio (1) • proportional constant in resistive force on phonon flow, $\text{kg m}^{-3} \text{s}^{-1}$ (12)
β_s	dimensionless pulse width (4)
β_0	dimensionless pulse duration (11)
χ	<ul style="list-style-type: none"> • coefficient in electron conductivity, $\text{W m}^{-1} \text{K}^{-1}$ (1) • dimensionless concentration (10)
Δ	<ul style="list-style-type: none"> • change of a quantity • dimensionless depth of thermal penetration (2) • Dirac-delta function (4) • average volume of the unit cell, m^3 (12)
$\Delta\xi, \Delta\beta$	size of space (ξ) and time (β) grids, dimensionless (13)
$\Delta\xi_i$	$i = 1, 2, 3$. Sizes of spatial grids in the direction of ξ_i , dimensionless (13)
δ	<ul style="list-style-type: none"> • dimensionless space (1, 2, 4, 5, 8, 11, 12); depth of thermal penetration, m (2) • Kronecker delta (3) • optical penetration depth, nm (5, 11) • delta function (7) • error amplitude in finite differencing, dimensionless (13)
ε	specific internal energy per unit mass, J kg^{-1} (3, 11)
ε_i	$i = 1, 2$, and 3. Radii of circles around the branch points, dimensionless (2)
Φ	amplitude function, K (8)
ϕ	<ul style="list-style-type: none"> • heat-flux/Lamé potential, W m^{-1} (2, 4) • azimuthal angle, rad (8) • dimensionless voltage (9, 13) • Lamé displacement potential, Pa m^2 (12)
φ	volume fraction, dimensionless (9, 10)
ϕ_n	$n = 1$ to ∞ . Spatial eigenfunctions of the undamped T wave (12)

Γ	<ul style="list-style-type: none"> time amplitude of temperature, $\text{K m}^{-\lambda}$ (8); K (12) Gamma function when noted, dimensionless (8) coefficient of phonon mismatch at the interface, dimensionless; nondimensional relaxation time (13)
γ	<ul style="list-style-type: none"> real axis in the Bromwich contour, dimensionless (2); s^{-1} when used with dimensions (6) transformation function, dimensionless (8) density ratio (saturated to ambient) (9) volumetric specific heat, $\text{J m}^{-3} \text{K}^{-1}$; Grüneisen constant, dimensionless (12)
η	<ul style="list-style-type: none"> coefficient in electron conductivity, dimensionless (1) dimensionless heat flux (2, 4, 7, 8, 12, 13) thermomechanical coupling factor, dimensionless (11, 12, 13); time interval, s (12)
η_0	elastic modulus (same as E_0 in Chapter 12), Pa (9)
κ	<ul style="list-style-type: none"> Boltzmann constant, J K^{-1} (1, 5, 11, 12) interfacial thermal conductance, $\text{W m}^{-2} \text{K}^{-4}$ (13)
$\kappa_{\epsilon, \sigma}$	coefficient of thermal expansion, K^{-1} (ϵ , strain) or Pa K^{-1} or $\text{J m}^{-3} \text{K}^{-1}$ (σ , stress) (11, 13)
Λ	<ul style="list-style-type: none"> intensity of hot-electron blast, $\text{N m}^{-2} \text{K}^{-2}$ (11, 13) time amplitude of the near-tip heat flux vector, $\text{W m}^{-(\lambda+2)}$ (8); defined constant (12)
λ	<ul style="list-style-type: none"> intrinsic length scale, m (1) positive coefficient, Pa or J m^{-3} (3) characteristic length, m (7) eigenvalues (r dependency) of the near-tip temperature, dimensionless (8) Lamé modulus in elasticity, Pa (11, 12) effective mean free path of phonons in the contact region, m (13)
λ_T, λ_q	nonlocal lengths of the temperature gradient (T) and the heat flux vector (q), m (12)
λ_1, λ_2	relaxation time constant in viscoelasticity, s (9)
μ	<ul style="list-style-type: none"> direction cosine (1) coefficient of viscosity, Pa s (3) shear modulus in elasticity, Pa (11, 12) Thomson coefficient, V K^{-1} (9, 13)
ν	<ul style="list-style-type: none"> vibration frequency of metal lattice, s^{-1} (1) Poisson ratio, dimensionless (11, 12, 13)
Ω	oscillating frequency, s^{-1} (12)
Π	<ul style="list-style-type: none"> configuration factor in the intrinsic length, dimensionless (1) Peltier coefficient (αT), V (9)
Θ	<ul style="list-style-type: none"> dynamic temperature, K (3) dimensionless temperature (8, 12)
θ	<ul style="list-style-type: none"> dimensionless temperature nonequilibrium temperature K (3) azimuthal angle, rad (8, 12)
θ_i	$i = 1, 2$. Boundary temperatures, dimensionless (13)
θ_j^n	nodal temperature at spatial node j and time note n (13)
ρ	<ul style="list-style-type: none"> integral variables, dimensionless (2) mass density, kg m^{-3} (3, 4, 9, 11, 12)
ρ_0	mass density of rest phonons, kg m^{-3} (12)

Σ	<ul style="list-style-type: none"> • entropy production rate per unit volume, $\text{W m}^{-3} \text{K}^{-1}$ (3) • dimensionless stress (11, 13) • dimensionless number in contact heat flux (13)
σ	<ul style="list-style-type: none"> • Cauchy stress tensor, Pa (3, 9, 11, 12) • electrical conductivity, $\text{A m}^{-1} \text{V}^{-1}$ (9) • Stefan–Boltzmann constant, $\text{W m}^{-2} \text{K}^{-4}$ (13)
σ_m	mean stress, Pa (11)
τ	<ul style="list-style-type: none"> • phase lag or relaxation time, s • mean free time or relaxation time, s (1, 3, 5, 8) • half-period of wave oscillations, s (2) • time delay between the heating and probing laser, s (5)
τ_j, ρ, C	phase lags of mass flux (j), density gradient (ρ) (9), and concentration gradient (10), s
τ_i	$i = R, N, B, I$. umklapp (R), normal (N), boundary (B) and impurity (I) relaxation time, s
$\tau_{R, N}$	Umklapp (R) and normal (N) relaxation time, s
ω	<ul style="list-style-type: none"> • frequency in the Fourier transform domain • phonon frequency, s^{-1} (1) • angular velocity of the running crack, m s^{-1} (8)
ω_n	$n = 1$ to ∞ . Frequency of the undamped T wave, s^{-1} (12)
ξ	dimensionless space variable (7, 8, 9, 12, 13)
ξ_i	<ul style="list-style-type: none"> • $i \equiv D, W$. Correlation length, m (7) • $i = 1, 2$. Material coordinates convecting with the crack tip (8)
ζ	transformation variable, dimensionless (8)
Ψ	Helmholtz potential, W m^{-1} (4)
ψ	<ul style="list-style-type: none"> • conductive temperature, K (3) • modal parameter in the autocorrelation function of laser pulses, s^{-1} (5)
∇	gradient operator, m^{-1}

Subscripts and Superscripts

0	<ul style="list-style-type: none"> • initial/reference value at $t = 0$ • equilibrium conditions (3, 5) • dimensionless quantity (11, 12)
a	<ul style="list-style-type: none"> • atom (1, 2, 5, 12) • arterial; air (10)
B	boundary (1)
b	<ul style="list-style-type: none"> • bulk • boundary (4, 5, 11, 12) • blood (10)
c	contact (13)
D	<ul style="list-style-type: none"> • diffusion (1, 7, 8) • Debye temperature or frequency (1, 12)
E	<ul style="list-style-type: none"> • equivalent quantity (5, 6) • free drug, extracellular (10) • elastic dilatation (11)

e	electron <ul style="list-style-type: none"> • internal energy (12)
F	quantities calculated at the Fermi surface; Fourier (9)
f	film, fluid
g	gaseous phase (6, 9)
I	<ul style="list-style-type: none"> • impurity (1) • intracellular (10)
l	<ul style="list-style-type: none"> • lattice (1, 2, 5, 12, 13) • fractal (7)
L	<ul style="list-style-type: none"> • liposome (10) • longitudinal waves (12)
LB	longitudinal branch in phonon scattering (1)
L^{-1}	inverse Laplace transform
M	<ul style="list-style-type: none"> • mechanical field (3) • thermal Mach wave (8) • nodal number in space (13)
max	maximum value
N	normal process of phonon collision (1, 2)
NL	nonlocal model
n	<ul style="list-style-type: none"> • normal viscous fluid component (4) • fracton (7) • $n = 1$ to ∞. Wave mode (12)
p	<ul style="list-style-type: none"> • isobaric (2, 4, 11) • parallel assembly (5, 12) • the full-width-at-half-maximum pulse (5, 11, 13)
q	heat flux
R	umklapp process of phonon collision (1, 2)
r	r component
S	solid phase (9)
s	<ul style="list-style-type: none"> • surface quantities (2, 4); Fourier transform (2) • superfluid (4) • pulse quantities (5, 6) • solid phase (6, 9, 10) • tissue (10)
(s)	steady state (8)
T	<ul style="list-style-type: none"> • temperature gradient (2, 4) • thermal field or temperature (3, 6, 7, 11, 12)
TM	thermomass (12)
TB	transverse branch in phonon scattering (1)
T_i	$i = 0, 1$. Boundary temperatures (1)
(t)	transient state (8)
v	<ul style="list-style-type: none"> • velocity space (3) • venous (10) • constant volume (11)
W	<ul style="list-style-type: none"> • wave (1, 9) • quantities at the wall (2, 4, 5, 12)

w	wire (1), wall (10)
\mathbf{X}	tensor X (3)
\bar{X}	vector X
\bar{X}	<ul style="list-style-type: none"> • Laplace transform of X • complex conjugate (12) • averaged value of X over the frequency domain (1)
\tilde{X}	averaged value of X over both the frequency and temperature domains (1)
\dot{X}	time derivative of X , $\partial X/\partial t$
X^*	<ul style="list-style-type: none"> • shifted, equivalent, or apparent quantities of X (2, 11) • dimensionless frequencies
	normalized with respect to τ_q (12)
X^+	approaching from the side greater than X (8)
X'	<ul style="list-style-type: none"> • deviatoric component (3) • derivative of X with respect to its argument (8, 11, 13)
$X_{,i}$	$\partial X/\partial x_i$, spatial derivatives (3)
$X_{(i)}$	$i = 1, 2$. Material properties of X in the i th layer (5)
(X)	$X = \text{I, II}$. Quantities in the subsystem (X)
$\langle X \rangle$	volumetric average of X (9)
X_i	the i th components of a vector (3)
ε	strain (11)
θ	θ component
σ	stress (11)