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Solutions Manual to Accompany LOSS MODELS

From Data to Decisions

Second Edition

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Loss Models

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Contents

1
Introduction

The solutions presented in this manual reflect the author's best attempt to provide insights and answers. While we have done our best to be complete and accurate, errors may occur and there may be more elegant solutions. Errata will be posted at the ftp site dedicated to the text and solutions manual:

ftp://ftp.wiley.com/public/sci_tech_med/loss_models/

Should you find errors or would like to provide improved solutions, please send your comments to Stuart Klugman at stuart.klugman@drake.edu.

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Chapter 2 solutions

2.1 SECTION 2.2

2.1 $F_5(x) = 1 - S_5(x) = \begin{cases} 0.01x, & 0 \le x < 50 \\ 0.02x - 0.5, & 50 \le x < 75. \end{cases}$

$f_5(x) = F_5'(x) = \begin{cases} 0.01, & 0 < x < 50 \\ 0.02, & 50 \le x < 75. \end{cases}$

$h_5(x) = \frac{f_5(x)}{S_5(x)} = \begin{cases} \frac{1}{100-x}, & 0 < x < 50 \\ \frac{1}{75-x}, & 50 \le x < 75. \end{cases}$

2.2 The requested plots appear below. The triangular spike at zero in the density function for Model 4 indicates the 0.7 of discrete probability at zero.

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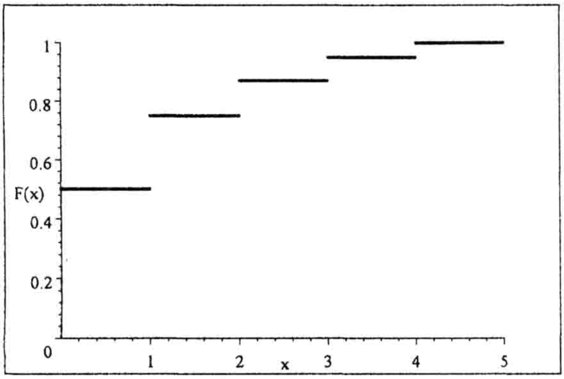
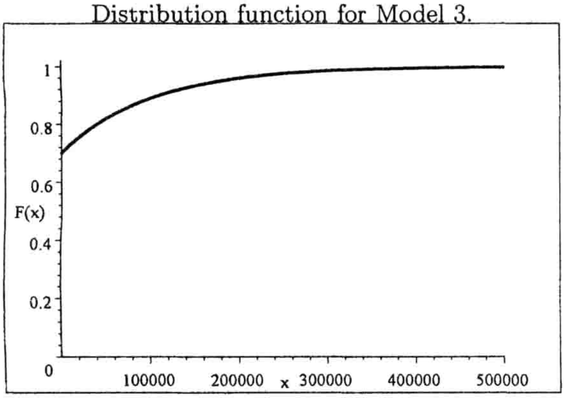
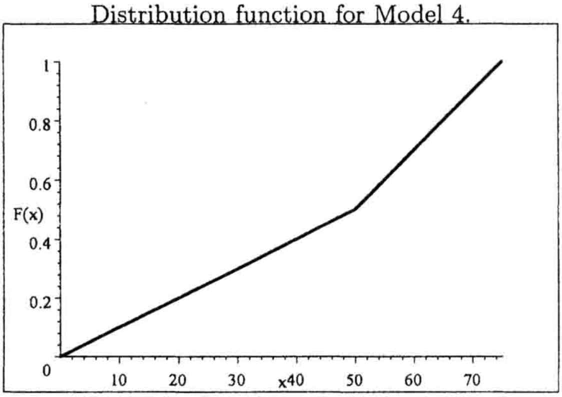


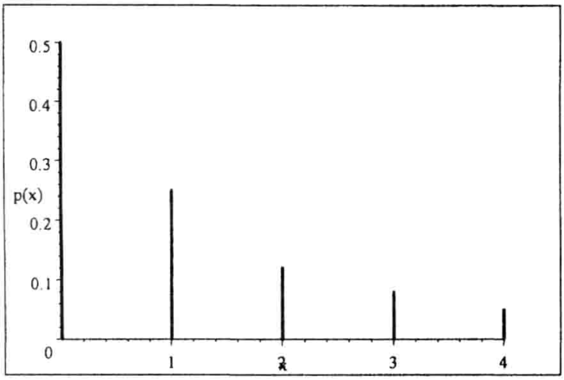
Distribution function for Model 3.



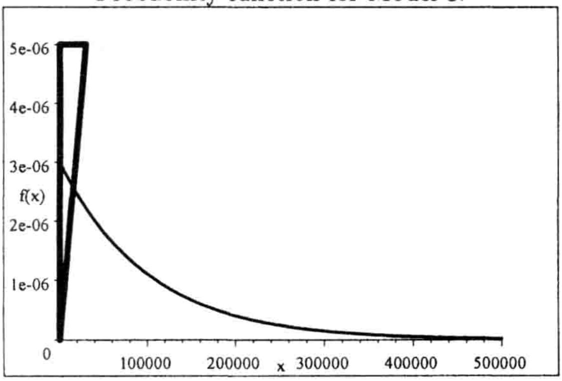
Distribution function for Model 4.



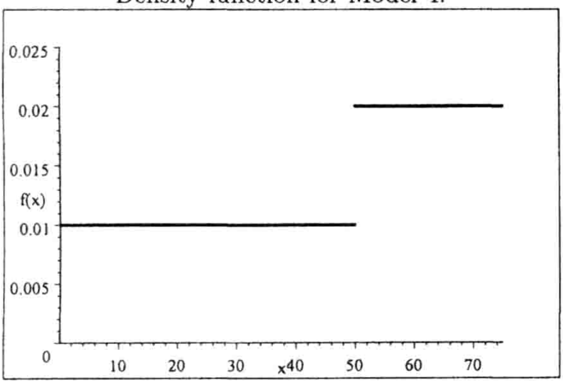
Distribution function for Model 5.



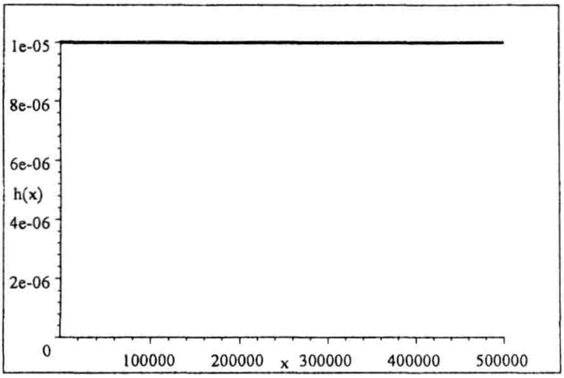
Probability function for Model 3.



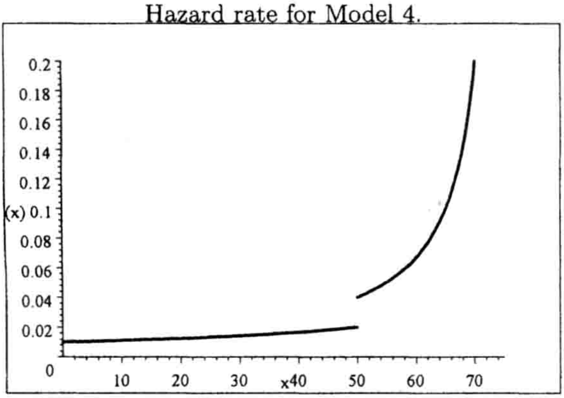
Density function for Model 4.



Density function for Model 5.



Hazard rate for Model 4.



Hazard rate for Model 5.

2.3 $f'(x) = 4(1+x^2)^{-3} - 24x^2(1+x^2)^{-4}$. Setting the derivative equal to zero and multiplying by $(1+x^2)^4$ gives the equation $4(1+x^2) - 24x^2 = 0$. This is equivalent to $x^2 = 1/5$. The only positive solution is the mode of $1/\sqrt{5}$.

2.4 The survival function can be recovered as

$$\begin{aligned} 0.5 &= S(0.4) = e^{-\int_0^{0.4} A+e^{2x}dx} \\ &= e^{-Ax-0.5e^{2x}\big|_0^{0.4}} \\ &= e^{-0.4A-0.5e^{0.8}+0.5}. \end{aligned}$$

Taking logarithms gives

$$-0.693147 = -0.4A - 1.112770 + 0.5$$

and thus $A = 0.2009$.

2.5 The ratio is

$$\begin{aligned} r &= \frac{\left(\frac{10{,}000}{10{,}000+d}\right)^2}{\left(\frac{20{,}000}{20{,}000+d^2}\right)^2} \\ &= \left(\frac{20{,}000+d^2}{20,000+2d}\right)^2 \\ &= \frac{20{,}000^2+40{,}000d^2+d^4}{20{,}000^2+80{,}000d+4d^2}. \end{aligned}$$

From observation, or two applications of L'Hôpital's rule, we see that the limit is infinity.

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Chapter 3 solutions

3.1 SECTION 3.1

3.1

$$\begin{aligned}\mu_3 &= \int_{-\infty}^{\infty}(x-\mu)^3 f(x)dx = \int_{-\infty}^{\infty}(x^3-3x^2\mu+3x\mu^2-\mu^3)f(x)dx \\ &= \mu_3' - 3\mu_2'\mu + 2\mu^3.\end{aligned}$$

$$\begin{aligned}\mu_4 &= \int_{-\infty}^{\infty}(x-\mu)^4 f(x)dx \\ &= \int_{-\infty}^{\infty}(x^4-4x^3\mu+6x^2\mu^2-4x\mu^3+\mu^4)f(x)dx \\ &= \mu_4' - 4\mu_3'\mu + 6\mu_2'\mu^2 - 3\mu^4.\end{aligned}$$

3.2 For Model 1, $\sigma^2 = 3{,}333.33 - 50^2 = 833.33$, $\sigma = 28.8675$.
$\mu_3' = \int_0^{100} x^3(.01)dx = 250{,}000$, $\mu_3 = 0$, $\gamma_1 = 0$.
$\mu_4' = \int_0^{100} x^4(.01)dx = 20{,}000{,}000$, $\mu_4 = 1{,}250{,}000$, $\gamma_2 = 1.8$.

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For Model 2, $\sigma^2 = 4{,}000{,}000 - 1000^2 = 3{,}000{,}000$, $\sigma = 1732.05$. μ_3' and μ_4' are both infinite so the skewness and kurtosis are not defined.

For Model 3, $\sigma^2 = 2.25 - .93^2 = 1.3851$, $\sigma = 1.1769$.
$\mu_3' = 0(.5)+1(.25)+8(.12)+27(.08)+64(.05) = 6.57$, $\mu_3 = 1.9012$, $\gamma_1 = 1.1663$.
$\mu_4' = 0(.5) + 1(.25) + 16(.12) + 81(.08) + 256(.05) = 21.45$, $\mu_4 = 6.4416$, $\gamma_2 = 3.3576$.

For Model 4, $\sigma^2 = 6{,}000{,}000{,}000 - 30{,}000^2 = 5{,}100{,}000{,}000$, $\sigma = 71{,}414$.
$\mu_3' = 0^3(.7) + \int_0^\infty x^3(.000003)e^{-.00001x}dx = 1.8 \times 10^{15}$, $\mu_3 = 1.314 \times 10^{15}$, $\gamma_1 = 3.6078$.
$\mu_4' = \int_0^\infty x^4(.000003)e^{-.00001x}dx = 7.2 \times 10^{20}$, $\mu_4 = 5.3397 \times 10^{20}$, $\gamma_2 = 20.5294$.

For Model 5, $\sigma^2 = 2395.83 - 43.75^2 = 481.77$, $\sigma = 21.95$.
$\mu_3' = \int_0^{50} x^3(.01)dx + \int_{50}^{75} x^3(.02)dx = 142{,}578.125$, $\mu_3 = -4394.53$, $\gamma_1 = -0.4156$.
$\mu_4' = \int_0^{50} x^4(.01)dx + \int_{50}^{75} x^4(.02)dx = 8{,}867{,}187.5$, $\mu_4 = 439{,}758.30$, $\gamma_2 = 1.8947$.

For Model 6, $\sigma^2 = 42.5 - 6.25^2 = 3.4375$, $\sigma = 1.8540$.
$\mu_3' = 310.75$, $\mu_3 = 2.15625$, $\gamma_1 = .3383$.
$\mu_4' = 2424.5$, $\mu_4 = 39.0508$, $\gamma_2 = 3.3048$.

3.3 The standard deviation is the mean times the coefficient of variation, or 4 and so the variance is 16. From (3.3) the second raw moment is $16 + 2^2 = 20$. The third central moment is (using Exercise 3.1) $136 - 3(20)(2) + 2(2)^3 = 32$. The skewness is the third central moment divided by the cube of the standard deviation, or $32/4^3 = 1/2$.

3.4 For a gamma distribution the mean is $\alpha\theta$. The second raw moment is $\alpha(\alpha + 1)\theta^2$ and so the variance is $\alpha\theta^2$. The coefficient of variation is $\sqrt{\alpha\theta^2}/\alpha\theta = \alpha^{-1/2} = 1$. Therefore $\alpha = 1$. The third raw moment is $\alpha(\alpha + 1)(\alpha + 2)\theta^3 = 6\theta^3$. From Exercise 3.1, the third central moment is $6\theta^3 - 3(2\theta^2)\theta + 2\theta^3 = 2\theta^3$ and the skewness is $2\theta^3/(\theta^2)^{3/2} = 2$.

3.5 For Model 1,

$$e(d) = \frac{\int_d^{100}(1 - .01x)dx}{1 - .01d} = \frac{100 - d}{2}.$$

For Model 2,

$$e(d) = \frac{\int_d^\infty \left(\frac{2000}{x+2000}\right)^3 dx}{\left(\frac{2000}{d+2000}\right)^3} = \frac{2000 + d}{2}$$