TRIGONOMETRY

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Trigonometry

Second Edition

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On the cover: The "Untitled" oil on paper monoprint on the cover is a diptych executed by Los Angeles artist Charles Arnoldi at the Garner Tullis Workshop in Santa Barbara, California, on November 18, 1988. Arnoldi's work may be seen at the Fuller Gross Gallery in San Francisco, the James Corcoran Gallery in Santa Monica, the Texas Gallery in Houston, and the Charles Cowles Gallery in New York City. His work is included in the permanent collections of the Albright-Knox Museum in Buffalo, New York, the Chicago Art Institute, the Museum of Modern Art in New York City, the Metropolitan Museum, the Los Angeles County Museum, and the San Francisco Museum of Modern Art.

Trigonometry

The Precalculus Series by Michael Sullivan:

College Algebra with Review
This text contains an in-depth review of intermediate algebra topics for those students who either may not be adequately prepared to begin a traditional college algebra course or desire to become reacquainted with certain prerequisite topics before beginning college algebra. After completing this book, the student will have covered the same mathematics at the same level as the student who has completed a traditional college algebra text.

College Algebra, Second Edition
This text contains a traditional approach to college algebra, with three chapters of review material preceding the chapter on functions and their graphs. After completing this book, the student will be adequately prepared to handle subsequent courses in finite mathematics, business mathematics, and engineering calculus.

College Algebra and Trigonometry, Second Edition
This text, in addition to all the features of *College Algebra*, also develops the trigonometric functions using a right triangle approach and showing how it leads to the unit circle approach. Graphing techniques are emphasized, including a thorough discussion of polar coordinates, parametric equations, and conics using polar coordinates.

Precalculus, Second Edition
This text contains one review chapter before covering the traditional precalculus topics of functions and their graphs, polynomial and rational functions, and exponential and logarithmic functions. The trigonometric functions are introduced using a unit circle approach and showing how it leads to the right triangle approach. Graphing techniques are emphasized, including a thorough discussion of polar coordinates, parametric equations, and conics using polar coordinates.

Trigonometry, Second Edition
This text, designed for stand-alone courses in trigonometry, develops the trigonometric functions using a unit circle approach and showing how it leads to the right triangle approach. Graphing techniques are emphasized, including a thorough discussion of polar coordinates, parametric equations, and conics using polar coordinates.

In memory of Joe and Rita

Preface

Intent/Purpose

As a professor at an urban public university for over 20 years, I am aware of the varied needs of students—students who range from being less well-prepared and insufficiently motivated to those who are well-prepared and highly motivated. As the author of an engineering calculus text as well as texts in finite mathematics and business mathematics, I understand what students must know in order to be successful in such courses. At the same time, I am also aware of, and very concerned about, the student who has decided this trigonometry course is to be a terminal mathematics course. Based on these experiences, I have written a book to serve students with varied backgrounds and goals.

For the student who requires it, a review of the basic material needed for trigonometry is contained in Chapter 1. To help further, concepts are viewed from different perspectives whenever possible, and alternate methods of solutions are given. Every effort has been made to be clear, precise, and consistent. Whenever it seemed appropriate, special encouragement has been offered; and whenever necessary, warnings have been given. To provide motivation, I have included understandable, realistic applications that are consistent with the abilities of the student. The problem sets have been carefully graded to build the student's confidence.

At the same time, a conscious attempt has been made to preview material that will be seen by students who intend to take courses such as physics or engineering calculus. These previews relate to important concepts, applications, examples, and exercises that will be encountered in later courses.

To summarize, my purpose in writing this book is to serve the different needs of each type of student through suitable motivation and material.

New to This Edition

The elements of the previous edition that proved so successful remain in this edition. Nevertheless, many changes—some obvious, others subtle—have been made. Virtually every change is the result of thoughtful comments and suggestions from colleagues and students who used the previous edition. As a result of this input, for which I am sincerely grateful, this edition will be an improved teaching device for professors and a better learning tool for students.

The list below, which is by no means exhaustive, should give you an idea of what has been done.

Illustrations. Every piece of art has been redrawn to improve accuracy and clarity. Many new illustrations have been inserted, especially in the exercises, to assist the student in visualizing concepts and to reinforce the graphical orientation of the text. This edition now contains over 500 illustrations.

New Material. A discussion of vector projection and its application to the work done by a constant force now appears in Section 7.4, "The Dot Product."

A new Appendix, "Completing the Square; the Quadratic Formula," which contains a derivation of the quadratic formula, has been added at the request of many instructors.

As part of the chapter review section at the end of each chapter, a new component called "Things to Know" has been added. This capsule summary of the important elements of the chapter should be of value to students as they prepare for examinations. The new "How To" section in the chapter review also can be used to identify the objectives of the chapter.

Revised/Reorganized Material. The change-of-scale method for graphing certain functions has been abandoned in favor of a compression/stretch approach in Section 1.5.

Chapter 2, Trigonometric Functions, has been reorganized and rewritten to improve clarity and provide for a more streamlined coverage of the material. Many applied exercises have been added.

The former Section 3.1 now becomes two sections: 3.1, "Graphs of the Six Trigonometric Functions," and 3.2, "Sinusoidal Graphs." This will allow for easier coverage of one section per class period.

Section 5.5, "Polar Equations and Graphs," has been completely rewritten. It now contains a clear and complete coverage of graphing in polar coordinates.

For emphasis, the chapter on analytic geometry now appears earlier, as Chapter 6.

The definition of a logarithmic function is now based on properties of inverse functions. Chapter 8, although now the last chapter in the text, may be covered at any time after Chapter 1.

Examples. New examples have been inserted where the level of content or the diversity of approach warranted. This edition contains over 300 examples.

Exercises. Many new exercises have been added. The majority fall into the categories of applied word problems, problems involving geometry, and problems to challenge the better student. This edition contains over 3000 exercises.

Calculator Usage. Earlier emphasis (see Section 1.1) has been given to the use of a calculator to provide concrete examples of certain properties of real numbers. Additional examples of calculator use in trigonometry have been inserted where appropriate. Also, references have been added to the new guides to using graphics calculators, which explain the use of a graphics calculator in trigonometry (see Supplementary Material).

Contents and Organization

Chapter 1 consists mainly of review material. For example, Section 1.1 contains a discussion of the Pythagorean Theorem and the formulas for perimeter and area of a rectangle, area and circumference of a circle, and volume of a sphere. The first discussion of calculators is also found here.

Chapter 1 also contains all the function concepts required for the study of the trigonometric functions. Special emphasis is placed on functions and the graphs of functions. Graphing is usually done in steps, all of which are illustrated. The graphing techniques introduced in Chapter 1 are utilized and reinforced in Chapters 3 and 8 using the functions introduced there.

With Chapter 2, we begin the study of trigonometry. The trigonometric functions of a real number are introduced using the unit circle approach. This is followed by a discussion of the properties of the trigonometric functions. Right triangle trigonometry and applications close out the chapter.

Chapter 3 continues the study of trigonometric functions with a detailed presentation of their graphs. Here, we use the graphing concepts introduced in Chapter 1, as well as some additional techniques. A discussion of inverse trigonometric functions, followed by an application to simple harmonic motion rounds out the chapter.

Chapter 4 treats the analytic side of trigonometry: identities; sum and difference, double-angle, half-angle, sum-to-product, and product-to-sum formulas; and trigonometric equations.

In Chapter 5, additional applications of trigonometry to solving a general triangle are given, including a rather complete discussion of the area of a triangle. Two full sections have been devoted to polar coordinates and graphing polar equations to provide a thorough development of this subject.

Chapter 6 contains topics from analytic geometry, beginning with the straight line in Section 6.1. In a detailed presentation of the conics (Sections 6.2–6.4), the parabola, ellipse, and hyperbola are defined using geometric (distance-based) means. After a discussion of rotation of axes and the general form of a conic, a unified definition of the conics using eccentricity is given. Lastly, plane curves and parametric equations are studied.

Chapter 7 is comprised of two parts that are independent of each other and may be covered in any order. Sections 7.1 and 7.2 begin with a discussion of complex numbers, including quadratic equations and the quadratic formula, and conclude with DeMoivre's Theorem. Sections 7.3 and 7.4 provide an introduction to vectors, including the dot product and applications.

Chapter 8 treats the exponential and logarithmic functions in the detail necessary and with language consistent for subsequent use in calculus. Applications to both compound interest and growth and decay are among those given.

Finally, a detailed discussion of completing the square and the derivation of the quadratic formula are provided in the Appendix.

Applications

As we mentioned earlier, every opportunity has been taken to present understandable, realistic applications consistent with the abilities of the student, drawing from such sources as tax rate tables, the *Guinness Book of World Records*, and newspaper articles. For added interest, some of the applied exercises have been adapted from textbooks the students may be using in other courses (for example, economics, chemistry, physics, etc.).

Historical Notes

William Schulz of Northern Arizona University has provided historical context and information in anecdotes that appear as introductory material and at the ends of many sections. In some cases, these comments also include exercises and discussion of comparative techniques.

Examples, Exercises, and Illustrations

The text includes over 300 examples and 3000 exercises, of which over 350 are applied problems. The examples are worked out in appropriate detail, starting with simple, reasonable problems and working gradually up to more challenging ones.

Exercises are numerous, well-balanced, and graduated. They usually begin with problems designed to build confidence, continue with drill-type problems that mimic worked-out examples, and conclude with prob-

lems that are more challenging. They include a number of exercises where the student will need a calculator for their solution; these are clearly marked with the symbol $\boxed{\text{C}}$. Answers are given in the back of the book for all the odd-numbered exercises.

Illustrations are abundant, numbering over 500. Full use is made of a second color to help clarify and highlight.

A Word about Format and Design

Each chapter-opening page contains a table of contents for that chapter and an overview—often historical—of the contents.

New terms appear in boldface type where they are defined. The most important definitions are shown in color.

Theorems are set with the word "Theorem" in the margin for easy identification; if it is a named theorem, the name also appears in the margin. When a theorem has a proof given, the word "Proof" appears in the margin to mark clearly the beginning of the proof, and the symbol ■ is used to indicate the end of the proof.

All important formulas and procedures are enclosed by a box and shown in color.

Examples are numbered within each section and identified clearly with the word "Example" in the margin. The solution to each example appears immediately following the example with the word "Solution" in the margin to identify it. The symbol ■ indicates the end of each solution.

Where appropriate in the text, instructions on the use of a calculator to do numerical computations are provided. In many instances, the instructions serve to reinforce the underlying mathematics. Any examples or exercises that require the use of a calculator are clearly marked using the symbol $\boxed{\text{C}}$.

Each section ends with an exercise set. Each chapter ends with a chapter review containing a list of "Things to Know" (a summary of the key elements of the chapter), a selection of fill-in-the-blank items (to test vocabulary and formulas), and a set of review exercises.

Supplementary Material

For the Instructor:

> *Instructor's Solution Manual to Accompany Trigonometry*, by Katy Sullivan and Marsha Vihon, contains worked-out solutions to all the even-numbered problems. If used in conjunction with the *Student's Solution Manual*, the instructor will have worked-out solutions for every problem in the book.

> *Instructor's Resource Material* contains approximately 100 transparency masters that duplicate important illustrations in the text.

Instructor's Test Battery, Dellen Test III, contains several printed samples of chapter and cumulative tests. Each chapter test contains 20–30 questions. Also included are the appropriate answer keys. From these, an instructor can construct appropriate quizzes, chapter tests, and cumulative examinations. Also included are the question models, documentation, and diskettes for both the Free Response and the Multiple Choice versions of Dellen Test III, an algorithm-based test generation system that utilizes an IBM PC℠ (or compatible). Using it, it is possible to generate a virtually unlimited number of test items comparable to the question models, each different from the other, yet each testing the same concept at the same level. This system thus provides a virtually unlimited number of quizzes, chapter tests or final examinations. The system also generates an answer key and a student worksheet with an answer column that exactly matches the column on the answer key. Graphing grids are included for all problems requiring graphs. The Multiple Choice version of Dellen Test III utilizes distractors that are representative of commonly made student errors; this version is machine-gradable.

Precalculus Explorer is IBM PC compatible software that enables the user to enter a function and see the graph on the screen.

For the Student:

Student's Solution Manual to Accompany Trigonometry, by Katy Sullivan and Marsha Vihon, contains complete step-by-step solutions to all the odd-numbered problems.

A Guide to Using the Casio Graphics Calculators with Michael Sullivan's Precalculus Series, by Joan Girard, provides a collection of activities that demonstrate the use of the Casio series of graphics calculators (FX7000G, FX8000G, FX7500G) to solve selected examples and exercises from the text.

A Guide to Using the Hewlett-Packard Graphics Calculator with Michael Sullivan's Precalculus Series, by Joan Girard, contains the same activities as the guide described above using the HP-28S graphics calculator.

Acknowledgments

Textbooks are written by an author, but evolve from an idea into final form through the efforts of many people. Before initial writing began, a survey was conducted which drew nearly 250 instructor responses. The manuscript then underwent a thorough and lengthy review process, including class-testing at Chicago State University. I would like to thank my colleagues and students at Chicago State, who cooperated and contributed to this text while it was being class-tested.

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