



# CALCULUS CONCEPTS

Brief First Edition

*An Informal Approach to the Mathematics of Change*

LATORRE : KENELLY : FETTA : CARPENTER : HARRIS

# Calculus Concepts

---

*An Informal Approach to the  
Mathematics of Change*

*Brief First Edition*

**Donald R. LaTorre**

**John W. Kenelly**

**Iris B. Fetta**

*Clemson University*

**Laurel L. Carpenter**

*Lansing, Michigan*

**Cynthia R. Harris**

*Reno, Nevada*

HOUGHTON MIFFLIN COMPANY

Boston

New York

EDITOR-IN-CHIEF: *Charlie Hartford*

ASSOCIATE EDITOR: *Elaine Page*

SENIOR PROJECT EDITOR: *Maria Morelli*

EDITORIAL ASSISTANT: *Christian Zabriskie*

SENIOR PRODUCTION/DESIGN COORDINATOR: *Carol Merrigan*

SENIOR MANUFACTURING COORDINATOR: *Sally Culler*

MARKETING MANAGER: *Ros Kane*

COVER DESIGN: Deborah Azerrad Savona

COVER IMAGE: New York, Midtown skyline from Queensboro Bridge. Photograph by Jon Ortner. © Tony Stone Images

Copyright © 1998 by Houghton Mifflin Company. All rights reserved.

No part of this work may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying and recording, or by any information storage or retrieval system without the prior written permission of Houghton Mifflin Company unless such copying is expressly permitted by federal copyright law. Address inquiries to College Permissions, Houghton Mifflin Company, 222 Berkeley Street, Boston, MA 02116.

Printed in the U.S.A.

Library of Congress Catalog Card Number: 97-72513

International Standard Book Numbers:

Text: 0-669-39859-4

Exam Copy: 0-669-45197-5

2 3 4 5 6 7 8 9-DC-02 01 00 99 98

# Calculus Concepts

---

*An Informal Approach to the  
Mathematics of Change*

*Brief First Edition*

# TO INSTRUCTORS

## What This Book is About

This book presents a fresh, new approach to the concepts of calculus for students in fields such as business, economics, liberal arts, management, and the social and life sciences. It is appropriate for courses generally known as “brief calculus” or “applied calculus.”

## Philosophy

Our overall goal is to improve learning of basic calculus concepts by involving students with new material in a way that is significantly different from traditional practice. The development of conceptual understanding, not mastery of algebraic skill and technique, is our guiding force coupled with a commitment to make calculus meaningful to the student. Thus, the material in this book is data-driven and technology-based, with a unique modeling approach. It considers the ability to correctly interpret the mathematics of real-life situations of equal importance to the understanding of the concepts of calculus in the context of change.

## Data-Driven

Many everyday, real-life situations involving change are discrete in nature and manifest themselves through data. Such situations often can be represented by continuous or piecewise continuous mathematical models so that the concepts, methods, and techniques of calculus can be brought to light. Thus we seek, when appropriate, to make real-life data a starting point for our investigations.

The use of real data and the search for appropriate models also exposes the students to the reality of uncertainty. We emphasize that sometimes there can be more than one appropriate model and that answers derived from models are only approximations. We believe that exposure to the reality that mathematics is not always right or wrong is valuable.

## Technology-Based

Calculus has traditionally relied upon a high level of algebraic manipulation. However, many non-technical students are not strong in algebraic skills, and an algebra-based approach tends to overwhelm them and stifle their progress. Today’s easy access to technology in the form of graphing calculators and microcomputers breaks down barriers to learning imposed by the traditional reliance on algebraic methods. It creates new opportunities for learning through graphical and numerical representations. We welcome these opportunities in this book by assuming continual and immediate access to technology.

This book requires that students use graphical representations (scatter plots of data and graphs of functions) freely, make numerical calculations routinely, and fit

functions to data. Thus, continual and immediate access to technology is absolutely essential. Because of their low cost, portability, and ability to personalize the mathematics, the authors prefer graphing calculators. These materials have also been successfully taught using microcomputer software (such as Maple) and spreadsheets.

It is worth noting that different technologies may give different model coefficients than those given in this book. We used a TI-83 graphing calculator to generate the models in the text and the answer key. Other technologies may use different fit criteria for some models than the criteria used by the TI-83.

## Modeling Approach

We believe that modeling is an important tool and introduce it at the outset. Both linear and nonlinear models of discrete data are used to obtain functional relationships between the variables of interest. The functions given by the models are the ones used by students to conduct their investigations of calculus concepts. It is the connection to real-life data that most students feel shows the relevance of the mathematics in this course to their lives and adds reality to the topics studied.

## Interpretation Emphasis

This book is substantially different from traditional texts, not only in the philosophy but also in its overall focus, level of activities, development of topics and attention to details. Interpretation of results is a key feature of this text that allows students to make sense of the mathematical concepts and appreciate the usefulness of those concepts in their lives.

## Informal Style

While we appreciate the formality and precision of mathematics, we also recognize that this alone can deter students from access to mathematics. Thus, we have sought to make our presentations as informal as possible by using non-technical terminology where appropriate and a conversational style of presentation.

## Projects

Projects included after each chapter are intended to be group projects with oral and/or written presentations. We recognize the importance of helping students develop the ability to work in groups, as well as hone presentation skills. The projects also give opportunity for students to practice the kind of writing that they will likely have to do in their future careers.

## Other Pedagogical Features

*Chapter Opener* Each chapter opens with a real-life situation and several questions about the situation that relate to the key concepts in the chapter.

*Concept Inventory* A Concept Inventory is listed at the end of each section, giving students a brief summary of the major ideas developed in that section.

*Section Activities* The Section Activities begin by cementing concepts followed by explorations of topics using, for the most part, actual data in a variety of real-world settings. Questions and interpretations pertinent to the data and the concepts are always included in these activities. The activities do not mimic the examples in the

chapter discussion and thus require more independent thinking on the part of the students. Possible answers to odd activities are given at the end of the book.

*Chapter Summary* A Chapter Summary connects the results of the chapter topics and further emphasizes the importance of knowing these results.

*Chapter Review Test* A Chapter Review Test at the end of each chapter provides practice with techniques and concepts. Answers to the Chapter Review Tests are included in the answer key.

## Supplements

The Instructor's Guide gives practical suggestions for using the text in the manner intended by the authors. It contains sample tests, ideas for in-class group work, suggestions for implementing and grading projects and complete activity solutions.

The technology supplements provide technology-specific instructions ordered to match the organization of the text chapters. An open-book icon appears at places in the text where a new concept or skill is presented in the technology supplements.

A Student Solutions Guide is also available. A Test Item File is available PC and Macintosh formats.

## Acknowledgments

We gratefully acknowledge the help and support of many people during the development of this book.

We express our gratitude to the Fund for the Improvement of Post-secondary Education (FIPSE), U.S. Department of Education, which provided substantial funding for this project. FIPSE, your dedication to innovation and improvement of American post-secondary education is genuinely appreciated.

We appreciate the thoughtful advice of several of our colleagues at Clemson University: P.M. Dearing, Matt Saltzman, Herman Senter, and Sherry Biggers, as well as that of Bruce Blackadar at University of Nevada Reno.

The many hours spent by the FIPSE project evaluators, Dr. Jim Wilson and Mary Beth Searcy of the University of Georgia, and Denise Johnson and Susan Kadles at Alma College, were extremely helpful in guiding the direction of the text. We sincerely appreciate all their advice.

The materials were used in hundreds of classes at Clemson since 1994. The authors appreciate the comments of the Clemson University students and graduate teaching assistants in these classes who provided valuable feedback as the material was being developed and tested.

Special thanks to April Haynes, Gloria Orr, Becky Singletary, and Karen McConkie who word processed the manuscript.

Class testing was also conducted by many different teachers at many different schools. Initial class testing was conducted by the teachers listed below. The authors would like to thank these instructors and their students for their many thoughtful comments and valuable contributions.

Ellen King, Kim Freeman, *Anderson College, SC*

John Haverhals, Mary Jane Sterling, Tom McKenzie, Mike McAsey, Chris Stewart,  
Libin Mou, *Bradley University, IL*

Linda Nash, Martha Wicker, Mary Stephens, Paul Myers, Catherine Aust, *Clayton State College, GA*

Nancy Mauldin, Gary Harrison, *College of Charleston, SC*

Daniel Alexander, Larry Naylor, *Drake University, IA*  
Heidi Staebler, *Texas A & M at Commerce*  
Joe Cieply, Gina Kietzmann, *Elmhurst College, IL*  
Jeffrey Clark, Terri Johnson, *Elon College, NC*  
Hugh Williams, Robin Baumgarner, Glenn Jacobs, *Greenville Technical College, SC*  
Marlene Sims, *Kennesaw State College, GA*  
Jerry Bolick, Vicki Schell, Lloyd Smith, Ron Butler, *Lenoir-Rhyne College, NC*  
Kathleen Bavelas, Kathy Peters, *Manchester Community Technical College, XX*  
Ann Preston, Cheryl Slayden, *Pellissippi State Technical Community College, TN*  
Siham Alfred, Lance Hemlow, *Raritan Valley Community College, NJ*  
Jacqueline Fernandez, *Santa Barbara City College, CA*  
Richard Sauvageau, *Staples High School, CT*  
Paul Ache III, Dan Lewis, Robert Main, Elton Lacey, Yvette Hester, *Texas A&M University, TX*  
Carollyne Guidera, *University College of the Fraser Valley, Canada*  
Mary Beth Searcy, *University of Georgia, GA*  
Bruce Blackadar, *University of Nevada Reno, NV*  
Suzanne Smith, John Thornton, *University of North Carolina Charlotte, NC*  
Stephen King, Frank Townsend, Patricia Brown, David Jaspers, Jack Leifer, Mike May *University of South Carolina Aiken, SC*  
Eddie Warren, Jim Harvey, *University of Texas Arlington, TX*  
Helen Read, *University of Vermont, VT*  
Audrey Borchardt, Robert DeVos, Bruce Pollack-Johnson, *Villanova University, PA*  
Jim Snodgrass, Bernd Rossa, Sheila Doran, Martha Holland, Danny Otero, *Xavier University, OH*

Special thanks go to Emily Keaton for her careful work in checking the text and answer key for accuracy. The authors express their sincere appreciation to Charlie Hartford, Elaine Page, Maria Morelli, and their staffs at Houghton Mifflin Company for all their work in bringing this first edition into print.



# TO STUDENTS

## What this Book is About

This book is written to help you understand the inner workings of how things change and to help you build systematic ways to use this understanding in everyday real-life situations that involve change. Indeed, a primary focus of the material is on change, since calculus is the mathematics of change.

Even if you have studied calculus before, this book is probably different from any other mathematics textbook that you have used. It is based on three premises:

1. Understanding is as important as the mastery of mathematical manipulations. Algebraic skill and the ability to manipulate expressions must be regularly practiced, or they will fade away. If you understand concepts, you will be able to explain some things in your life forever.
2. Mathematics is present in all sorts of real-life situations. It is not just an abstract subject in textbooks. In real life, mathematics is often messy and not at all like the tidy, neat equations that you were taught to factor and solve. Speaking of equations, where do they come from? Nature seldom whispers an equation into our ears.
3. The new graphics technology in today's calculators and computers is a powerful tool that can help you understand important mathematical connections. Like many tools in various fields, technology frees you from tedious, unproductive work; enables you to engage situations more realistically; and lets you focus on what you do best . . . think and reason.

## How to Use this Book

- Begin by throwing away any preconceived notions that you may have about what calculus is and any notion that you are “not good” in mathematics.
- Make a commitment to learn the material: not just a good intention, but a genuine commitment.
- Study this book. Notice that we said “study”, not “read”. Reading is a part of study, but study involves much more. You should not only read (and re-read) the discussions, but work through each example to understand its development.
- Use paper, pencil and your graphing calculator or computer when you study. These are your basic tools, and you cannot study effectively without them.
- Find a study partner, if at all possible. Each of you will be able to help the other learn. Communicating within mathematics, and about mathematics, is important to your overall development toward understanding mathematics.
- Write. Write your solutions clearly and legibly, being certain to interpret all of your answers with complete sentences using proper grammar. Careful writing will help you sort through your ideas and focus your learning.

- Make every effort not to fall behind. You know the dangers, of course, but we remind you nevertheless.
- Finally, remember that there is no substitute for effective study. You have your most valuable resource with you at all times—your mind. Use it.

# CONTENTS

<i>To Instructors</i>	<i>xvii</i>
<i>To Students</i>	<i>xxi</i>

## **1** Ingredients of Change: Functions and Linear Models    1

<b>1.1</b>	<b>Fundamentals of Modeling</b>	<b>2</b>
	What Is Mathematical Modeling?	2
	The Role of Graphs	3
	Modeling in Business	4
	Using Models in Real Situations	6
	<i>Concept Inventory 1.1</i>	<i>11</i>
	<i>Activities for Section 1.1</i>	<i>11</i>
<b>1.2</b>	<b>Functions and Graphs</b>	<b>16</b>
	Determining Outputs	18
	Recognizing Functions	20
	Discrete versus Continuous	20
	<i>Concept Inventory 1.2</i>	<i>26</i>
	<i>Activities for Section 1.2</i>	<i>26</i>

<b>1.3</b>	<b>Constructed Functions</b>	30
	Inverse Functions	30
	Combining Functions	32
	Piecewise Continuous Functions	37
	<i>Concept Inventory 1.3</i>	39
	<i>Activities for Section 1.3</i>	39
<b>1.4</b>	<b>Linear Functions and Models</b>	42
	Representations of a Linear Model	42
	Finding a Linear Model	44
	What Is “Best Fit”?	51
	Numerical Considerations	53
	<i>Concept Inventory 1.4</i>	57
	<i>Activities for Section 1.4</i>	57
	<b>Chapter 1 Summary</b>	65
	<b>Chapter 1 Review Test</b>	67
	<b>Project 1.1</b> Tuition Fees	68
	<b>Project 1.2</b> Finding Data	69
<b>2</b>	<b>Ingredients of Change: Nonlinear Models</b>	71
<b>2.1</b>	<b>Exponential Functions and Models</b>	72
	Applications of Exponential Models	73
	Percentage Change in Exponential Models	76
	Exponential Growth with Constraints	79
	<i>Concept Inventory 2.1</i>	84
	<i>Activities for Section 2.1</i>	84

<b>2.2 Exponential Models in Finance</b>	93
Compound Interest	93
Continuous Compounding and the Number $e$	94
Present and Future Value	97
<i>Concept Inventory 2.2</i>	98
<i>Activities for Section 2.2</i>	98
<b>2.3 Polynomial Functions and Models</b>	100
Quadratic Modeling	100
Quadratic or Exponential?	103
Cubic Modeling	107
<i>Concept Inventory 2.3</i>	111
<i>Activities for Section 2.3</i>	111
<b>2.4 Choosing a Model</b>	119
General Guidelines	119
Examining Scatter Plots	120
Four Steps in Choosing a Model	123
<i>Concept Inventory 2.4</i>	128
<i>Activities for Section 2.4</i>	128
<b>Chapter 2 Summary</b>	135
<b>Chapter 2 Review Test</b>	137
<b>Project 2.1</b> Compulsory School Laws	139
<b>Project 2.2</b> Fund-Raising Campaign	141

## 3 Describing Change: Rates 143

### 3.1 Average Rates of Change 144

Finding Average Rates of Change Using Secant Lines 145

Determining Average Rates of Change Using an Equation 149

*Concept Inventory 3.1* 150

*Activities for Section 3.1* 150

### 3.2 Instantaneous Rates of Change 155

The Importance of Continuous Models 155

Tangent Lines and Rates of Change 158

*Concept Inventory 3.2* 161

*Activities for Section 3.2* 161

### 3.3 Tangent Lines 163

Lines Tangent to Circles 163

Lines Tangent to a Curve 165

Local Linearity 169

Where Does the Instantaneous Rate of Change Exist? 170

Approximating Instantaneous Rates of Change 173

*Concept Inventory 3.3* 174

*Activities for Section 3.3* 174

### 3.4 Derivatives 179

Derivative Terminology and Notation 179

Approximating with Derivatives 183

Does *Instantaneous* Refer to Time? 185

*Concept Inventory 3.4* 186

*Activities for Section 3.4* 186

**3.5 Percentage Change and Percentage Rates of Change** 191*Concept Inventory 3.5* 194*Activities for Section 3.5* 194**Chapter 3 Summary** 199**Chapter 3 Review Test** 201**Project 3.1** Fee-Refund Schedules 203**Project 3.2** Doubling Time 204**4 Determining Change: Derivatives** 205**4.1 Numerically Finding Slopes** 206

Finding Slopes by the Numerical Method 206

Finding Slopes by Using the Algebraic Method 208

Finding Slopes of Piecewise Functions 210

*Concept Inventory 4.1* 213*Activities for Section 4.1* 213**4.2 Drawing Slope Graphs** 218

A Detailed Look at the Slope Graph 220

Points of Undefined Slope 223

*Concept Inventory 4.2* 224*Activities for Section 4.2* 225

<b>4.3</b>	<b>Slope Formulas</b>	227	
	Using Derivative Notation for Slope Formulas		228
	A General Formula for Derivatives	233	
	<i>Concept Inventory 4.3</i>	235	
	<i>Activities for Section 4.3</i>	235	
<b>4.4</b>	<b>The Sum Rule</b>	239	
	The Constant Multiplier Rule	239	
	The Sum Rule	241	
	<i>Concept Inventory 4.4</i>	244	
	<i>Activities for Section 4.4</i>	244	
<b>4.5</b>	<b>The Chain Rule</b>	249	
	The First Form of the Chain Rule	249	
	The Second Form of the Chain Rule	251	
	<i>Concept Inventory 4.5</i>	254	
	<i>Activities for Section 4.5</i>	254	
<b>4.6</b>	<b>The Product Rule</b>	258	
	<i>Concept Inventory 4.6</i>	262	
	<i>Activities for Section 4.6</i>	262	
	<b>Chapter 4 Summary</b>	268	
	<b>Chapter 4 Review Test</b>	270	
	<b>Project 4.1</b> Fertility Rates	272	
	<b>Project 4.2</b> Superhighway	273	



## **5 Analyzing Change: Extrema and Points of Inflection**      275

### **5.1 Optimization**      276

*Concept Inventory 5.1*      283

*Activities for Section 5.1*      283

### **5.2 Inflection Points**      289

The Second Derivative      290

*Concept Inventory 5.2*      296

*Activities for Section 5.2*      296

### **5.3 Approximating Change**      302

Marginal Analysis      308

*Concept Inventory 5.3*      309

*Activities for Section 5.3*      309

**Chapter 5 Summary**      315

**Chapter 5 Review Test**      317

**Project 5.1** Hunting License Fees      318

**Project 5.2** Fund-Raising Campaign      319