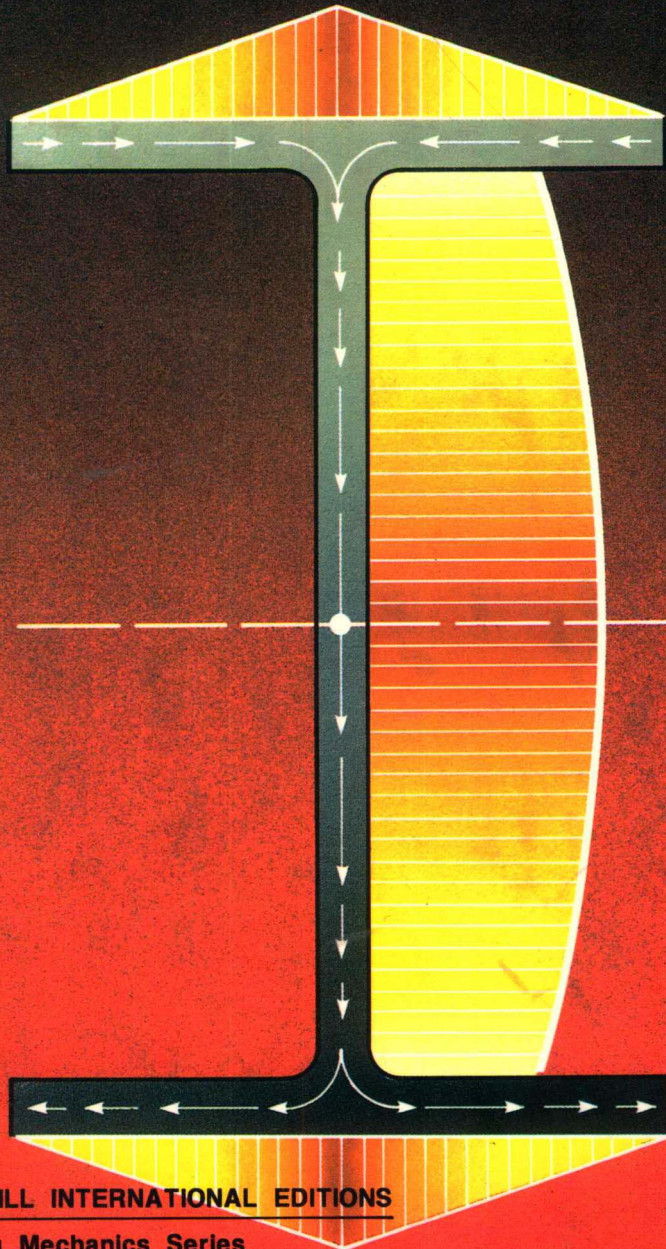


# MECHANICS OF MATERIALS

A. C. UGURAL



McGRAW-HILL INTERNATIONAL EDITIONS

Engineering Mechanics Series

# **MECHANICS OF MATERIALS**

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### Conversion Factors for SI Units and U.S. Customary Units

Quantity	SI Unit (U.S. Customary Unit)	U.S. Equivalent (SI Equivalent)
Area	m <sup>2</sup> (ft <sup>2</sup> )	10.76 ft <sup>2</sup> (0.0929 m <sup>2</sup> )
	m <sup>2</sup> (in. <sup>2</sup> )	1550 in. <sup>2</sup> (0.645 × 10 <sup>-3</sup> m <sup>2</sup> )
Force	N (lb)	0.2248 lb (4.4482 N)
Length	m (ft)	3.2808 ft (0.3048 m)
	m (in.)	39.3701 in. (0.0254 m)
Mass	kg (lb)	2.2051 lb (0.4535 kg)
Moment of a force, torque	N·m (lb·ft)	0.7376 ft·lb (1.3558 N·m)
	N·m (lb·in.)	8.8496 lb·in. (0.1130 N·m)
Moment of inertia of a plane area	m <sup>4</sup> (in. <sup>4</sup> )	2.4025 × 10 <sup>6</sup> in. <sup>4</sup> (0.4162 μm <sup>4</sup> )
Power	W (ft·lb/s)	0.7376 ft·lb/s (1.3558 W)
	kW (hp)	1.3410 hp (0.7457 kW)
Pressure or stress	Pa (psi)	0.145 × 10 <sup>-3</sup> psi (6.895 kPa)
Velocity	m/s (ft/s)	3.2808 ft/s (0.3048 m/s)
Volume	m <sup>3</sup> (ft <sup>3</sup> )	35.3147 ft <sup>3</sup> (0.0283 m <sup>3</sup> )
Work or energy	J (ft·lb)	0.7376 ft·lb (1.3558 J)

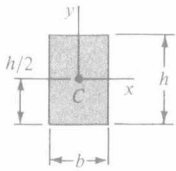
### SI Unit Prefixes

Prefix	Symbol	Factor
tera	T	10 <sup>12</sup> = 1 000 000 000 000
giga	G	10 <sup>9</sup> = 1 000 000 000
mega	M	10 <sup>6</sup> = 1 000 000
kilo	k	10 <sup>3</sup> = 1 000
hecto	h	10 <sup>2</sup> = 100
deka	da	10 <sup>1</sup> = 10
deci	d	10 <sup>-1</sup> = 0.1
centi	c	10 <sup>-2</sup> = 0.01
milli	m	10 <sup>-3</sup> = 0.001
micro	μ	10 <sup>-6</sup> = 0.000 001
nano	n	10 <sup>-9</sup> = 0.000 000 001
pico	p	10 <sup>-12</sup> = 0.000 000 000 001

*Note:* The use of the prefixes hecto, deka, deci, and centi is not recommended but they are sometimes encountered.

## Useful Properties of Common Areas

### Rectangle

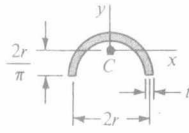


$$A = bh$$

$$I_x = \frac{bh^3}{12}$$

$$I_y = \frac{hb^3}{12}$$

### Half of thin tube

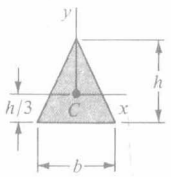


$$A = \pi r t$$

$$I_x \approx 0.095 \pi r^3 t$$

$$I_y = 0.5 \pi r^3 t$$

### Triangle

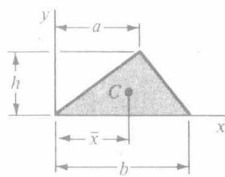


$$A = \frac{bh}{2}$$

$$I_x = \frac{bh^3}{36}$$

$$I_y = \frac{hb^3}{48}$$

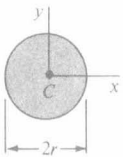
### Triangle



$$A = \frac{bh}{2}$$

$$\bar{x} = \frac{(a+b)}{3}$$

### Circle

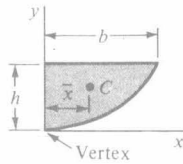


$$A = \pi r^2$$

$$I_x = I_y = \frac{\pi r^4}{4}$$

$$J_C = \frac{\pi r^4}{2}$$

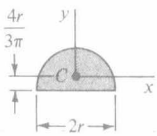
### Parabola ( $y = kx^2$ )



$$A = \frac{2bh}{3}$$

$$\bar{x} = \frac{3b}{8}$$

### Semicircle

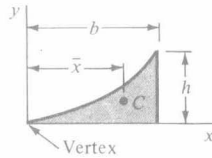


$$A = \frac{\pi r^2}{2}$$

$$I_x \approx 0.11 r^4$$

$$I_y = \frac{\pi r^4}{8}$$

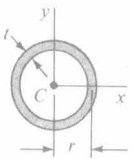
### Parabolic spandrel ( $y = kx^2$ )



$$A = \frac{bh}{3}$$

$$\bar{x} = \frac{3b}{4}$$

### Thin tube

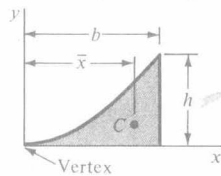


$$A = 2\pi r t$$

$$I_x = I_y = \pi r^3 t$$

$$J_C = 2\pi r^3 t$$

### General spandrel ( $y = kx^n$ )



$$A = \frac{bh}{n+1}$$

$$\bar{x} = \frac{n+1}{n+2} b$$

# **MECHANICS OF MATERIALS**

## ABOUT THE AUTHOR

Ansel C. Ugural is Adjunct Professor of Mechanics and Materials Science at the College of Engineering at Rutgers University. He received his Ph.D. in engineering mechanics from the University of Wisconsin—Madison. Dr. Ugural has taught at the University of Wisconsin as well as at Fairleigh Dickinson University, where he also served as Professor and Chairman of Mechanical Engineering from 1966 to 1990. He has considerable and varied industrial experience in both full-time and consulting capacities. A member of several professional societies, including the American Society of Mechanical Engineers and the American Society of Engineering Education, he is currently listed in *Who's Who in Engineering*. Dr. Ugural is the author of *Stresses in Plates and Shells* (McGraw-Hill, 1981), a coauthor (with S. K. Fenster) of *Advanced Strength and Applied Elasticity* (Elsevier, 1987), and has published numerous articles in the trade and professional journals.

# PREFACE

This volume is designed for an undergraduate-level engineering course in strength, or mechanics, of materials, although the author has endeavored to make it useful as a reference for engineering professionals as well. Fundamentals of the subject, the three aspects of solid mechanics problems, and the applications necessary to prepare students for more advanced study and for engineering practice are emphasized throughout.

The text offers a simple, comprehensive, and methodical presentation of the basic concepts in the analysis of members subjected to axial loads, torsion, bending, and pressure. The coverage presumes a knowledge of statics; however, topics that are particularly significant to an understanding of the subject are reviewed as they are taken up.

To enhance the student's overall understanding of the multiple aspects of structural strength, the behavior of various types of members is discussed, using design criteria such as yielding, plastic collapse, fatigue, and fracture. Failure criteria are employed to predict the behavior of structures under combined loadings. Analyses of members made of isotropic as well as composite and inelastic materials under ordinary and/or high-temperature loads are presented. Applications of the equilibrium, numerical, and energy methods are described in detail, and a treatment of the column-stability problem is included.

The rational design procedure—and its applications to axially loaded and twisted bars and beams—is described in order to make clear the relation of mechanics of materials to design. Equations of elasticity are presented to acquaint the reader with the essentially simple nature of the foundations of an important field in the mechanics of solids. These formulations are used to make critical evaluations of the force-deformation relations of the strength of materials. The text attempts to fill what the author believes to be a void in this regard.

The material presented here strikes a balance between the theory necessary to gain insight into mechanics and numerical solutions, both so useful in performing stress analysis in a realistic setting. Above all, an effort has been made to provide a visual interpretation of the basic equations and of the means by which the loads are resisted in typical members. Finally,



attention is given to analytical procedures, formulations, and numerical techniques suitable for computer programming.

Emphasis is placed upon the illustration of important principles and applications through examples, and a broad range of practical problems is provided for solution by the student. This volume offers more than 135 illustrative examples, fully worked out; more than 935 problem sets, many of which are drawn from engineering practice; and a multitude of formulas and tabulations (Appendix B) from which preliminary design calculations can be made. The properties of moments and centroids of areas are also described (Appendix A).

Both the International System of Units (SI) and the U.S. Customary System of units are used; but since in practice the former is replacing the latter, this book places a greater emphasis on SI units. (The conversion factors for common SI units and U.S. customary units are listed in a table inside the back cover of this text.) A sign convention that is consistent with vector mechanics is employed throughout for loads, internal forces, and stresses. This convention has been carefully chosen to conform to that used in most classical strength of materials texts as well as to that most often employed in numerical analysis of complex structures.

Most chapters are independent of one another and are self-contained. Hence the order of presentation can be smoothly altered to meet the instructor's preference. Optional sections that can be deleted without destroying the continuity of the text are identified by an asterisk.

The text is accompanied by a supporting package of instructional materials: a solutions manual to the problems and three diskettes containing the TK Solver designed for the IBM-PC, XT, AT, PS/2, or any compatible computer. The TK Solver is a powerful tool that builds and solves mathematical equations and models. It plots, uses graphics and charts, and has spreadsheet functions. Its purpose is to provide students with a tool for analyzing and solving a wide variety of problems in mechanics of materials. Thirty-one documented models address specific problems from all but the first chapter of the text. They were prepared by the author, programmed by Universal Technical Systems, Inc., and should be easy to use and understand.

Instructors should contact their local McGraw-Hill sales representatives to obtain the complete supplementary package. Students can purchase their TK Solver through the college bookstore.

## ACKNOWLEDGMENTS

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*A. C. Ugural*

# LIST OF SYMBOLS

$A$	area	$R$	reaction, force
$b$	breadth, width	$r$	radius, radius of gyration
$C$	centroid	$S$	elastic section modulus
$c$	distance from neutral axis to extreme fiber, radius	$s$	distance, length along a line
$D$	diameter	$T$	torque, temperature
$d$	diameter, distance, depth, dimension	$t$	thickness, width, tangential deviation
$E$	modulus of elasticity	$U$	strain energy
$e$	eccentricity, dilatation, distance	$u, v, w$	displacement components
$F$	force	$V$	shearing force, volume
$f$	frequency, flexibility, shape factor	$v$	velocity
$f_s$	factor of safety	$W$	work, weight
$G$	modulus of rigidity, shear modulus of elasticity	$w$	load per unit length
$g$	acceleration of gravity ( $\approx 9.81 \text{ m/s}^2$ )	$Z$	plastic section modulus
$h$	height, depth of beam	$x, y, z$	rectangular coordinates, distances
$I$	moment of inertia of area	$\alpha$ (alpha)	angle, coefficient of thermal expansion, form factor for shear
$J$	polar moment of inertia of area	$\gamma$ (gamma)	shearing strain, specific weight
$K$	stress concentration factor, impact factor	$\delta, \Delta$ (delta)	deformation, displacement, finite difference
$k$	spring constant, stiffness, shear coefficient	$\varepsilon$ (epsilon)	normal strain
$L$	length, span	$\theta$ (theta)	angle, slope
$M$	bending moment, couple	$\kappa$ (kappa)	curvature
$m$	mass	$\mu$ (mu)	micro
$N$	number of cycles	$\nu$ (nu)	Poisson's ratio
$n$	modular ratio, number of coils	$\rho$ (rho)	radius, radius of curvature, density
$P$	force, concentrated load	$\sigma$ (sigma)	normal stress
$p$	pressure	$\tau$ (tau)	shearing stress
$Q$	first moment of area, force	$\phi$ (phi)	angle, total angle of twist
$q$	shear force per unit length, shear flow	$\omega$ (omega)	angular velocity

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