ART IN THE LIFE OF MATHEMATICIANS

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ART IN THE LIFE OF MATHEMATICIANS

PREFACE

The mathematician's best work is art, a high perfect art, as daring as the most secret dreams of imagination, clear and limpid. Mathematical genius and artistic genius touch one another.

—Gosta Mittag-Leffler

The idea for this collection was born in 2010, when I organized an exhibition, entitled "Art in the World of Mathematicians," for the 70th birthday of my husband, Endre Szemerédi.

The exhibition's theme of "mathematics and art" generated such great public interest, from mathematicians and non-mathematicians alike, that I decided to create a book gathering the writings of mathematicians on art.

It occurred to me: Why not let mathematicians speak for themselves and express their thoughts on art? I was interested to hear what art means to them.

The color palette is vast. Mathematicians range from being devout opera fans and art enthusiasts to being playwrights, musicians, painters, photographers, actors, and dancers themselves, often at a professional level.

I asked world-class mathematicians to contribute to this book and many have graciously consented. Working with them has been a thrilling experience.

Naturally, many other mathematicians could have contributed to this book but did not due to various reasons such as time constraints or simply the fact that I did not know of their artistic interests and therefore did not contact them. I hope that some of them will be inspired by this book and will share their perspective in a future edition.

Acknowledgements. I cannot thank enough all of the brilliant mathematicians who have been eager to share their priceless thoughts in writing. I am

immensely grateful for their time, patience, and the enthusiasm with which they have contributed to this project.

I would like to express special thanks to Sergei Gelfand for having faith in my idea for this book and his help in bringing it to publication.

I greatly thank my husband, Endre Szemerédi, for his support and comments. I am grateful too for the opportunity to meet all his great colleagues and friends, many of whom have contributed to this book.

Finally, I would like to thank my daughters for their patience, time, and sometimes unvarnished critical comments while I was working on this collection.

I dedicate this book to my grandchildren hoping to teach them that mathematics is everywhere.

Anna Kepes Szemerédi

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Assaf Naor and AnnMarie Perl

INTRODUCTION

No attempt will be made here to systematically account for the enduring relationship of two areas as vast and vital as mathematics and art, or even to specifically introduce the variously autobiographical and philosophical articles of this volume, whose diversity, striking sincerity and passion may itself serve to better indicate the breadth and relevance of this subject. Instead, we highlight some aspects of this relationship that seem most important to us with the aim of bringing these obviously disparate disciplines into the closest possible comparison, and with the warning that, as a mathematician and an art historian, deeply immersed in our own disciplines, neither of us are specialists of this wholly other subject.

Firstly, it must be admitted that the relationship between mathematics and art is asymmetrical, insofar as, throughout history, artists have depended upon mathematics, whereas, to the best of our knowledge, no mathematician has ever used art to create new mathematics. If we focus on the visual arts, for which we have the most complete historical record, instead of on the performing arts, we can nevertheless observe some basic tendencies in common. These may help to explain the affinity of mathematicians for art, or at least why mathematicians often turn to art as a metaphor or an analogy, especially in conversation with the broader public, in order to make clear the invisible aesthetic qualities of their conceptual discoveries and the special kind of satisfaction that they derive from them. Even amongst themselves, mathematicians speak of beauty and elegance when confronted with those results that do much more than introduce important questions or solve difficult problems, but that, in addition, yield a nearly extra-intellectual, but obviously intellectual pleasure, which is paradoxically experienced as very much sensual. It is this particular pleasure, and its relation to art, that we will ultimately try to describe.

Tendency toward idealization. The most primary of shared properties between mathematics and art is perhaps the tendency toward idealization. Just as mathematics occurs in the world of ideas, in history, most of art has also reached for or depicted the worlds of the gods or heroes. In ancient Greek sculpture for instance, it is emphatically not nature or man that is being represented, because nature and man are low and base, full of faults and ugliness, easily accessible and reproducible, and almost totally without interest. (In anticipation, we may note that this idealizing tendency in art is one of the reasons why the arrival of photography could not and did not kill the appeal of art.) The figures represented in ancient Greek sculpture are not individual, historical human beings. These figures are exemplary and eternal, by far superior, divine beings, awesomely materialized in sculpture through mathematical reasoning.[1]

The lost *Canon* of the influential sculptor Polykleitos, for example, developed a method for representing the whole of the human body by repeating one measured part, which served throughout as a basic module.[2] Still today, art students are often taught to draw the human face's greatest spans by repeating the height of the forehead three times vertically and the width of the eye five times horizontally. While students can easily adapt these rules to accommodate the larger foreheads or widely-spaced eyes of their subjects, the ancient Greeks were not interested in lifelike portraits of people, but rather, as in the *Canon*, in ideal forms, whose internal consistency, independence from reality and isolation of the essential and general over and against the idiosyncratic and specific bear some resemblance to the main characteristics of mathematics. Indeed, mathematics was so potent an ally of art in ancient Greece that in a structure as singularly significant as the Parthenon it could be used not only for the basic design but also, on a more subtle level, to anticipate and correct for the inevitable weaknesses of human sense perception when confronted with a monument of such scale: The final structure included very slight, deliberately measured variations from purely straight lines, so that overall the building would appear to be ideal, that is, not in fact or in reality, but to man.[3]

When Plato vilified especially artists in Book X of *The Republic* as ignorant, arrogant, imitative, superficial and deceptive, he was criticizing them in very con-

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[1] Their nudity, we may note, is only a further sign of their divinity and distinction from humans, who must hide and clothe their inevitably imperfect bodies in public life, or otherwise offend their communities and risk their reputations.

[2] Richard Tobin, "The Canon of Polykleitos," *American Journal of Archaeology*, Vol. 79, No. 4 (Oct., 1975), pp. 307–321. Andrew Stewart, "The Canon of Polykleitos: A Question of Evidence," *The Journal of Hellenic Studies*, Vol. 98, (1987), pp. 122–131.

[3] On the Parthenon and the main theories of its departures from straight lines, see J.J. Pollitt, *Art and Experiences in Classical Greece*, (Cambridge: Cambridge University Press, 1972), pp. 71–78.

temporary terms, for famed artists like Polykleitos had arguably been knowledgeable in the advanced fields of anatomy and mathematics, well-trained in their crafts, highly specialized, creative, rational and, contrary to Plato's claims, virtuous. The arts, Plato argued, are merely pleasureful, and not beneficial to society or the individual, as mathematics are, Plato reasoned, for military strategy, the refinement of the mind and the pursuit of truth itself. Which is why, barring some unforeseen new arguments, Plato banned poetry and art from his Republic (his ideal society), while making mathematics foundational. When mathematicians today attempt to convince outsiders that mathematics is aesthetic and full of pleasure, they are inverting ancient values, judgments and oppositions with ongoing influence, not to mention contradicting most people's painful experience of mathematics in primary and secondary school.

Distinct pleasure in perfect forms. A second, related quality common to mathematics and art is the pleasure that is produced by encountering a perfect, simple, irreducible form, whether in the mind, through vision, or physically, as in large-scale sculpture or architecture. Although this pleasure is distinct, sometimes it is difficult to perceive as such, since the encounter is rarely as pure, obvious or articulable as seeing a circle or a tetrahedron. Normally, these uniquely defined forms do not exist in nature, but they are knowingly combined, modified and manipulated in art, so that they function subliminally and resist easy recognition. Compare the gratification that a mathematician experiences in finding the most meaningful and simplest possible pattern, connection or organizing principle between disparate data, where there previously seemed to be none, to an artist's creation of a dynamic, balanced and cohesive pictorial composition, or to a viewer's analysis of how this artist's arrangement of unlike elements operates formally as a unit and, furthermore, how this formal operation relates to the subject matter being depicted. Depending upon one view's of mathematics as invention or discovery, the mathematician here is a creative artist or a deeply engaged, even participatory viewer. While the execution of art is manual, and indeed computations are likewise necessary in mathematics, much of the work in both art and mathematics is conceptual, as is the resultant pleasure. It may not be necessary to mention the obvious: namely, that the more challenging the perception of the perfect form, the more rewarding the encounter with it; easy mathematics or easy art is not great mathematics or great art (ease has nothing to do with simplicity, which is hard-fought).

As a consequence of the tendency toward idealization, perfect forms or forms that are as simple, accurate and complete as possible are the preferred way in which space is endowed with structure in mathematics and art. Neither mathematics nor art would allow space to remain unarticulated or unknown;

neither discipline would accept that kind of chaos or cluelessness. Compare, for example, the notion of a group in mathematics to the central plan in architecture, and the special relationship of both to symmetry and their internal and overall consistency. The ubiquity of groups in contrast to the relative rarity of central-plan architecture, despite its potentially wide applicability and outstanding aesthetic effect of geometry on view, highlights an essential difference between mathematics, which floats freely, and architecture, which is grounded and very much bound to human function. It is not by coincidence that buildings as perfectly proportioned and scaled as the Pantheon or the Tempietto in Rome were erected as religious monuments, or, further, that Bramante's central plan for St. Peter's Basilica was heavily adapted in order to accommodate the diverse and specialized needs of the population that it had to serve. Maybe the Palladian villas outside of Venice provide us with the best examples of perfect forms in domestic use, but that such structures are rare luxuries and yet still demand considerable sacrifices of convenience demonstrates the relative freedom of mathematics from material and practical constraints.[4] Of course, while mathematics is free from physical constraints, insofar as it exceeds three dimensions and indeed often operates in the realm of unbounded or infinite dimensions, the powerful presence of art in the physical world is its greatest asset, since everyone who confronts art must contend with it, even if in the worst of circumstances only to dismiss it out of hand.

Abstraction and the avant-garde. The most obvious connection between mathematics and art may be their reliance upon abstraction, which in both cases is a later historical development that has yielded new information and connections, while creating new languages that require prolonged immersion (more than just reading or looking) in order to fully understand. Although within mathematics and art such new languages have put previously distinct groups of specialists into dialogue, advancing our overall knowledge greatly, they have also limited the access of and alienated non-speakers, which is perhaps the most pressing P.R. problem of both mathematics and art (some may consider it no problem at all). As mathematicians are accused of irrelevance, artists have been charged with the more serious crime of charlatanism, which assumes malicious intentions and deleterious social effects. In the case of art, it is also at least partially a paradox, since many artists employed

[4] Alternatively, under the most trying of circumstances, there is the profusion of a kind of central-plan architecture in tipis and igloos, but then these structures, in which function and aesthetics could be seen to be fortuitously aligned, are valued for their supreme functionality primarily, and not their prefectly conical and half-spherical shapes. Conventionally then, our appreciation of architecture is biased by preferences for monumental scale, permanence and uniqueness. Also, perfect forms should not be overly dominant or evident; for the greatest pleasure, they must be embedded in more complex forms and so discovered with delay.