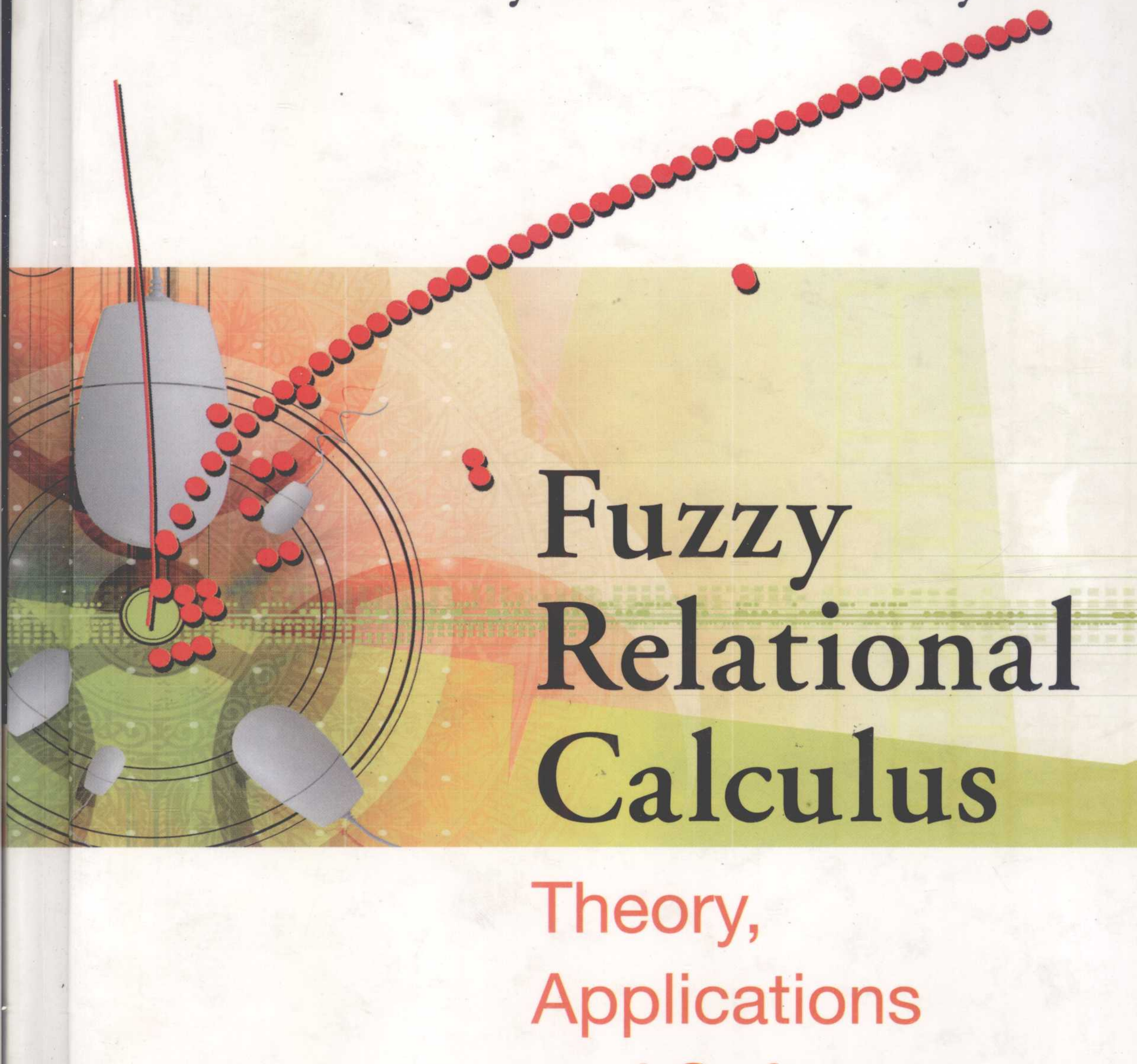
Advances in Fuzzy Systems — Applications and Theory – Vol. 22

Ketty Peeva & Yordan Kyosev



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Theory, Applications and Software

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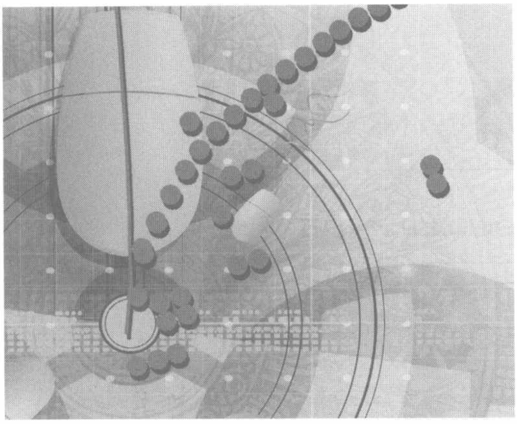

Fuzzy Relational Calculus

Theory, Applications and Software

(With CD-Rom)

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World Scientific

NEW JERSEY • LONDON • SINGAPORE • BEIJING • SHANGHAI • HONG KONG • TAIPEI • CHENNAI

Published by

World Scientific Publishing Co. Pte. Ltd.
5 Toh Tuck Link, Singapore 596224
USA office: 27 Warren Street, Suite 401–402, Hackensack, NJ 07601
UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

Advances in Fuzzy Systems — Applications and Theory, Vol. 22
FUZZY RELATIONAL CALCULUS
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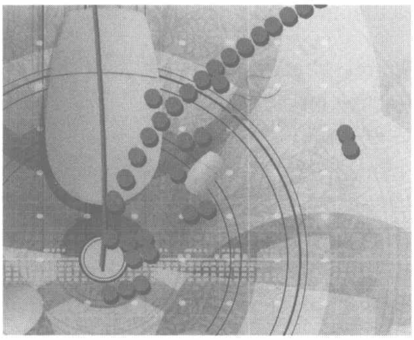
ISBN 981-256-076-9

Printed by Multiprint Services



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Preface

Fuzzy relations appear as a natural generalization of crisp relations. While a crisp relation determines the presence or absence of interconnectedness between the elements of two or more sets, fuzzy relations supply additional information for degrees of membership, strengths of associations, interaction between elements. Advantage of fuzzy relations is also that they permit to manipulate values that can be specified in linguistic terms.

As [Zadeh and Desoer (1963)] show, in the general study of systems the relationship between input and output parameters can be modelled by fuzzy relation between input and output spaces. Investigating behavior of such systems presumes a powerful fuzzy relational calculus. The importance of the theory of fuzzy relational equations is best described by Zadeh in the preface of the monograph by [Di Nola *et al.* (1989)]: *"Human knowledge may be viewed as a collection of facts and rules, each of which may be represented as the assignment of a fuzzy relation to the unconditional or conditional possibility distribution of a variable. What this implies is that the knowledge may be viewed as a system of fuzzy relational equations. In this perspective, then, inference from a body of knowledge reduces to the solution of a system of fuzzy relational equations."*

Fuzzy relations and fuzzy relational calculus have many reasonable applications in pure and applied mathematics. The basic operations with fuzzy relations correspond to the key operations in fuzzy logic. They are implemented in all inference forward or backward chain reasoning schemes as described for instance in [Bezdek (1999), Bien and Min (1995), Dubois and Prade (2000), Dubois *et al.* (1999), Klir and Yuan (1995), Zadeh *et al.* (1996)]. The most valuable implementations are in expert systems and in artificial intelligence areas – approximate reasoning, inference systems, knowledge representation, knowledge acquisition and validation, learning, in information processing, in pattern analysis and classification, in fuzzy system science for fuzzy control and modelling, in decision making, in engineering for fault detection and diagnosis, in management, etc.

Implementing fuzziness requires developing fuzzy relational calculus, special mathematical skills, ability to operate with modern mathematics and software.

In Part 1 we propose methodology and universal algorithms for direct and inverse problem resolution in fuzzy relational calculus. They include in unified frame fuzzy linear systems of equations, fuzzy relational equations, fuzzy relational inequalities and fuzzy relational inclusions upon various compositions in bounded chain. Then intuitionistic fuzzy relational calculus is reasonably developed. Computational complexity of the problems is investigated, numerical estimations are obtained. Based on these methods and algorithms, in Part 2 we solve open problems in the theory of fuzzy machines, fuzzy languages and syntactic fuzzy pattern recognition, we propose applications in modules of expert systems. The corresponding software for fuzzy relational calculus is described in Part 3. The Appendix and CD complete the exposition.

Theoretical background is presented in *Part 1*, Chapters 1 – 6.

In *Chapter 1* we include in unified frame the recent results in fuzzy relational calculus for compositions of fuzzy relations and solving fuzzy relational equations and also outline the place of investigations in this book.

Compositions of fuzzy relations and direct problem resolution are studied in *Chapter 2*. Attention is paid on their properties, the interconnection between fuzzy relation and its representative membership matrix, as well as between compositions of fuzzy relations and matrix multiplications.

Models based on fuzzy logic require methods and algorithms for solving fuzzy relational equations. Fuzzy relational equations stay in the heart of fuzzy relational calculus. They are subject of Chapters 3 – 6.

Chapter 3 is devoted to fuzzy linear systems of equations and fuzzy relational equations over a bounded chain, when the composition is the standard one (in particular max – min). Methods and algorithms are proposed for finding the complete solution set. Solvability criterion is proved. Analytical expressions are given for determining the solutions, if the system is consistent. In case of inconsistency, the connections, that can not be satisfied simultaneously with the other connections, are marked. We also investigate algorithmical solvability and computational complexity of the problems in this subject.

Chapter 4 covers inverse problem resolution for fuzzy linear systems of inequalities and fuzzy relational inclusions. Solvability condition is proved and analytical expressions are given for the solutions. Algorithms are proposed for solving fuzzy linear systems of inequalities and fuzzy relational inclusions. Applications in fuzzy linear programming are described.

Chapters 5 and 6 are reasonable extension of the previous two chapters. We study inverse problem resolution for co-standard (in particular min – max) composition of fuzzy relational equations and for intuitionistic fuzzy relational equations thereby solving open problems in fuzzy relational calculus.

In *Chapter 5* we investigate fuzzy relational equations over a bounded chain, if the composition is the co-standard one. We solve the inverse problem developing a conventional approach, based on the methodology of Chapter 3. Specially for the min − max composition on the real closed interval [0, 1] we propose another approach, implementing duality.

In *Chapter 6* we introduce and investigate direct and inverse problems in intuitionistic fuzzy relational calculus. Their resolution is provided by the methods and algorithms developed for standard and for co-standard compositions in previous chapters.

Fuzzy relational calculus, as presented in Part 1, provides a powerful theoretical background for dealing with fuzzy machines, fuzzy languages, pattern recognition, expert systems and other artificial intelligence areas, subject of *Part 2*, Chapters 7, 8 and 9.

Several types of fuzzy finite machines are studied in *Chapter 7*, all of them over a bounded chain. We investigate fuzzy finite machines with behavior obtained upon the standard or upon the co-standard law of composition. Both classes are reasonably joined in the case of intuitionistic fuzzy finite machines. Investigation of behavior, reduction and minimization problems of all these classes of fuzzy finite machines is provided by the fuzzy algebra theory as developed in the first part of this book. We express the behavior and study various equivalences, reduction and minimization problems and their algorithmical solvability, applying direct and inverse problem resolutions, as well as the algorithms from Part 1.

In *Chapter 8* we propose how to use regular fuzzy languages for syntactic pattern recognition and classification of distorted images. We introduce intuitionistic fuzzy languages and implement them for pattern recognition and classification.

In *Chapter 9* we give engineering applications of direct and inverse problem resolution in modules of expert systems for diagnosis, testing, validation, learning, fault detection and monitoring.

Chapter 10 in *Part 3* concerns the implemented in MATLAB Fuzzy Relational Calculus Toolbox. It describes realization of functions and algorithms, as presented in Part 1, for the fuzzy algebra $\mathbb{I} = ([0,1], \max, \min)$ without any restrictions about the size of the instant and about right-hand side constants (some of them or all may be equal). Working examples are also included.

Bibliographical notes at the end of each chapter include the most substantial theoretical and applied papers and monographs and also show the place of this investigation.

The book is based on original authors' results following mainly [Peeva (2002b)] and the recent publications by Peeva and Kyosev.

Available with this book is a CD, that contains functions, described in the book, as well as a lot of examples with solutions. The software has been tested on systems of dimension 50×50, but in principal the dimension is not limited. The examples are presented with input data, complete solution set and computational solution time. The CD contains also a restricted demo version of an alternative fast algorithm (which is not described in this book) for solving fuzzy linear systems. Tests are made on a PC with CPU 900 MHz, but some of them are also tested on machines with CPU 200 MHz up to 2400 MHz. The toolbox should be of use for both teaching and research.

The toolbox is distributed under the terms of the GNU General Public License (http://www.gnu.org/copyleft/gpl.html, version 2 of the License, or any later version) as published by the Free Software Foundation.

The tools used to prepare the book are the MikTEX (http://www.miktex.org/), including its excellent YAP previewer.

The authors are grateful to MATLAB for modern computer aids with the MathWorks Book Program.

We wish to thank all of the editorial and production staff of World Scientific publishing company. Special thanks are due to Prof. Elie Sanchez, Editor in the book series Advances in Fuzzy Systems.

We are also grateful to Prof. Krassimir Atanassov and to Dr. Tony Croft for their valuable comments and proposals that help to improve the presentation.

Ketty Peeva
Yordan Kyosev

Contents

PART 1

FUZZY RELATIONAL CALCULUS

Chapter 1

Introduction

In this chapter we outline the most important subjects in fuzzy relational calculus and point out the place of the problems that we solve in the book.

The presentation in Sections 1.1, 1.2, 1.3 and 1.4 follows in essence the well-established monographs or chapters in monographs by [De Baets (2000), Di Nola *et al.* (1989), Klir *et al.* (1997), Klir and Yuan (1995)]. Di Nola and co-authors in 1989 published the first monograph for fuzzy relational equations that is still actual. De Baets presents in a unified frame the basic problems for various relational compositions, fundamental results, applications and modern aspects. Klir and Yuan in their monograph give fundamental concepts, enriched with conceptual treatments, motivations, when and why the problems have appeared and mention various applied subjects specially for fuzzy relational calculus.

1.1 Basic Concepts

Here we recall some notions from algebra, lattice theory and fuzzy set theory. The terminology for algebra and lattice theory is according to [Grätzer (1978), MacLane and Birkhoff (1979), Davey and Priestley (1990)], for fuzzy sets and fuzzy relations we follow [Klir and Yuan (1995), De Baets (2000)].

The reader is supposed to be familiar with the basics of set theory. Nevertheless, some notations and notions follow.

The crisp set of all elements x of a given set B such that x satisfies the property P, is denoted by A,

$$A = \{x \mid x \in B \ \wedge \ P(x)\}.$$

An arbitrary crisp subset A of a given universal set E may be introduced by assigning the number 1 to each element of E that is element of A and by assigning the number 0 to the remaining elements of E. This assignment is known as *characteristic function* of the set A. The numbers 1 and 0 are used here only as convenient symbols and they do not have any numerical significance.

Universal sets are always supposed to be crisp, regardless of whether we deal with their crisp or fuzzy subsets.

Each fuzzy set $\tilde{A}$ is introduced in terms of a relevant crisp universal set E by a *membership function*. The membership function of a fuzzy set $\tilde{A}$ is usually denoted by μ_A and it has the form:

$$\mu_A : E \to [0, 1].$$

The value $\mu_A(x)$ is the *degree of membership* of the element $x \in E$ in $\tilde{A}$.

If X and Y are crisp sets, we let $X \times Y$ denote the Cartesian product of X and Y:

$$X \times Y = \{(x, y) \mid x \in X,\ y \in Y\}.$$

A *binary relation* R from a set X into a set Y is defined as a subset of $X \times Y$, written as $R \subseteq X \times Y$. For $(x, y) \in R$ we also write xRy. A *binary fuzzy relation* $\tilde{R}$ from a set X into a set Y is defined as a fuzzy subset of $X \times Y$.

In this book we operate with binary relations only. Next 'relation' is used instead of 'binary relation' and 'fuzzy relation' is used instead of 'binary fuzzy relation' with 'binary' being always suppressed.

If $X = Y$ we call R a *relation on* X.

When the set (relation, respectively) is taken in conventional sense, we call it *crisp set* (*crisp relation*, respectively).

Let $R \subseteq X \times Y$ be a crisp relation. The *domain* of R is defined to be the set

$$\mathrm{Dom}(R) = \{x \mid x \in X \wedge (\exists y \in Y)(xRy)\}.$$

The *image* (or *range*) of R is defined to be the set

$$\mathrm{Im}(R) = \{y \mid y \in Y \wedge (\exists x \in X)(xRy)\}.$$

A crisp relation R on the set P is called:

Reflexive: if aRa for all $a \in P$.
Antisymmetric: if aRb and bRa implies $a = b$.
Transitive: if aRb and bRc implies aRc.

These three properties are extended for fuzzy relations as well, see for details [Klir and Yuan (1995)].

A reflexive, antisymmetric and transitive crisp relation on P is called *partial order relation* and the corresponding set P is called *partially ordered set* or *poset.*

We use the symbol $\leq$ for partial order relation on a poset P. When P is a poset with respect to $\leq$, we write also $(P, \leq)$ or simply P.

A partial order relation R on P is called *total* (or *linear*) *order* if for each $x, y \in P$, either $(x, y) \in R$ or $(y, x) \in R$.

Let $(P, \leq)$ be a poset and $H \subseteq P$.

i) An element $a \in P$ is called an *upper bound* of the subset H if $h \leq a$ for all $h \in H$.

ii) An upper bound $a \in P$ of H is called *least upper bound* (l.u.b) of H if whenever $a' \in P$ is an upper bound of H, it holds $a \leq a'$, in symbols $a = \sup H$ or $a = \vee H$.

iii) An element $b \in P$ is called a *lower bound* of the subset H if $b \leq h$ for all $h \in H$.

iv) A lower bound $b \in P$ of H is called *greatest lower bound* (g.l.b) of H if whenever $b' \in P$ is a lower bound of H, it holds $b' \leq b$. In this case we write $b = \inf H$ or $b = \wedge H$.

v) By a *greatest element* of a poset P we mean an element $c \in P$ such that $x \leq c$ for all $x \in P$; the *least element* of P is defined dually.

vi) The (unique) least and greatest elements of P, when they exist, are called *universal bounds* of P and are denoted by 0 and 1, respectively.

Definition 1.1

i) A *lattice* is a poset L any two of whose elements x and y have a *greatest lower bound* or *meet* denoted by $x \wedge y$ or by $\wedge(x, y)$, and a *least upper bound* or *join* denoted by $x \vee y$ or by $\vee(x, y)$.

ii) A totally ordered poset is called a *chain.*

iii) A chain with universal bounds 0 and 1 is called *bounded chain.*

iv) A lattice is called *complete* if any of its subsets X has a l.u.b., denoted by $\sup X$ or $\vee X$ and a g.l.b. denoted by $\inf X$ or $\wedge X$.

v) A lattice L is called *distributive* if for all x, y, $z \in L$ the following equality holds: $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$.

vi) A *Brouwerian lattice* is a lattice L in which for any given elements a and b the set of all $x \in L$ such that $a \wedge x \leq b$ contains a greatest element, denoted $a \,\alpha\, b$ (called the *relative pseudocomplement* of a in b). In a *dually Brouwerian lattice* for any given elements a and b the set of all $x \in L$ such that $a \vee x \geq b$ contains a least element, denoted $a \,\varepsilon\, b$ (*dually relative pseudocomplement* of a in b).

vii) A lattice L with universal bounds 0 and 1 is called *complemented* if each element $x \in L$ has a complement.

We say that an element $y \in L$ is a *complement* of $x \in L$, if

$$x \wedge y = 0 \quad \text{and} \quad x \vee y = 1$$

hold.

In this book we work on complete lattices because we have always to compute the g.l.b. and the l.u.b. of compositions of relations.

We write

$$\mathbb{L} = (L, \vee, \wedge, 0, 1)$$