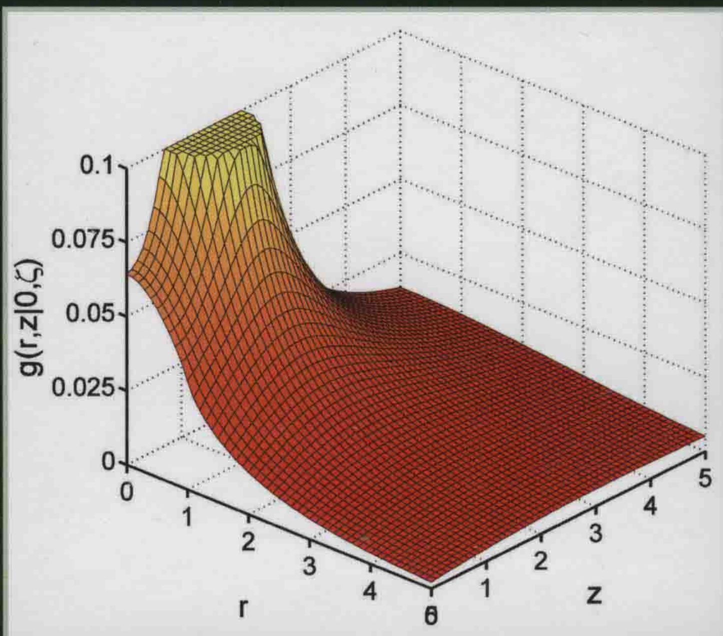
Advances in Applied Mathematics

GREEN'S FUNCTIONS WITH APPLICATIONS

Second Edition



DEAN G. DUFFY

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Advances in Applied Mathematics

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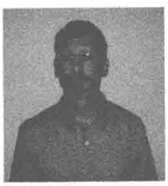
Dedicated to My Dear Friends Pete, Philippe,

Ed, Dave, Karl, Michael, El, John, and Ralph

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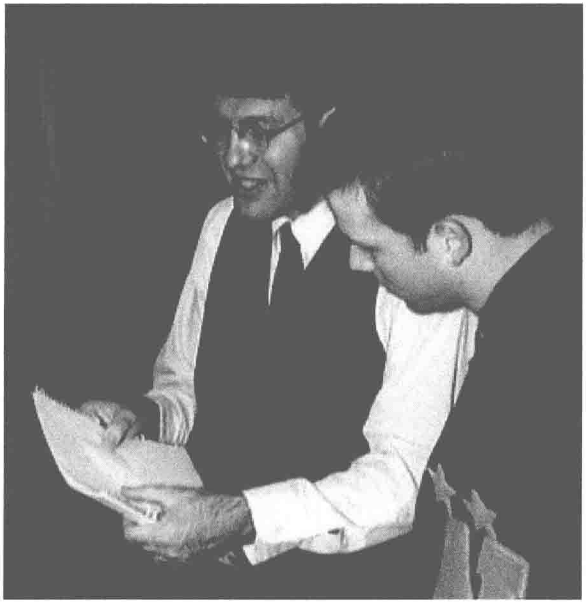
Special thanks go to Prof. Michael D. Marcozzi for his many useful suggestions for improving this book. Dr. Tim DelSole provided outstanding guidance in the section on convective/absolute instability. Drs. Chris Linton and Geordie D. McBain made several useful suggestions regarding Section 6.9 and the numerical methods chapter, respectively. Many of the plots and calculations were done using MATLAB®.



Author

Dean G. Duffy received his bachelor of science in geophysics from Case Institute of Technology (Cleveland, Ohio) and his doctorate of science in meteorology from the Massachusetts Institute of Technology (Cambridge, Massachusetts). He served in the United States Air Force from September 1975 to December 1979 as a numerical weather prediction officer. After his military service, he began a twenty-five year (1980 to 2005) association with NASA at the Goddard Space Flight Center (Greenbelt, Maryland) where he focused on numerical weather prediction, oceanic wave modeling and dynamical meteorology. He also wrote papers in the areas of Laplace transforms, antenna theory, railroad tracks, and heat conduction. In addition to his NASA duties he taught engineering mathematics, differential equations and calculus at the United States Naval Academy (Annapolis, Maryland) and the United States Military Academy (West Point, New York). Drawing from his teaching experience, he has written several books on transform methods, engineering mathematics, Green's functions and mixed boundary value problems.

Preface

This book had its origin in some electronic mail that I received from William S. Price a number of years ago. He needed to construct a Green's function and asked me if I knew a good book that might assist him. In suggesting several standard texts, I could not help but think that, based on my own experiences utilizing Green's functions alone and in conjunction with numerical solvers, I had my own ideas on how to present this material. It was this thought that ultimately led to the development of this monograph.

Since its original publication 14 years ago, the purpose of this book has remained the same: to provide applied scientists and engineers with a systematic presentation of the various methods available for deriving a Green's function. However, I now understand many of the weaknesses of my original text and have sought to address these flaws in this new edition.

The book opens with a new chapter on the historical development of the Green's function. References are made to many seminal papers and books that helped establish Green's function in its current use. The early history was slow and difficult until the mathematical concepts of transform methods and the Dirac delta function became well established.

All books are written with assumptions about the background of the reader. Chapter 2 was written so that everyone would start on a flat playing field with regards to Fourier and Laplace transforms and the classical special functions of Bessel functions and Legendre polynomials. We also include a review of the Dirac delta function. We close this chapter by anticipating future results and introducing some simple applications of Green's function.

Chapters 3 through 6 are the heart of the book. For each class of differential equation (ordinary differential, wave, heat and Helmholtz equations) we present Green's functions according to the number of spatial dimensions and the geometry of the domain. In each chapter I added solutions which I missed in my first edition and new results that have appeared since 2001. Of particular interest is the use of Green's functions in the life sciences.

Each chapter closes with special sections devoted to topics where Green's functions are particularly useful. For example, in the case of the wave equation, Green's functions are very useful in describing diffraction and wave waves. These chapters also include problem sets. They are there so that the reader can master Green's function as well as a source of additional results.

A new aspect to this book is the use of the symbol □ to denote the end of an example, proof, or remark. The only exception occurs if an example concludes a section. In this case its use is superfluous.

A few years after the publication of the original book, I received an email from a graduate student asking about numerical methods for computing Green's functions for an aeronautical problem that he was solving. This incident motivated me to write the chapter on numerical methods that closes this book. Clearly this is an underdeveloped area of study and further work is expected.

Definitions of the Most Commonly Used Functions

Function	Definition
$\delta(t-a)$	$= \begin{cases} \infty, & t = a, \\ 0, & t \neq a, \end{cases} \qquad \int_{-\infty}^{\infty} \delta(t-a)\,dt = 1$
$H(t-a)$	$= \begin{cases} 1, & t > a, \\ 0, & t < a, \end{cases} \qquad a \geq 0$
$I_n(x)$	modified Bessel function of the first kind and order n
$J_n(x)$	Bessel function of the first kind and order n
$K_n(x)$	modified Bessel function of the second kind and order n
$P_n(x)$	Legendre polynomial of order n
$r_<$	$= \min(r,\rho)$
$r_>$	$= \max(r,\rho)$
$x_<$	$= \min(x,\xi)$
$x_>$	$= \max(x,\xi)$
$Y_n(x)$	Bessel function of the second kind and order n
$y_<$	$= \min(y,\eta)$
$y_>$	$= \max(y,\eta)$
$z_<$	$= \min(z,\zeta)$
$z_>$	$= \max(z,\zeta)$

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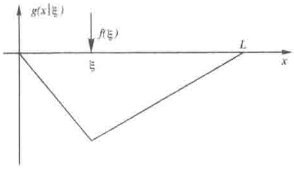

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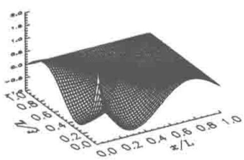

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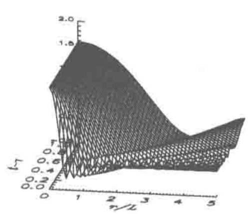

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