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Theory and Algorithms

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Preface

The idea for this book grew out of a course given at a winter school of the International Doctoral Program "Identification, Optimization and Control with Applications in Modern Technologies" in Schloss Thurnau in March 2009. Initially, the main purpose of this course was to present results on stability and performance analysis of nonlinear model predictive control algorithms, which had at that time recently been obtained by ourselves and coauthors. However, we soon realized that both the course and even more the book would be inevitably incomplete without a comprehensive coverage of classical results in the area of nonlinear model predictive control and without the discussion of important topics beyond stability and performance, like feasibility, robustness, and numerical methods.

As a result, this book has become a mixture between a research monograph and an advanced textbook. On the one hand, the book presents original research results obtained by ourselves and coauthors during the last five years in a comprehensive and self contained way. On the other hand, the book also presents a number of results—both classical and more recent—of other authors. Furthermore, we have included a lot of background information from mathematical systems theory, optimal control, numerical analysis and optimization to make the book accessible to graduate students—on PhD and Master level—from applied mathematics and control engineering alike. Finally, via our web page www.nmpc-book.com we provide MATLAB and C++ software for all examples in this book, which enables the reader to perform his or her own numerical experiments. For reading this book, we assume a basic familiarity with control systems, their state space representation as well as with concepts like feedback and stability as provided, e.g., in undergraduate courses on control engineering or in courses on mathematical systems and control theory in an applied mathematics curriculum. However, no particular knowledge of nonlinear systems theory is assumed. Substantial parts of the systems theoretic chapters of the book have been used by us for a lecture on nonlinear model predictive control for master students in applied mathematics and we believe that the book is well suited for this purpose. More advanced concepts like time varying formulations or peculiarities of sampled data systems can be easily skipped if only time invariant problems or discrete time systems shall be treated.

The book centers around two main topics: systems theoretic properties of nonlinear model predictive control schemes on the one hand and numerical algorithms on the other hand; for a comprehensive description of the contents we refer to Sect. 1.3. As such, the book is somewhat more theoretical than engineering or application oriented monographs on nonlinear model predictive control, which are furthermore often focused on linear methods.

Within the nonlinear model predictive control literature, distinctive features of this book are the comprehensive treatment of schemes without stabilizing terminal constraints and the in depth discussion of performance issues via infinite horizon suboptimality estimates, both with and without stabilizing terminal constraints. The key for the analysis in the systems theoretic part of this book is a uniform way of interpreting both classes of schemes as relaxed versions of infinite horizon optimal control problems. The *relaxed dynamic programming* framework developed in Chap. 4 is thus a cornerstone of this book, even though we do not use dynamic programming for actually solving nonlinear model predictive control problems; for this task we prefer direct optimization methods as described in the last chapter of this book, since they also allow for the numerical treatment of high dimensional systems.

There are many people whom we have to thank for their help in one or the other way. For pleasant and fruitful collaboration within joint research projects and on joint papers—of which many have been used as the basis for this book—we are grateful to Frank Allgöwer, Nils Altmüller, Rolf Findeisen, Marcus von Lossow, Dragan Nešić, Anders Rantzer, Martin Seehafer, Paolo Varutti and Karl Worthmann. For enlightening talks, inspiring discussions, for organizing workshops and minisymposia (and inviting us) and, last but not least, for pointing out valuable references to the literature we would like to thank David Angeli, Moritz Diehl, Knut Graichen, Peter Hokayem, Achim Ilchmann, Andreas Kugi, Daniel Limón, Jan Lunze, Lalo Magni, Manfred Morari, Davide Raimondo, Saša Raković, Jörg Rambau, Jim Rawlings, Markus Reble, Oana Serea and Andy Teel, and we apologize to everyone who is missing in this list although he or she should have been mentioned. Without the proof reading of Nils Altmüller, Robert Baier, Thomas Jahn, Marcus von Lossow, Florian Müller and Karl Worthmann the book would contain even more typos and inaccuracies than it probably does—of course, the responsibility for all remaining errors lies entirely with us and we appreciate all comments on errors, typos, missing references and the like. Beyond proof reading, we are grateful to Thomas Jahn for his help with writing the software supporting this book and to Karl Worthmann for his contributions to many results in Chaps. 6 and 7, most importantly the proof of Proposition 6.17. Finally, we would like to thank Oliver Jackson and Charlotte Cross from Springer-Verlag for their excellent support.

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Contents

Chapter 1
Introduction

1.1 What Is Nonlinear Model Predictive Control?

Nonlinear model predictive control (henceforth abbreviated as NMPC) is an optimization based method for the feedback control of nonlinear systems. Its primary applications are *stabilization* and *tracking* problems, which we briefly introduce in order to describe the basic idea of model predictive control.

Suppose we are given a controlled process whose state $x(n)$ is measured at discrete time instants t_n, $n = 0, 1, 2, \ldots$. "Controlled" means that at each time instant we can select a control input $u(n)$ which influences the future behavior of the state of the system. In tracking control, the task is to determine the control inputs $u(n)$ such that $x(n)$ follows a given *reference* $x^{\text{ref}}(n)$ as good as possible. This means that if the current state is far away from the reference then we want to control the system towards the reference and if the current state is already close to the reference then we want to keep it there. In order to keep this introduction technically simple, we consider $x(n) \in X = \mathbb{R}^d$ and $u(n) \in U = \mathbb{R}^m$, furthermore we consider a reference which is constant and equal to $x_* = 0$, i.e., $x^{\text{ref}}(n) = x_* = 0$ for all $n \geq 0$. With such a constant reference the tracking problem reduces to a stabilization problem; in its full generality the tracking problem will be considered in Sect. 3.3.

Since we want to be able to react to the current deviation of $x(n)$ from the reference value $x_* = 0$, we would like to have $u(n)$ in *feedback form*, i.e., in the form $u(n) = \mu(x(n))$ for some map μ mapping the state $x \in X$ into the set U of control values.

The idea of model predictive control—linear or nonlinear—is now to utilize a model of the process in order to predict and optimize the future system behavior. In this book, we will use models of the form

$$x^+ = f(x, u) \tag{1.1}$$

where $f : X \times U \to X$ is a known and in general nonlinear map which assigns to a state x and a control value u the successor state x^+ at the next time instant. Starting from the current state $x(n)$, for any given control sequence $u(0), \ldots, u(N-1)$ with

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horizon length $N \geq 2$, we can now iterate (1.1) in order to construct a prediction trajectory x_u defined by

$$x_u(0) = x(n), \qquad x_u(k+1) = f\big(x_u(k), u(k)\big), \quad k = 0, \ldots, N-1. \quad (1.2)$$

Proceeding this way, we obtain predictions $x_u(k)$ for the state of the system $x(n+k)$ at time t_{n+k} in the future. Hence, we obtain a prediction of the behavior of the system on the discrete interval $t_n, \ldots, t_{n+N}$ depending on the chosen control sequence $u(0), \ldots, u(N-1)$.

Now we use optimal control in order to determine $u(0), \ldots, u(N-1)$ such that x_u is as close as possible to $x_* = 0$. To this end, we measure the distance between $x_u(k)$ and $x_* = 0$ for $k = 0, \ldots, N-1$ by a function $\ell(x_u(k), u(k))$. Here, we not only allow for penalizing the deviation of the state from the reference but also—if desired—the distance of the control values $u(k)$ to a reference control u_*, which here we also choose as $u_* = 0$. A common and popular choice for this purpose is the quadratic function

$$\ell\big(x_u(k), u(k)\big) = \big\|x_u(k)\big\|^2 + \lambda\big\|u(k)\big\|^2,$$

where $\|\cdot\|$ denotes the usual Euclidean norm and $\lambda \geq 0$ is a weighting parameter for the control, which could also be chosen as 0 if no control penalization is desired. The optimal control problem now reads

$$\text{minimize} \quad J\big(x(n), u(\cdot)\big) := \sum_{k=0}^{N-1} \ell\big(x_u(k), u(k)\big)$$

with respect to all admissible[1] control sequences $u(0), \ldots, u(N-1)$ with x_u generated by (1.2).

Let us assume that this optimal control problem has a solution which is given by the minimizing control sequence $u^\star(0), \ldots, u^\star(N-1)$, i.e.,

$$\min_{u(0),\ldots,u(N-1)} J\big(x(n), u(\cdot)\big) = \sum_{k=0}^{N-1} \ell\big(x_{u^\star}(k), u^\star(k)\big).$$

In order to get the desired feedback value $\mu(x(n))$, we now set $\mu(x(n)) := u^\star(0)$, i.e., we apply the first element of the optimal control sequence. This procedure is sketched in Fig. 1.1.

At the following time instants $t_{n+1}, t_{n+2}, \ldots$ we repeat the procedure with the new measurements $x(n+1), x(n+2), \ldots$ in order to derive the feedback values $\mu(x(n+1)), \mu(x(n+2)), \ldots$. In other words, we obtain the feedback law μ by an *iterative online optimization* over the predictions generated by our model (1.1).[2] This is the first key feature of model predictive control.

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[1] The meaning of "admissible" will be defined in Sect. 3.2.

[2] Attentive readers may already have noticed that this description is mathematically idealized since we neglected the computation time needed to solve the optimization problem. In practice, when the measurement $x(n)$ is provided to the optimizer the feedback value $\mu(x(n))$ will only be available after some delay. For simplicity of exposition, throughout our theoretical investigations we will assume that this delay is negligible. We will come back to this problem in Sect. 7.6.

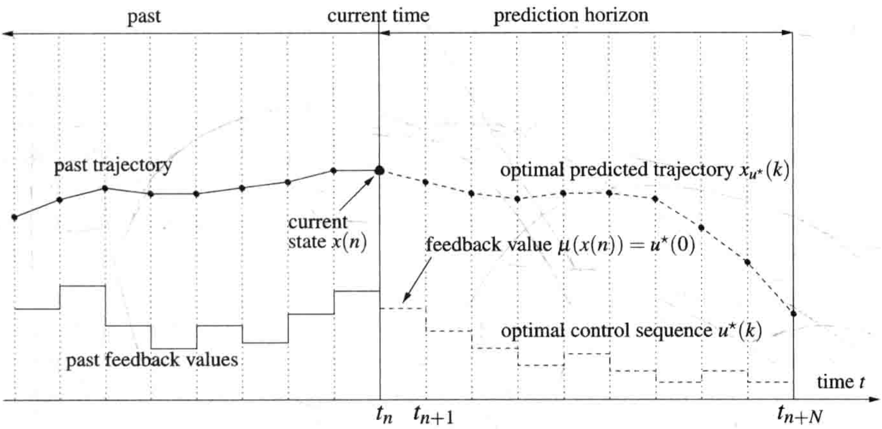

Fig. 1.1 Illustration of the NMPC step at time t_n

From the prediction horizon point of view, proceeding this iterative way the trajectories $x_u(k)$, $k = 0, \ldots, N$ provide a prediction on the discrete interval $t_n, \ldots, t_{n+N}$ at time t_n, on the interval $t_{n+1}, \ldots, t_{n+N+1}$ at time t_{n+1}, on the interval $t_{n+2}, \ldots, t_{n+N+2}$ at time t_{n+2}, and so on. Hence, the prediction horizon is moving and this *moving horizon* is the second key feature of model predictive control.

Regarding terminology, another term which is often used alternatively to *model predictive control* is *receding horizon control*. While the former expression stresses the use of model based predictions, the latter emphasizes the moving horizon idea. Despite these slightly different literal meanings, we prefer and follow the common practice to use these names synonymously. The additional term *nonlinear* indicates that our model (1.1) need not be a linear map.

1.2 Where Did NMPC Come from?

Due to the vast amount of literature, the brief history of NMPC we provide in this section is inevitably incomplete and focused on those references in the literature from which we ourselves learned about the various NMPC techniques. Furthermore, we focus on the systems theoretic aspects of NMPC and on the academic development; some remarks on numerical methods specifically designed for NMPC can be found in Sect. 10.7. Information about the use of linear and nonlinear MPC in practical applications can be found in many articles, books and proceedings volumes, e.g., in [15, 22, 24].

Nonlinear model predictive control grew out of the theory of optimal control which had been developed in the middle of the 20th century with seminal contributions like the maximum principle of Pontryagin, Boltyanskii, Gamkrelidze and Mishchenko [20] and the dynamic programming method developed by Bellman [2]. The first paper we are aware of in which the central idea of model predictive control—for discrete time linear systems—is formulated was published by Propoĭ [21] in the early 1960s. Interestingly enough, in this paper neither Pontryagin's maximum principle nor dynamic programming is used in order to solve the optimal control problem. Rather, the paper already proposed the method which is predominant nowadays in NMPC, in which the optimal control problem is transformed into a static optimization problem, in this case a linear one. For nonlinear systems, the idea of model predictive control can be found in the book by Lee and Markus [14] from 1967 on page 423:

> One technique for obtaining a feedback controller synthesis from knowledge of open-loop controllers is to measure the current control process state and then compute very rapidly for the open-loop control function. The first portion of this function is then used during a short time interval, after which a new measurement of the process state is made and a new open-loop control function is computed for this new measurement. The procedure is then repeated.

Due to the fact that neither computer hardware nor software for the necessary "very rapid" computation were available at that time, for a while this observation had little practical impact.

In the late 1970s, due to the progress in algorithms for solving constrained linear and quadratic optimization problems, MPC for linear systems became popular in control engineering. Richalet, Rault, Testud and Papon [25] and Cutler and Ramaker [6] were among the first to propose this method in the area of process control, in which the processes to be controlled are often slow enough in order to allow for an online optimization, even with the computer technology available at that time. It is interesting to note that in [25] the method was described as a "new method of digital process control" and earlier references were not mentioned; it appears that the basic MPC principle was re-invented several times. Systematic stability investigations appeared a little bit later; an account of early results in that direction for linear MPC can, e.g., be found in the survey paper of García, Prett and Morari [10] or in the monograph by Bitmead, Gevers and Wertz [3]. Many of the techniques which later turned out to be useful for NMPC, like Lyapunov function based stability proofs or stabilizing terminal constraints were in fact first developed for linear MPC and later carried over to the nonlinear setting.

The earliest paper we were able to find which analyzes an NMPC algorithm similar to the ones used today is an article by Chen and Shaw [4] from 1982. In this paper, stability of an NMPC scheme with equilibrium terminal constraint in continuous time is proved using Lyapunov function techniques, however, the whole optimal control function on the optimization horizon is applied to the plant, as opposed to only the first part as in our NMPC paradigm. For NMPC algorithms meeting this paradigm, first comprehensive stability studies for schemes with equilibrium terminal constraint were given in 1988 by Keerthi and Gilbert [13] in discrete time and in 1990 by Mayne and Michalska [17] in continuous time. The fact that for nonlinear systems equilibrium terminal constraints may cause severe numerical difficulties subsequently motivated the investigation of alternative techniques. Regional terminal constraints in combination with appropriate terminal costs turned out to be a suitable tool for this purpose and in the second half of the 1990s there was a rapid development of such techniques with contributions by De Nicolao, Magni and Scattolini [7, 8], Magni and Sepulchre [16] or Chen and Allgöwer [5], both in discrete and continuous time. This development eventually led to the formulation of a widely accepted "axiomatic" stability framework for NMPC schemes with stabilizing terminal constraints as formulated in discrete time in the survey article by Mayne, Rawlings, Rao and Scokaert [18] in 2000, which is also an excellent source for more detailed information on the history of various NMPC variants not mentioned here. This framework also forms the core of our stability analysis of such schemes in Chap. 5 of this book. A continuous time version of such a framework was given by Fontes [9] in 2001.

All stability results discussed so far add terminal constraints as additional state constraints to the finite horizon optimization in order to ensure stability. Among the first who provided a rigorous stability result of an NMPC scheme without such constraints were Parisini and Zoppoli [19] and Alamir and Bornard [1], both in 1995 and for discrete time systems. Parisini and Zoppoli [19], however, still needed a terminal cost with specific properties similar to the one used in [5]. Alamir and Bonnard [1] were able to prove stability without such a terminal cost by imposing a rank condition on the linearization on the system. Under less restrictive conditions, stability results were provided in 2005 by Grimm, Messina, Tuna and Teel [11] for discrete time systems and by Jadbabaie and Hauser [12] for continuous time systems. The results presented in Chap. 6 of this book are qualitatively similar to these references but use slightly different assumptions and a different proof technique which allows for quantitatively tighter results; for more details we refer to the discussions in Sects. 6.1 and 6.9.

After the basic systems theoretic principles of NMPC had been clarified, more advanced topics like robustness of stability and feasibility under perturbations, performance estimates and efficiency of numerical algorithms were addressed. For a discussion of these more recent issues including a number of references we refer to the final sections of the respective chapters of this book.

1.3 How Is This Book Organized?

The book consists of two main parts, which cover systems theoretic aspects of NMPC in Chaps. 2–8 on the one hand and numerical and algorithmic aspects in Chaps. 9–10 on the other hand. These parts are, however, not strictly separated; in particular, many of the theoretical and structural properties of NMPC developed in the first part are used when looking at the performance of numerical algorithms.

The basic theme of the first part of the book is the systems theoretic analysis of stability, performance, feasibility and robustness of NMPC schemes. This part starts with the introduction of the *class of systems* and the presentation of *background material* from Lyapunov stability theory in Chap. 2 and proceeds with a *detailed* *description of different NMPC algorithms* as well as related background information on dynamic programming in Chap. 3.

A distinctive feature of this book is that both schemes with stabilizing terminal constraints as well as schemes without such constraints are considered and treated in a uniform way. This "uniform way" consists of interpreting both classes of schemes as relaxed versions of *infinite horizon optimal control*. To this end, Chap. 4 first develops the theory of infinite horizon optimal control and shows by means of dynamic programming and Lyapunov function arguments that infinite horizon optimal feedback laws are actually asymptotically stabilizing feedback laws. The main building block of our subsequent analysis is the development of a *relaxed dynamic programming* framework in Sect. 4.3. Roughly speaking, Theorems 4.11 and 4.14 in this section extract the main structural properties of the infinite horizon optimal control problem, which ensure

- asymptotic or practical asymptotic stability of the closed loop,
- admissibility, i.e., maintaining the imposed state constraints,
- a guaranteed bound on the infinite horizon performance of the closed loop,
- applicability to NMPC schemes with and without stabilizing terminal constraints.

The application of these theorems does not necessarily require that the feedback law to be analyzed is close to an infinite horizon optimal feedback law in some quantitative sense. Rather, it requires that the two feedback laws share certain properties which are sufficient in order to conclude asymptotic or practical asymptotic stability and admissibility for the closed loop. While our approach allows for investigating the infinite horizon performance of the closed loop for most schemes under consideration—which we regard as an important feature of the approach in this book—we would like to emphasize that near optimal infinite horizon performance is not needed for ensuring stability and admissibility.

The results from Sect. 4.3 are then used in the subsequent Chaps. 5 and 6 in order to analyze stability, admissibility and infinite horizon performance properties for NMPC schemes with and without stabilizing terminal constraints, respectively. Here, the results for *NMPC schemes with stabilizing terminal constraints* in Chap. 5 can by now be considered as classical and thus mainly summarize what can be found in the literature, although some results—like, e.g., Theorems 5.21 and 5.22—generalize known results. In contrast to this, the results for *NMPC schemes without stabilizing terminal constraints* in Chap. 6 were mainly developed by ourselves and coauthors and have not been presented before in this way.

While most of the results in this book are formulated and proved in a mathematically rigorous way, Chap. 7 deviates from this practice and presents a couple of *variants and extensions* of the basic NMPC schemes considered before in a more survey like manner. Here, proofs are occasionally only sketched with appropriate references to the literature.

In Chap. 8 we return to the more rigorous style and discuss *feasibility and robustness* issues. In particular, in Sects. 8.1–8.3 we present feasibility results for NMPC schemes without stabilizing terminal constraints and without imposing viability assumptions on the state constraints which are, to the best of our knowledge, either