

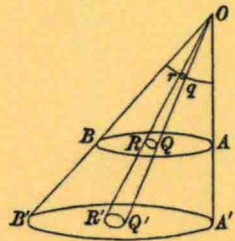
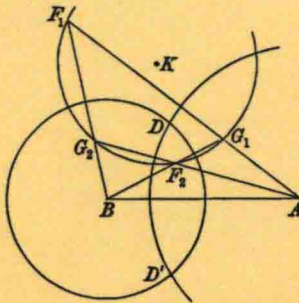
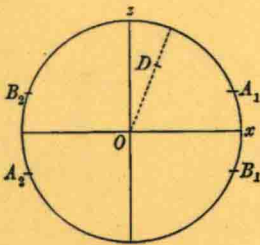
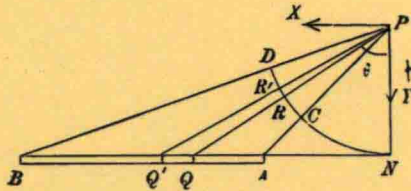
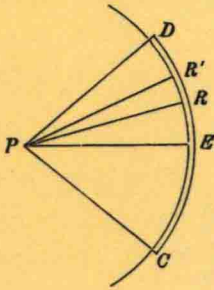
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A TREATISE ON ANALYTICAL STATICS

WITH NUMEROUS EXAMPLES

VOLUME 1

EDWARD JOHN ROUTH



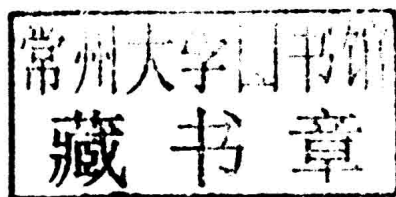
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A Treatise on Analytical Statics

With Numerous Examples

VOLUME 1

EDWARD JOHN ROUTH



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A TREATISE ON
ANALYTICAL STATICS

WITH NUMEROUS EXAMPLES

BY

EDWARD JOHN ROUTH, Sc.D., LL.D., M.A., F.R.S., &c.,

HON. FELLOW OF PETERHOUSE, CAMBRIDGE;

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PREFACE.

DURING many years it has been my duty and pleasure to give courses of lectures on various Mathematical subjects to successive generations of students. The course on Statics has been made the groundwork of the present treatise. It has however been necessary to make many additions; for in a treatise all parts of the subject must be discussed in a connected form, while in a series of lectures a suitable choice has to be made.

A portion only of the science of Statics has been included in this volume. It is felt that such subjects as Attractions, Astatics, and the Bending of rods could not be adequately treated at the end of a treatise without either making the volume too bulky or requiring the other parts to be unduly curtailed. These remaining portions appear in the second volume.

In order to learn Statics it is essential to the student to work numerous examples. Besides some of my own construction, I have collected a large number from the University and College Examination papers. Some of these are so good as to deserve to rank among the theorems of the science rather than among the examples. Solutions have been given to many of the examples, sometimes at length and in other cases in the form of hints when these appeared sufficient.

I have endeavoured to refer each result to its original author. I have however found that it is a very difficult task to effect this

with any completeness. The references will show that I have searched many of the older books and memoirs as well as some of those of recent date to discover the first mention of a theorem.

In this edition I have made many additions and have also omitted several things which on after consideration appeared to be of minor importance. The explanations also have been simplified wherever there appeared to be any obscurity. For the convenience of reference I have retained the order of the articles as far as that was possible.

The latter part of the chapter on forces in three dimensions has been enlarged by the addition of several theorems and the portions on five and six forces re-arranged. The chapter on graphical statics also has been almost entirely rewritten.

An index has been added which it is hoped will be found useful.

EDWARD J. ROUTH.

PETERHOUSE,
May, 1896.

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