Mechanics of Machines

Third Edition

V. Ramamurti

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Preface to the Third Edition

This third edition has the following three major additions: Expressions are derived for the length of belt required to connect two shafts at a given center distance with pulley sizes known. Unlike the case of flat belts, vee belts of any length cannot be procured since they are endless. Standard sizes of vee belts are listed to decide on the belt to be used.

The second is the inclusion of salient features of commercial balancing machines. The speed range of operation of these machines manufactured with reasons are indicated. Their limitations on usage is highlighted. The distinction between rigid rotors and flexible rotors is presented and the third is the dynamics of diesel electric locomotive. The severe drawbacks of steam locomotive, with specific reference to dynamics is presented. The presence of variation in tractive effort and hammer blow limits the speed of operation of these locomotives. This is totally eliminated in diesel electric locomotives, especially with the introduction of multicylinder vee configuration.

It is hoped that these additions would be useful to students and professionals in the industry.

V. RAMAMURTI

Preface to the First Edition

The book has been written to update the information on this important subject of interest to mechanical engineers. It serves both the purposes—a good textbook for undergraduate students in mechanical engineering and a reference for practicing engineers. Some chapters have been completely altered from the material presented in conventional textbooks, e.g. kinematics of plane and space motion. Emphasis is given to analytical method of solution with relevant computer programs. The use of computers in the chapter on dynamics is a deviation from the classical presentation. Center distance modification in involute gears, efficiency of gears trains, vector approach to epicyclic gear trains and field balancing of rotors with computer programs are novelties in this book. Throughout emphasis is on the industrial significance of study of this subject.

I take this opportunity to thank the authorities of IIT Madras where I have taught this course for nearly 30 years. Constant interaction with my brilliant students has moulded my thought process. I would also like to express my gratitude to my students Dr. C. Saravanan, Dr. K.V. Gangadharan, Mr. Ravikiran Kadoli and Mr. Padala Tejesu for the commendable job of preparing the manuscript at short notice.

V. RAMAMURTI

Contents

1

Definitions

1.1 Introduction

Machines performing useful work in industries include power, automobile, steel and chemical plants sectors, to mention a few. For their design and performance they have some requirements to meet. The physical concepts governing their behaviour have to be clearly understood. The presentation in this book is not merely a subject to be learnt by a student aiming at acquiring a degree in mechanical engineering but also fundamental requirement for a practising engineer to decide on the criteria for ordering new machines or to analyse the root cause of problems arising out of malfunctioning of existing equipment.

This book presents the basic theory of such machines. To start with it is necessary to understand what is meant by a machine. Machine can be loosely defined as a device which receives energy and uses it to do a particular kind of work, e.g. a diesel engine is a machine that uses heat energy derived from the combustion of fuel to move a vehicle. Similarly, a reciprocating pump is a machine which takes the energy from the electric motor and converts it into potential energy by lifting water to an overhead tank. Steam turbine in a power plant takes energy from the coal burnt in the boilers and converts it into electrical energy when the turbine is connected to the generator.

The theory of machines comprises the study of relative motion between parts of the machine and forces which act on these parts due to constrained motion. The study of relative motion alone is termed as kinematics while that of forces acting on these parts is called dynamics. Broadly, dynamics can be considered to consist of two parts: (i) forces which are not time dependent (statics) and the (ii) forces which are time dependent (kinetics) dealing with the inertia forces and torques due to the motion of masses of different parts. This definition clearly indicates that kinematics does not depend on the density of the machine parts, but on physical dimensions like length, kind of connections between moving parts, largely a problem in geometry [1–5].

The output speeds encountered in mechanical engineering vary from as high as 120,000 revolutions per minute (here afterwards referred to as rpm) in turbochargers to as low as a fraction of an rpm. The speed of a kiln in a cement plant is around 1 rpm. There are several instances of output speeds of process equipment in fractions of one rpm. Input speeds of prime movers vary widely. They are 1440 rpm (induction motor with 4 poles, 50 cps), 3000 rpm for normal steam turbines, 1500 to 6000 rpm for IC engines and 100 to 3000 rpm for d.c. motors. This book discusses means of achieving output speeds from the available input speeds. Also presented are pure reciprocating and oscillating motions of operation. The resulting forces due to the dynamics associated with these motions and consequences thereof are also presented.

To explain relative motion consider the case of a reciprocating pump. The piston reciprocates in cylinder due to the crank shaft which receives uniform rotation from a motor coupled to it. The pressure drop created inside the cylinder enables water to get in during forward stroke and to be pumped out during return stroke. The rotary motion of the crankshaft (which is the input) gets converted to output reciprocating motion of the piston P through the crank OC and connecting rod OP (Fig. 1.1). So far as the relative motion between the parts is concerned it is only necessary to represent each part by its centre line. Only the lengths of a crank and connecting rod, the angular position of the crank and the speed of the crank shaft are necessary to determine the kinematics (velocities and accelerations of the different parts) of the problem. But for every position of the piston, one can compute the force on the piston due to water and the reactions of different parts by the laws of statics. For this, different parts are considered as pin jointed frames. All possible positions of this machine are analysed before determining the maximum magnitudes of the force on the piston and the reactions thereof. Afterwards it is necessary to compute the additional forces and torques on these (kinetics) parts due to motion arising out of the masses and mass moment of inertia of the moving parts. The net force on every part is a combination of static and dynamic analysis. The complete study may require identifying the cross section of some parts to withstand these loads.

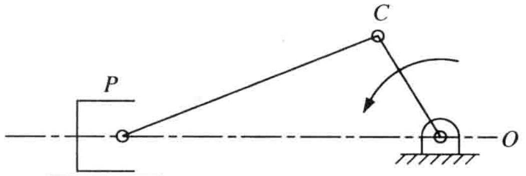

Fig. 1.1 Reciprocating engine mechanism

1.2 Link (Element)

Part of a machine which has motion relative to any other part is termed as link or an element. It is possible that each link consists of several parts connected to each other but having no relative motion between them. In a reciprocating engine, piston, connecting rod, crank and cylinder are four different elements. The bed forms part of the element identified as cylinder since these two have no relative motion between them. Similarly, flywheel, crank and crankshaft form the same element.

1.3 Kinematic Pair

Two links which are connected together in such a way that their relative motion is completely constrained forms a kinematic pair. This constraint may be *complete* or successful. These two terms are explained through the following examples.

In Fig. 1.2(i) the shaft *A* can rotate or translate in the housing *B*. In Fig. 1.2(ii) it can only translate relative to *B* (sliding pair) since the feather key prevents rotation. Figs. 1.2(iii) and (iv) show that it can only rotate (turning pair) since the collars on either side prevent translation. In Fig. 1.2(v) known rotation about the axis of *A*, results in known translation of *A* since the screw inside the nut has a definite pitch. Fig. 1.2(i) is an incomplete constraint. Fig. 1.2(ii) to (v) are cases of complete constraint and hence can be called kinematic pairs.

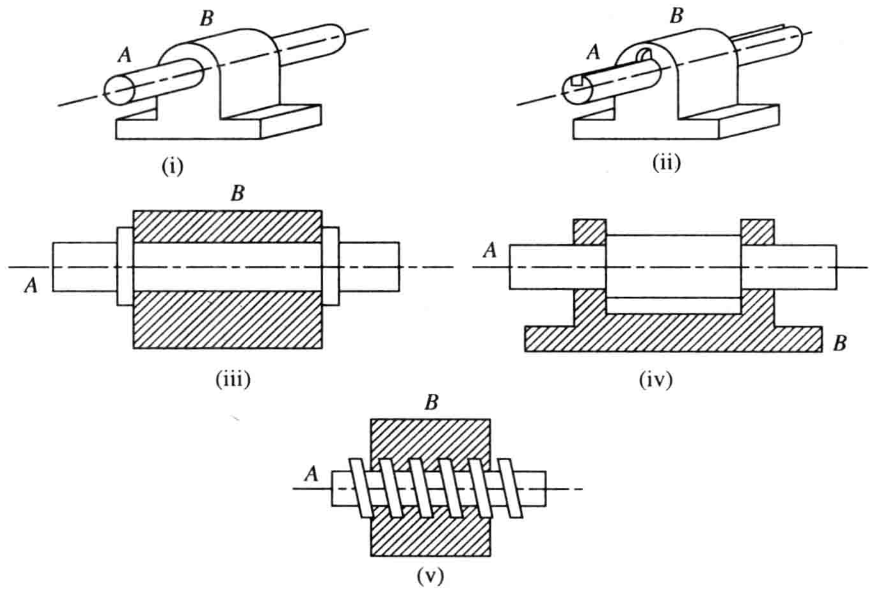

Fig. 1.2 Different types of pairs

In cases of Fig. 1.2(iii) and (iv), if there is no chance of an axial force in one direction, one of the collars can be removed. This is a case of successful constraint as in turbine rotors with vertical shafts (Fig. 1.3). Since the dead load of the rotor is vertically downward, the shaft cannot move upwards. Similarly, the piston inside the cylinder can rotate (Fig. 1.1) as well as translate. But since the plane of rotation of the crank is fixed the piston can only translate in that plane. This is also a case of *successful constraint*. The case of complete constraint is also known as *self-closure* and the case of successful constraint as *forced closure*.

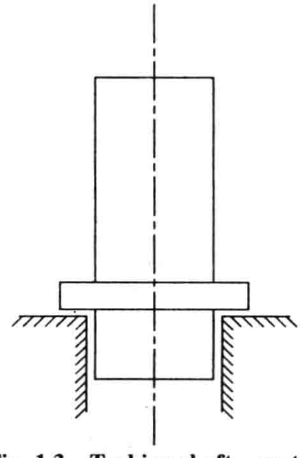

Fig. 1.3 Turbine shaft—vertical

One common feature to all these kinematic pairs is that the two elements have surface of contact, when in motion one surface moves relative to the other. Such pairs are known as *lower pairs*. All other pairs (discussed later on) will have line or point contact and will be called higher pairs, e.g. cam and follower and a pair of gears in mesh.

1.4 Kinematic Chain

It is a combination of kinematic pairs in which each link forms part of two kinematic pairs with motion completely constrained. Let us examine some combinations of links shown in Fig. 1.4. Fig. 1.4(i) shows three links pin jointed at their ends which cannot move relative to each other since the three lengths form the three sides of a unique triangle. In Fig.1.4(ii), if link *AD* is fixed, for every position of *B*, there is a distinct position for *C*. Hence, the motion is completely constrained if one of the links is fixed. Then this kinematic chain has one degree of freedom. In Fig. 1.4(iii), where there are five links and five turning pairs, the position of *C* cannot be (when *AE* is fixed) located unless the position of *B* and *D* are independently located. This chain has two degrees of freedom.

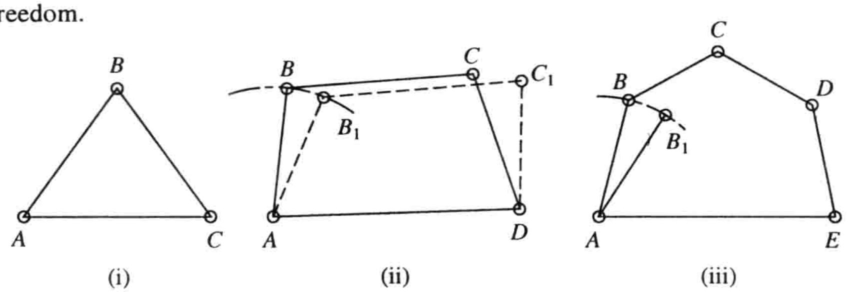

Fig. 1.4 Combination of links

Usually a kinematic chain is called a mechanism if one link is fixed. We can now generalise our discussion by deriving the relationship between the number of links and number of pairs in a mechanism.

1.5 Relationship for Constrained Motion in a Mechanism in Planar Motion

The relationship for constrained planar motion is known as Grubbler's criterion[1].

Let there be n links, l lower and h higher pairs in a mechanism. Every link, if not constrained, will have three degrees of freedom in plane motion. One end of every link will have two degrees of freedom and the end relative to the first will have one degree of freedom. Since in a mechanism, one link will be fixed, the total number of degrees of freedom f for n links when not connected to each other will be $3(n - 1)$.

But for every lower pair (turning or sliding), only one degree of freedom exists or two are getting lost. For l lower pairs $2l$ degrees of freedom will be lost. If higher pair is defined as one, which has two degrees of freedom between two links constituting the pair (will be explained later), the number of degrees of freedom lost for the higher pairs will be equal to h.

Hence, the net degrees of freedom the mechanism will possess will be

$$f = 3(n - 1) - 2l - h \tag{1.1}$$

Equation (1.1) implies that, if a mechanism has one degree of freedom and if it consists of lower pairs only, the number of links (n) must be even. Let us demonstrate it through the foregoing examples (Fig. 1.5).

Fig.1.5(i) shows a mechanism with $n = 3$, $l = 3$ (one link is fixed). It is a structure, since $f = 3(3 - 1) - (2 \times 3) = 0$.

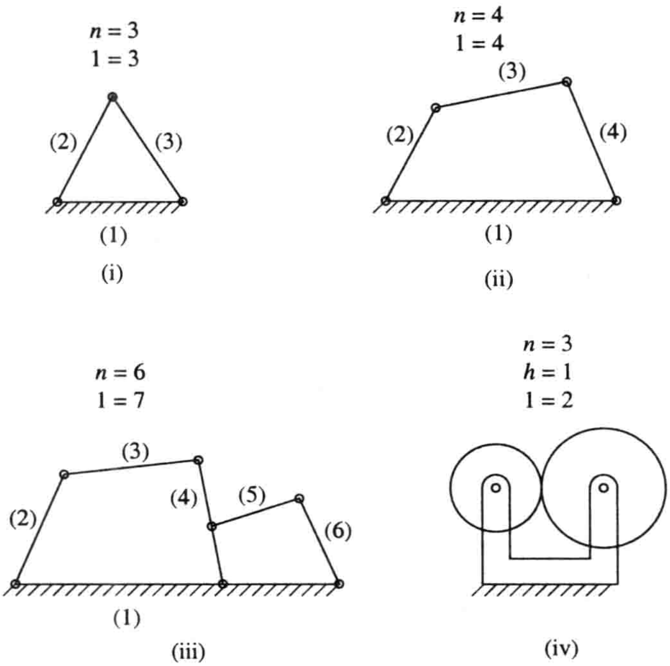

Fig. 1.5 Combination of links

In the second case, $n = 4$, $l = 4$; therefore $f = 3(4 - 1) - (2 \times 4) = 1$. This is a mechanism with one degree of freedom.

In the third example, $n = 6$, $l = 7$. Hence $f = 3(6 - 1) - (2 \times 7) = 1$. In the fourth example of a gear and a pinion, $n = 3$, $h = 1$, $l = 2$ assuming the gear pinion combination to be a higher pair. Hence $f = 1$.

All these four case of mechanisms have one degree of freedom each.

Other cases illustrating constrained motion are reported in Exercises 1.1 and 6.18.

1.6 Inversions of Mechanism

A mechanism is defined as a machine if the different links are designed to withstand the forces that act on them. Then the machine will be capable of transmitting available energy to do a particular kind of work. A kinematic chain becomes a mechanism when one of the links is fixed. If the different links of the same mechanism are fixed, one at a time, it is possible to obtain mechanisms with different characteristics. Each mechanism is defined as an inversion of the same kinematic chain. Let us now study this aspect in detail.

1.6.1 Quadric cycle chain

Let us consider a kinematic chain with four links and four kinematic pairs, all turning. This will, henceforth be called quadric cycle chain (as shown in Fig. 1.6). Consider a quadric cycle chain (Fig. 1.6) with the four links designated s, 1, p and q. Let s be the shortest link, 1 the longest, p and q the other two links. Two cases of problems can be encountered with respect to the length of the links.

$$(s + 1) < (p + q)$$

$$(s + 1) > (p + q) \tag{1.2}$$

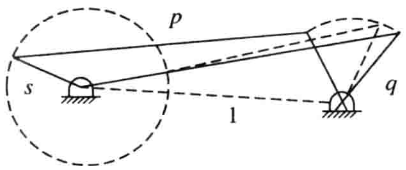

Fig. 1.6 Crank and rocker mechanism

casel is known as Grashof's type I and case 2 Type II. In type I, three distinct mechanisms are possible. [2, 18].

If s is the link adjacent to the fixed link, the resulting mechanism will be a crank rocker mechanism as shown in Fig. 1.6 and Fig. 1.7 (ii). (The term crank is used for a link which can execute a complete revolution and the term rocker for a link oscillating between limits).

Instead, if the shortest link is the coupler, (the link whose both ends are not directly connected to the fixed link), one would get a double rocker mechanism.

If, on the contrary, the shortest link is fixed, one would obtain a double crank mechanism.

In type II, the resulting mechanism will bve always a double rocker as shown in Fig. 1.7 (iii).

The applications of these inversions are shown in Fig. 1.7. Fig. 1.7(i) is the coupling rod of a locomotive, Fig. 1.7(ii) is a beam engine. Feed mechanism of a shaper is also a crank rocker mechanism. Fig. 1.7(iii) is Ackermann's steering gear of an automobile. This enables the car to take the position shown in the third figure while it takes a turn. It is seen that the same quadric cyclic chain gives rise to three different characteristics, in the first (coupling rod), two links complete full revolutions (double crank mechanism), in the second (feed mechanism of a shaper or beam engine), one completes one revolution and another rocks (crank rocker mechanism) and in the third (steering gear), both rock between two limits (double rocker mechanism). All the three inversions are from the same quadric cycle chain.

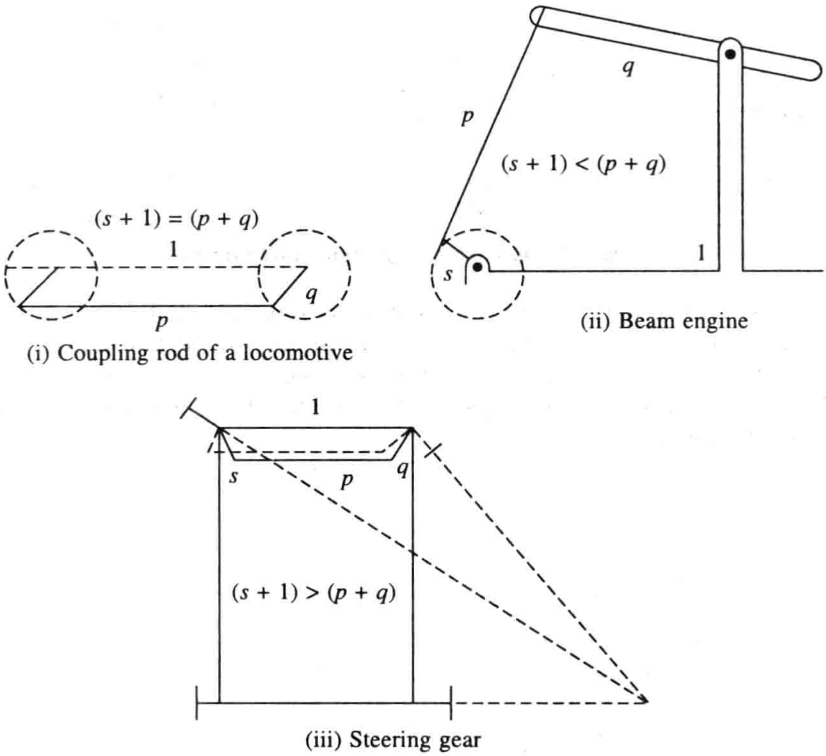

Fig. 1.7 Inversions of quadric cycle chain

Example 1.1 (Grashof's inequality) Shown in Fig. 1.4 (ii) is a four bar chain with $AB = 3$ cm, $BC = 4$ cm, $CD = 4.5$ cm and $AD = 5$ cm. AD is the fixed link. What kind of mechanism is this?

From the dimensions given, s the shortest is 3cm, 1, the longest is 5 cm, p and q are 4 and 4.5 cm respectively.

$$(s + 1) = 8 \text{ cm}, (p + q) = 8.5 \text{ cm. Hence } (s + 1) < (p + q)$$

This is Grashof's type I. Besides s, the shortest link is adjacent to the fixed link. Hence this is a crank rocker mechanism.

1.6.2 Single slider crank chain

If in the four pairs mentioned earlier, one of them becomes a sliding pair the chain is known as a single slider crank chain. Let us consider one such chain as shown in Fig. 1.8. There are four links. The pair between links (1) and (4) is a sliding pair whereas the one between links (1) and (2), links (2) and (3) and links (3) and (4) are turning pairs. Length of link (2) is the shortest. When link (1) is fixed link (2) can complete a full rotation (crank) whereas the link (4) can only reciprocates (slider).

Let us consider two other inversions of this chain by fixing links (2) or (3).