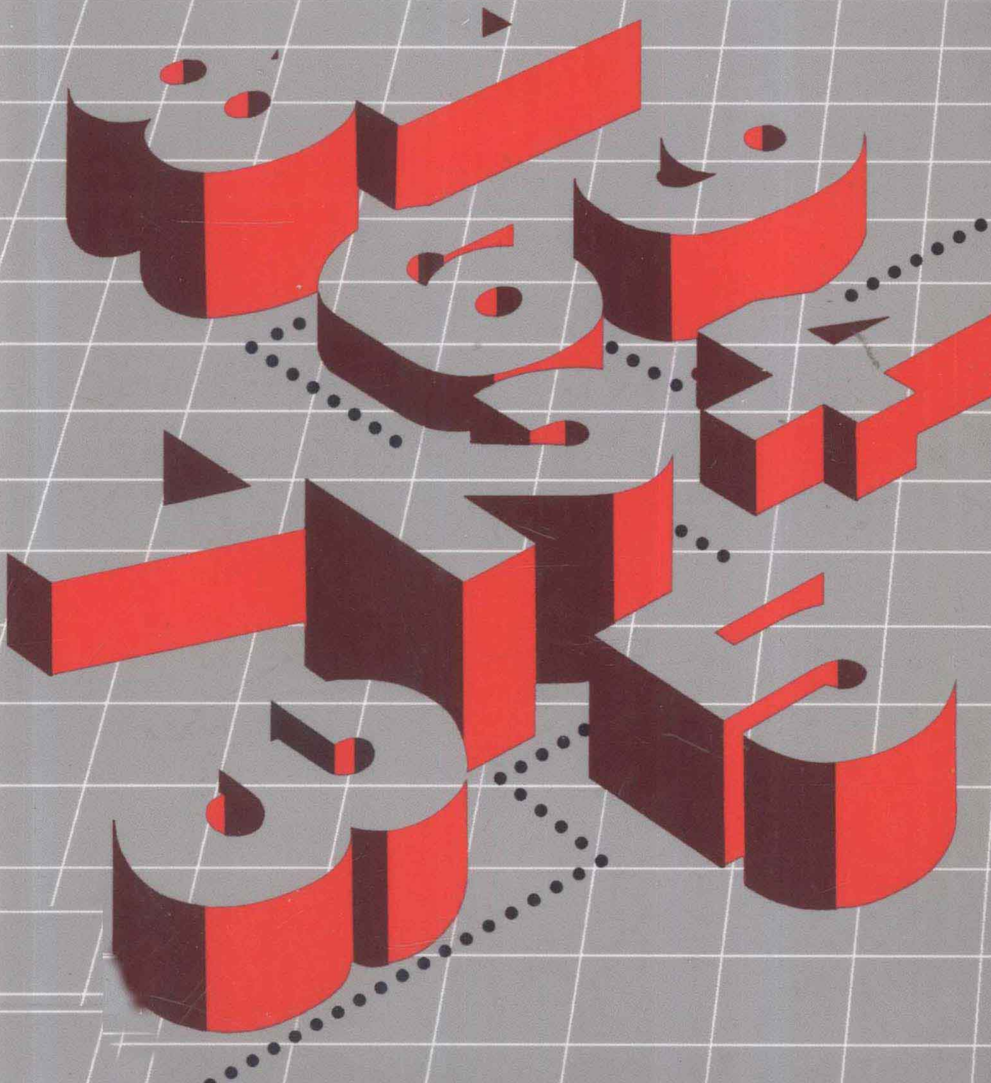
MATHEMATICS AND THE SEARCH FOR KNOWLEDGE



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Preface

How do we acquire knowledge about our physical world? All of us are obliged to rely on our sense perceptions—hearing, sight, touch, taste, and smell—to perform our daily tasks and enjoy some pleasures. These perceptions tell us a good deal about our physical world, yet in general they are crude. As Descartes put it, perhaps too strongly, sense perception is sense deception. It is true that modern instruments such as telescopes extend our perceptions immensely, yet such instruments have limited applicability.

Major phenomena of our physical world are not perceived at all by the senses. They do not tell us that the Earth is rotating on its axis and revolving around the sun. They do not tell us the nature of the force that keeps all the planets revolving around the sun. Nor do they tell us anything about electromagnetic waves that enable us to receive radio and television programs originating hundreds and even thousands of miles away.

This book will not be much concerned with what one might characterize as mundane applications of mathematics such as finding the precise height of a fifty-story building. The reader will be made aware of the limitations of sense perceptions, but our chief concern will be to describe what is known about the realities of our physical world *only* through the medium of mathematics. Rather than presenting much of the actual mathematics, I shall describe what mathematics reveals about major phenomena in our modern world. Of course, experience and experimentation play a role in our investigation of nature; however, as will become evident, these measures are in many areas minor.

In the seventeenth century, Blaise Pascal bemoaned human helplessness. Yet today a tremendously powerful weapon of our own creation—namely, mathematics—has given us knowledge and mastery of major areas of our physical world. In his address in 1900 at the International Congress of Mathematicians, David Hilbert, the foremost mathematician of our era, said: "Mathematics is the foundation of all exact knowledge of natural phenomena." One can justifiably add that, for many vital phenomena, mathematics provides the only knowledge we have. In fact, some sciences are made up solely of a collection of mathematical theories adorned with a few physical facts.

Contrary to the impression students acquire in school, mathematics is not just a series of techniques. Mathematics tells us what we have never known or even suspected about notable phenomena and in some instances even contradicts perception. It is the essence of our knowledge of the physical world. It not only transcends perception but outclasses it.

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Historical Overview: Is There an External World?

A philosopher is someone who knows something about knowing which nobody else knows so well. Descartes

There is nothing so absurd that it has not been said by philosophers. Cicero

Is not the whole of philosophy like writing in honey? It looks wonderful at first sight. But when you look again it is all gone. Only the smear is left. Einstein

Is there a physical world independent of humanity? Are there mountains, trees, land, sea, and sky, all of which exist whether or not human beings are there to perceive these objects? The question seems silly. Of course there is. Do we not observe this world constantly? Do not our senses give us continuous evidence of the existence of this world? However, thoughtful people are not above questioning the obvious even if only to confirm it.

Let us begin by turning to the philosophers, the lovers of wisdom, who have for centuries pondered all matters concerning humanity and our world but who like unrequited lovers often felt rejected. Some of the greatest philosophers have considered this very subject of the existence of an external world. Some have denied it; others have admitted its existence but have seriously doubted how much we can know about the external world and how reliable our knowledge is. Although Bertrand Russell, himself an outstanding philosopher, said in *Our Knowledge of the External World,* "Philosophy, from the earliest times, has made greater claims and achieved fewer results than any other branch of learning," we should at least examine what some of the philosophers have had to say. We shall be concerned primarily with those who have seriously questioned our knowledge of the external world.

The first of the ancient Greek philosophers to deal with this question was Heraclitus, who lived about 500 B.C. Heraclitus did not deny that there is an external world but maintained that everything in the world is constantly changing. As he put it, one cannot step twice into the same river, so whatever facts we may think we have gathered about the physical world no longer obtain in the very next instant.

Epicurus (341–270 B.C.), in contrast, held to the fundamental principle that our senses are the infallible guide to truth. They tell us that matter exists, that motion occurs, and that the ultimate realities are bodies composed of atoms existing in a void. They have existed forever and cannot be destroyed. They are indivisible and unchangeable.

Plato (427–347 B.C.), the most influential philosopher of all time, was also interested in this problem. He admitted the existence of an external world but came to the conclusion that the world perceived by the senses is motley, protean, ever-changing, and unreliable. The true world is the world of ideas, which is unchangeable and incorruptible. Yet this world of ideas is not accessible to the senses but only to the mind. Observations are useless. Thus, in the *Republic,* Plato states clearly that the reality behind appearances, what is inherently true of them, is mathematical; to understand reality is to elicit it from appearances, not to impose it on them. Mathematics is the foundation of true being, the "eternally real." In emphasizing the importance of mathematics Plato conceived of it as a part of a more general system of abstract, nonmaterial, ideal *Ideas.* These are models of perfection that everything in the universe—the material, the ethical, and the aesthetic—strives to attain. Plato says in the *Republic:*

> But if anyone tries to learn about things of the sense, whether gaping up or blinking down, I would never say that he really learns—for nothing of the kind admits of true knowledge—nor would I say that his soul looks up, but down, even though he study floating on his back on sea or land.

Plutarch relates in his "Life of Marcellus" that Eudoxus and Archytas, famous contemporaries of Plato, used physical arguments to "prove" mathematical results. However, Plato indignantly denounced such proofs as a corruption of geometry; they utilized sensuous facts in place of pure reasoning.

Plato's attitude toward astronomy illustrates his position on the knowledge that we should seek. This science, he said, is not concerned with the movements of the visible heavenly bodies. The arrangement of the stars in the heavens and their apparent movements are indeed wonderful and beautiful to behold, but mere observations and explanations of the motions fall far short of true astronomy. Before we can attain to this true science we "must leave the heavens alone," for true astronomy deals with the laws of motion of true stars in a mathematical heaven of which the visible heaven is but an imperfect expression. Through Socrates, in words now famous, Plato tells us the astronomer's true concerns:

> These sparks that paint the sky, since they are decorations on a visible surface, we must regard, to be sure, as the fairest and most exact of material things but we must recognize that they fall far short of the truth, the movements, namely, of real speed and real slowness in true number and in all true figures. . . . These can be apprehended only by reason and thought, but not by sight. [Therefore] we must use the blazonry of the heavens [merely] as patterns to aid in the study of those realities, if we are to have a part in the true science of astronomy.

This conception of astronomy staggers the modern mind, and scholars have not hesitated to indict Plato's downgrading of sensory experience as a grave disservice to the advancement of science. We should recognize, however, that the role here assigned to the astronomer parallels precisely the course followed successfully by the geometrician, who studies mental idealizations of, for example, triangles rather than particular triangular objects. In Plato's time observational astronomy had been pursued virtually to the limits then attainable, and he may have thought that further progress now awaited some hard thinking and theorizing on the assembled information.

Plato's conception of abstract ideals did unfortunately retard progress in experimental science for centuries. It implied that true knowledge could be acquired only through philosophical contemplation of abstract ideas and not through observation of the accidental and imperfect things in the real world.

However, there were and are philosophers who accepted the existence of an external world and believed that we can acquire sound knowledge from our sensations. Aristotle, in opposition to Plato, not only affirmed the existence of a world external to humanity but also maintained that our ideas about the world are obtained by abstracting from it ideas common to various classes of material objects we perceive such as triangles, spheres, foliage, and mountains. He criticized Plato's otherworldliness and his reduction of science to mathematics. Aristotle, physicist in the literal sense of the word, believed in material things as the primary substance and source of reality. Physics, and science generally, must study the physical world and obtain truths from it. Thus, genuine knowledge is obtained from sense experience, by intuition and by abstraction. These abstractions have no existence independent of human minds.

To arrive at truths, Aristotle used what he called universals—general qualities that are abstracted from real things. In his words, we must "start with things which are knowable and observable to us and proceed toward those things which are clearer and more knowable by nature." He took the obvious sensuous qualities of objects, hypostatized them, and elevated them to independent mental concepts. Specifically, above the central Earth, which includes the whole volume of water, comes the region occupied by air; higher still, extending as far as the moon, is a substance that we call fire, although in reality it is a mixture of fire and air. They owe their existence, says Aristotle, to "four principles": hot, cold, dry, and moist (see Chapters V and X). These can combine in pairs in six ways, but two of the combinations—hot with cold, dry with moist—are inherently impossible; the remaining four pairs generate the four elements. Thus, earth is cold and dry; water, cold and moist; air, hot and moist; and fire, hot and dry. The elements are not eternal; on the contrary, matter is continually passing from one form to another. All the universe, from Earth out as far as the moon, is a region undergoing constant change, corruption, mortality, and decay, as the phenomena of climate and geology vividly attest.

Although their influence is unmistakable, one might be inclined to dismiss the views of Greek philosophers, because while their culture emphasized mathematics, they lived in what may justifiably be called a prescientific world. They did not experiment much and, on the whole, lived apart from the world of science as we know it today.

During the Middle Ages concern about the external world was unimportant; theology was the supreme concern. Not until the Renaissance did philosophers turn with heightened interest to the physical world. In Western Europe especially, we can see the beginning of modern philosophy and with it, a new interest in science.

René Descartes (1596–1650) is the founder of modern philosophy. His *Discourse on the Method of Rightly Conducting the Reason and Seeking Truth in the Sciences* (1637), which contained three appendices, "Geometry," "Dioptric," and "Meteors," is a classic. Although Descartes thought that his philosophical and scientific doctrines subverted Aristotelianism and scholasticism, he was at heart a scholastic or Aristotelian. He followed in the footsteps of Aristotle and drew from his own mind propositions about the nature of being and reality. Perhaps for this very reason his writings influenced the seventeenth century more extensively than the researches of those who had begun to draw truths from observation and experimentation—sources that were wholly at variance with the traditional ones.

Seeing that there was the logical possibility that all his beliefs were false, Descartes sought a solid base on which he might build an edifice of truth. He found but one fact he could be sure of—*Cogito, ergo sum* (I think, therefore I am). Because he recognized that he was finite and imperfect, he reasoned that this very sense of his limitations implied that there had to be an infinite and perfect being against whom he measured himself. This being, God, had to exist because He would not be perfect if he lacked the essential attribute of existence. To Descartes this result, God's existence, was more important for science than for theology, for it afforded the possibility of solving the central problem of the existence of an objective world.

Because all our knowledge of a world external to our minds comes to us through sense impressions, the question arises whether there exists anything other than just these, or whether objective reality is an illusion. To this question Descartes answered that God, being perfect, would not be a deceiver; He would not make us believe in the existence of a material universe if it were not real.

This objective reality can be grasped primarily through the physical attribute of extension. This is innate in the very notion of matter, which itself is not derived from the senses. Therefore, no knowledge of the material world is, save in an indirect manner, derived from the senses. Descartes was also able to classify his observations of material objects into primary and secondary qualities. Thus, color is secondary because it is perceived only by the senses, whereas extension and motion are primary.

To Descartes, the entire physical universe is a great machine operating according to laws that may be discovered by human reason, particularly mathematical reasoning. He deprecated experimentation, although he did experiment in biology.

Responding directly to the knowledge that was being acquired in mathematics and science, the philosopher Thomas Hobbes (1588–1679) affirmed in his *Leviathan* (1651) that external to us there is only matter in motion. External bodies press against our sense organs and by purely mechanical processes produce sensations in our brains. All knowledge is derived from these sensations, which then become images in the brain. When a train of images arrives, it recalls others already received—as, for example, the image of an apple recalls that of a tree. Thought is the organization of chains of images. Specifically, names are attached to bodies and properties of bodies as they appear in images, and thought consists in connecting these names by assertions and in seeking the relations that necessarily hold among these assertions.

Hobbes, in his book *Human Nature* (1650), says that ideas are images or memories of what is received through the senses. There are no innate ideas or ideals; no universals or abstract ideas. Triangle means merely the idea (image) of all triangles perceived. All substance that gives rise to ideas is material or corporeal. In fact, the mind too is substance. Language (for example, the language of science and mathematics) consists only of symbols or names for experiences. All knowledge is but remembrance, and the mind works with words that are but names for things. True and false are attributes of names, not of things. That humans are living creatures is true because whatever is called human is also called a living creature.

Knowledge is obtained when regularities are discovered by the brain as it organizes and relates the assertions about physical objects. And mathematical activity produces just such regularities. Hence the mathematical activity of the brain produces genuine knowledge of the physical world, and mathematical knowledge *is* truth. In fact, reality is accessible to us only in the form of mathematics.

So strongly did Hobbes defend the exclusive right of mathematics to truth that even the mathematicians objected. In a letter to a leading physicist of the age, Christian Huygens, the mathematician John Wallis wrote of Hobbes:

> Our Leviathan is furiously attacking and destroying our Universities (and not only ours but all) and especially ministers and the clergy and all religion, as though the Christian world had no sound knowledge, none that was not ridiculous either in philosophy or religion, and as though men could not understand religion if they did not understand philosophy, nor philosophy unless they knew mathematics.

The emphasis placed by Hobbes on the purely physical origin of sensation and on the limited action of the brain in reasoning shocked many philosophers to whom the mind was more than a mass of matter acting mechanically. In his *Essay Concerning Human Understanding*, published in 1690, John Locke (1632–1704) began somewhat as Hobbes did, but unlike Descartes, by asserting that there are no innate ideas in humans; men are born with minds as empty as blank tablets.

Experience, through the media of the sense organs, writes on those tablets and produces simple ideas. Some simple ideas are exact resemblances of qualities actually inhering in bodies. These qualities, which he called primary, are exemplified by solidity, extension, figure (shape), motion or rest, and number. Such properties exist whether or not anyone perceives them. Other ideas that arise from sensations are the effects of the real properties of objects on the mind, but these ideas do not correspond to actual properties. Among such secondary qualities are color, taste, smell, and sound.

Locke's aim in the *Essay* was to discover the limit or boundary between the knowable and the unknowable, the "horizon . . . which sets the bounds between the enlightened and dark parts of things." By so doing, he would refute both the skeptics, who "question everything, and disclaim all knowledge, because some things are not to be understood," and at the opposite extreme those overconfident reasoners who presume that the whole vast ocean of being is "the natural and undoubted possession of our understanding, wherein there was nothing exempt from its decisions, or that escaped its comprehension." More positively and constructively, his purpose was to establish the grounds of knowledge and opinion, and the measures by which truth might be attained or approximated in all things that the human understanding has the capacity to comprehend.

The plan or design of the *Essay,* as Locke explained in the introduction, was to "inquire into the origin, certainty, and extent of *human knowledge;* together with the grounds and degrees of *belief, opinion,* and *assent.*" Following a "historical, plain method," he gave an account of the origin of our ideas, then showed what knowledge the understanding had through those ideas, and finally inquired into the nature and grounds of faith or opinion.

Although the mind cannot invent or frame any simple idea, it does have the power to reflect on, compare, and unite simple ideas and thus form complex ideas. Here Locke departed from Hobbes. In addition, he said, the mind does not know reality itself, but only ideas of reality, and it works with these. Knowledge concerns the connection of ideas such as their agreement or inconsistency. Truth consists in knowledge that conforms to the reality of things.

Basic mathematical ideas are constructed by the mind, although they are ultimately traceable to experience; however, some ideas are not traceable to real entities. These latter, more abstract mathematical ideas are constructed from the former by repeating, combining, and