



Introduction to Modern Nonparametric Statistics

James J. Higgins

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James J. Higgins

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Preface

Modern nonparametric statistics comprises a broad range of methods for data analysis. Rank-based methods, permutation tests, bootstrap methods, and curve smoothing fall under this heading. Most normal-theory methods have nonparametric counterparts that may be used when the normal-theory assumptions are violated. Nonparametric methods may also solve problems for which conventional methods do not apply or are difficult to implement. These problems include the analysis of sparse contingency tables, tests for ordered alternatives, one-sided multivariate tests, and the analysis of censored data.

This book presents a wide array of nonparametric methods that researchers will find useful in analyzing their data. It is appropriate for advanced undergraduates and beginning graduate students in fields such as the life sciences, engineering, medicine, social sciences, and statistics. Students are expected to be familiar with topics typically covered in an introductory methods course such as t tests, inferences for proportions, simple linear regression, and one-way analysis of variance. A few topics in the latter part of the book require a more advanced background, including multiple regression and two-way analysis of variance.

At one time nonparametric methods were thought of as quick, hand-calculation methods, suitable only for simple designs and small data sets. However, the availability of high-speed computing to carry out time-consuming or difficult computations has enabled nonparametric statistics to come into its own as a field of statistics. Computationally intensive methods such as permutation tests, bootstrap estimation, curve smoothing, and robust methods, along with the classic methods based on ranks, have become part of the statistician's tool kit.

The selection of material for this book was guided by data analysis problems that commonly arise in practice. In addition to the familiar one-sample, two-sample, and k -sample procedures, nonparametric counterparts to analysis of variance, multiple regression, and multivariate analysis are considered. Categorical data methods include exact permutation tests for contingency tables and tests for tables with ordinal classifications. A chapter on censored data covers common nonparametric techniques

for the analysis of survival data. Bootstrap methods, nonparametric density estimation, and curve smoothing are also considered.

Where possible, the connection among methods is stressed. For instance, rank tests are introduced as special cases of permutation tests applied to ranks, and methods for censored data are treated as special cases of permutation tests applied to general scores. Some statistical theory is sketched where it is felt that it would help illuminate the methods. In most cases, the theory is set apart from the methods and may be omitted at the option of the instructor. Selected computer output and code show how methods are implemented. Although generally not needed in light of the availability of statistical packages, statistical tables are given for some of the more common tests.

Chapter 1 treats one-sample methods based on the binomial distribution. The issue of the power of statistical tests is discussed, and an example is given to show how a simple nonparametric test for a location parameter can have substantially greater power than the one-sample t test.

Chapter 2 sets a pattern that is used for the next six chapters. A permutation test is introduced to solve a problem, and then rank tests or other tests are developed as special cases or modifications of the permutation test. Chapter 2 introduces two-sample methods. K -sample methods, including multiple comparisons, follow in Chapter 3. Then come paired comparisons and blocked designs in Chapter 4, tests for trends and association including contingency table analysis in Chapter 5, multivariate tests in Chapter 6, and tests for censored data in Chapter 7. Asymptotic approximations are introduced in Chapter 2 and appear throughout. However, the need for such approximations is not as compelling as it once was as a result of the availability of computer software to carry out exact tests.

Chapter 8 introduces bootstrap methods for one-sample, two-sample, k -sample, and regression problems. In Chapter 9, both bootstrap and aligned-rank methods are given for two-factor analysis of variance with the extension to the multifactor setting being straightforward. A section on lattice-ordered alternatives extends tests for ordered alternatives to the multifactor setting. In Chapter 10, nonparametric density estimation, curve smoothing, and robust model fitting are introduced. The availability of computer software to carry out the computations makes this material accessible to the practitioner.

The material in this text is sufficient for a one-semester course. Problems at the end of each chapter are included to provide practice with the methods and, in some cases, to extend theory or methods not covered in the text.

Statistical software is indispensable for the implementation of modern statistical methods, and the area of nonparametrics is no exception. We feature four statistical packages in this book: Resampling Stats, StatXact, S-Plus, and MINITAB.

Resampling Stats

Resampling Stats comes in three versions: a stand-alone package, an add-in to Microsoft Excel, and an add-in to Matlab. The stand-alone version is a simple-to-use programming language that is specifically designed to do the type of re-

sampling of data needed for permutation tests and bootstrap sampling. The programming language has functions that enable programmers to construct a variety of statistical procedures. With intuitive commands such as “shuffle,” “sample,” “repeat,” and “score,” Resampling Stats code may also serve as pseudocode for describing the steps needed to program nonparametric procedures in other languages. The add-ins are designed to work in conjunction with the features of the host software. Most of the methods in Chapters 2–7 can be done with this software. If students have the option of only one statistical software package to use with this book, Resampling Stats is the one to choose. The web address is www.resample.com.

StatXact

StatXact is a powerful, simple-to-use, menu-driven program that has more than 80 nonparametric procedures for continuous and categorical data. The software also has data manipulation capabilities. Power and sample size computations are available for tests for one and two binomial populations. Most of the procedures in Chapters 2–5 and Chapter 7 are included in this software package, as well as a number of procedures that we do not cover. Those who make use of exact nonparametric procedures in their statistical consulting and research will find this program extremely useful. The web address is www.cytel.com.

S-Plus

S-Plus is a powerful programming language that can be used to carry out the procedures in this book. We limit our use to those procedures that can be accessed through the S-Plus menu options. This program is especially useful for the smoothing techniques and robust methods discussed in Chapter 10. The web address is www.insightful.com.

MINITAB

MINITAB has long been a favorite of statisticians for both instruction and consulting. Some of the standard statistical procedures in Chapters 1–4 and Chapter 7 are included in the MINITAB menu. Data manipulation capabilities are included. MINITAB also has commands for carrying out rank-based regression as discussed in Section 10.3, although these are not presently documented in the help menu. The web address is www.minitab.com.

Other

The SAS® programming language, a standard for applied statisticians, has several capabilities for nonparametric statistics. PROC NPAR1WAY has a variety of two-sample and k -sample tests, and there is an option for doing exact tests. PROC FREQ has an option to do exact tests for contingency tables. PROC MULTTEST allows

bootstrap and permutation sampling and may be applied in the multivariate setting. The web address is www.sas.com.

One other source of software that we have used is at a web site created by J. W. McKean. Here one can carry out rank-based regression and analysis of variance online. The web address is www.stat.wmich.edu/slab/RGLM.

Most sections of the book have a subsection entitled “Computer Analysis” that outlines the use of selected statistical packages. We do not intend this to be a comprehensive discussion, but rather it is to give an indication of the capabilities of various software packages.

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James J. Higgins

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