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Erhan Çinlar

Probability and Stochastics

概率和随机

Springer

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PREFACE

This is an introduction to the modern theory of probability and stochastic processes. The aim is to enable the student to have access to the many excellent research monographs in the literature. It might be regarded as an updated version of the textbooks by Breiman, Chung, and Neveu, just to name three.

The book is based on the lecture notes for a two-semester course which I have offered for many years. The course is fairly popular and attracts graduate students in engineering, economics, physics, and mathematics, and a few overachieving undergraduates. Most of the students had familiarity with elementary probability, but it was safer to introduce each concept carefully and in a uniform style.

As Martin Barlow put it once, mathematics attracts us because the need to memorize is minimal. So, only the more fundamental facts are labeled as theorems; they are worth memorizing. Most other results are put as propositions, comments, or exercises. Also put as exercises are results that can be understood only by doing the tedious work necessary. I believe in the Chinese proverb: I hear, I forget; I see, I remember; I do, I know.

I have been considerate: I do not assume that the reader will go through the book line by line from the beginning to the end. Some things are recalled or re-introduced when they are needed. In each chapter or section, the essential material is put first, technical material is put toward the end. Subheadings are used to introduce the subjects and results; the reader should have a quick overview by flipping the pages and reading the headings.

The style and coverage is geared toward the theory of stochastic processes, but with some attention to the applications. The reader will find many instances where the gist of the problem is introduced in practical, everyday language, and then is made precise in mathematical form. Conversely, many a theoretical point is re-stated in heuristic terms in order to develop the intuition and to provide some experience in stochastic modeling.

The first four chapters are on the classical probability theory: random variables, expectations, conditional expectations, independence, and the classical limit theorems. This is more or less the minimum required in a course at graduate level probability. There follow chapters on martingales, Poisson random measures, Lévy processes, Brownian motion, and Markov processes.

The first chapter is a review of measure and integration. The treatment is in tune with the modern literature on probability and stochastic processes. The second chapter introduces probability spaces as special measure spaces, but with an entirely different emotional effect; sigma-algebras are equated to bodies of information, and measurability to determinability by the given information. Chapter III is on convergence; it is routinely classical; it goes through the definitions of different modes of convergence, their connections to each other, and the classical limit theorems. Chapter IV is on conditional expectations as estimates given some information, as projection operators, and as Radon-Nikodym derivatives. Also in this chapter is the construction of probability spaces using conditional probabilities as the initial data.

Martingales are introduced in Chapter V in the form initiated by P.-A. Meyer, except that the treatment of continuous martingales seems to contain an improvement, achieved through the introduction of a "Doob martingale", a stopped martingale that is uniformly integrable. Also in this chapter are two great theorems: martingale characterization of Brownian motion due to Lévy and the martingale characterization of Poisson process due to Watanabe.

Poisson random measures are developed in Chapter VI with some care. The treatment is from the point of view of their uses in the study of point processes, discontinuous martingales, Markov processes with jumps, and, especially, of Lévy processes. As the modern theory pays more attention to processes with jumps, this chapter should fulfill an important need. Various uses of them occur in the remaining three chapters.

Chapter VII is on Lévy processes. They are treated as additive processes just as Lévy and Itô thought of them. Itô-Lévy decomposition is presented fully, by following Itô's method, thus laying bare the roles of Brownian motion and Poisson random measures in the structure of Lévy processes and, with a little extra thought, the structure of most Markov processes. Subordination of processes and the hitting times of subordinators are given extra attention.

Chapter VIII on Brownian motion is mostly on the standard material: hitting times, the maximum process, local times, and excursions. Poisson random measures are used to clarify the structure of local times and Itô's characterization of excursions. Also, Bessel processes and some other Markov processes related to Brownian motion are introduced; they help explain the recurrence properties of Brownian motion, and they become examples for the Markov processes to be introduced in the last chapter.

Chapter IX is the last, on Markov processes. Itô diffusions and jump-diffusions are introduced via stochastic integral equations, thus displaying the process as an integral path in a field of Lévy processes. For such processes, we derive the classical relationships between martingales, generators, resolvents, and transition functions, thus introducing the analytic theory of them. Then we re-introduce Markov processes in the modern setting and explain, for Hunt processes, the meaning and implications of the strong Markov property and quasi-left-continuity.

Over the years, I have acquired indebtedness to many students for their enthusiastic search for errors in the manuscript. In particular, Semih Sezer and Yury Polyanskiy were helpful with corrections and improved proofs. The manuscript was formatted by Emmanuel Sharef in his junior year, and Willie Wong typed the first six chapters during his junior and senior years. Siu-Tang Leung typed the seventh chapter, free of charge, out of sheer kindness. Evan Papageorgiou prepared the figures on Brownian motion and managed the latex files for me. Finally, Springer has shown much patience as I missed deadline after deadline, and the staff there did an excellent job with the production. Many thanks to all.

Frequently Used Notation

$\mathbb{N} = \{0, 1, \ldots\}$, $\overline{\mathbb{N}} = \{0, 1, \ldots, +\infty\}$, $\mathbb{N}^* = \{1, 2, \ldots\}$.

$\mathbb{R} = (-\infty, +\infty)$, $\overline{\mathbb{R}} = [-\infty, +\infty]$, $\mathbb{R}_+ = [0, \infty)$, $\overline{\mathbb{R}}_+ = [0, +\infty]$.

$(a,\ b)$ is the open interval with endpoints a and b; the closed version is $[a,\ b]$; the left-open right-closed version is $(a,\ b]$.

$\exp\ x = e^x$, $\exp\_\ x = e^{-x}$, Leb is the Lebesgue measure.

$\mathbb{R}^d$ is the d-dimensional Euclidean space, for x and y in it,

$$x \cdot y = x_1 y_1 + \cdots + x_d y_d, \quad |x| = \sqrt{x \cdot x} \quad .$$

$(E, \mathcal{E})$ denotes a measurable space, $\mathcal{E}$ is also the set of all $\mathcal{E}$-measurable functions from E into $\overline{\mathbb{R}}$, and $\mathcal{E}_+$ is the set of positive functions in $\mathcal{E}$.

$1_A(x) = \delta_x(A) = I(x, A)$ is equal to 1 if $x \in A$ and to 0 otherwise.

$\mathcal{B}_E$ is the Borel σ-algebra on E when E is topological.

$C(E \mapsto F)$ is the set of all continuous functions from E into F.

$\mathcal{C}_K^2 = C_K^2(\mathbb{R}^d \mapsto \mathbb{R})$ is the set of twice continuously differentiable functions, from $\mathbb{R}^d$ into $\mathbb{R}$, with compact support.

$\mathbb{E}(X|\mathcal{G})$ is the conditional expectation of X given the σ-algebra $\mathcal{G}$.

$\mathbb{E}_t X = \mathbb{E}(X|\mathcal{F}_t)$ when the filtration $(\mathcal{F}_t)$ is held fixed.

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