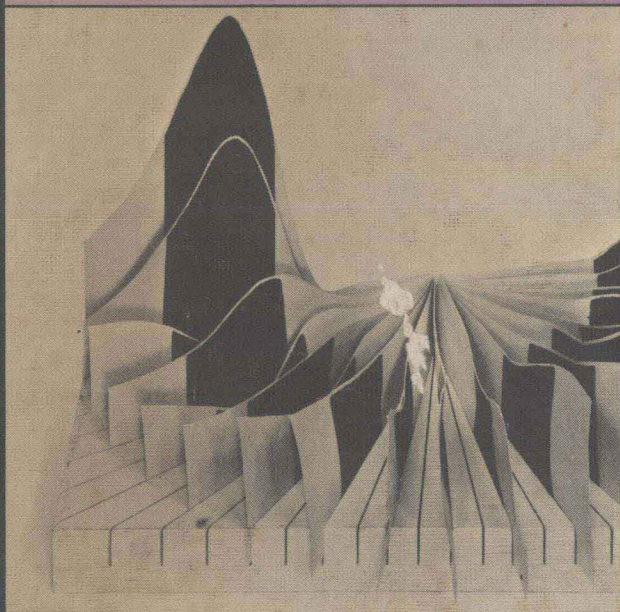


Selected Readings in Physics

Nuclear Reactions

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NUCLEAR REACTIONS

BY

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SELECTED READINGS IN PHYSICS

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NUCLEAR REACTIONS

Preface

FOR over fifty years nuclear reactions have been the primary source of a rapidly evolving mass of information about nuclei. The study of the mechanisms of the reactions themselves is a fascinating one, not only because of the information about nuclei that it yields, but because of the great diversity of quantum phenomena that must be understood and reconciled. In fact, until very recent experimental developments in atomic physics, nuclear physics was the only field in which energy measurements could be made with sufficient precision to study quantum scattering phenomena in terms of pure states. We have now reached the stage where there is hope that certain reactions can be understood largely in terms of the basic forces between nucleons.

In this book I have tried to explain the development of the understanding of nuclear reactions in a way that is intelligible to undergraduate students after a first course in quantum mechanics. The theory of nuclear reactions involves quite sophisticated applications of quantum mechanics, so that this is a very difficult task. However, I have tackled the problem head-on. Rather than attempt over-simplified explanations, I have attempted to develop the required knowledge and feeling for quantum mechanics in the course of explaining the most important phenomena. The task is simplified by the extremely interesting nature of the subject itself and of the original papers in Part 2, which are usually meant to be read concurrently with the text.

The choice of original papers has been motivated only partly by the requirements of space and simplicity. The key papers in the subject are long and require detailed study. The text has been written as an aide to this detailed study. The examples of the modern study of direct interactions are chosen with more regard

to their power of explanation than to their place in the chronological order of the subject.

The present volume is intended as a companion to the earlier book in this series by Brink entitled *Nuclear Forces*, in the sense that I have referred to Brink's book whenever the question of the details of nuclear forces arises. However, it is self-contained if the most important properties of nuclear forces, which are mentioned here, are taken for granted.

I would like to thank my friends and colleagues whose help and encouragement have made the writing of this book possible. I would like to thank the publishers of the following journals for permission to reproduce the original articles in Part 2: *The Proceedings of the Royal Society*, *Nature*, *The Physical Review*, *The Philosophical Magazine*, *Reviews of Modern Physics*, also the University of Toronto Press. I am grateful to the authors of these articles and to Professor A. Bohr for confirming the permission.

The book was written concurrently with my research program sponsored by the Air Force of Scientific Research, Office of Aerospace Research, United States Air Force. I am grateful to this agency for its support.

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PART 1

I

Early Successes and Difficulties

THE early history of nuclear physics may be divided into two periods of approximately twenty years. The first period began with the discovery of the nucleus by Rutherford at Manchester in 1911. It was characterized by years of hard, painstaking work and brilliant insight, inspired largely by Rutherford. Monoenergetic beams of α -particles from radioactive sources were used as probes for nuclear properties. Together with detailed information about the mass relationships between nuclei, these early reaction studies laid the foundation of knowledge about the nucleus. The second period began in 1932 with the discovery of the neutron and the invention of accelerators which enabled different probes to be used. During this period a basic understanding of nuclear reactions and structure in terms of quantum mechanics was achieved, but technology had not reached the stage where critical experiments to test the understanding could be performed or analysed. In the early 1950's more sophisticated accelerators, counting and data recording techniques and computing methods enabled the quantitative phase of nuclear physics to begin.

The earliest information about nuclei was obtained by studying the deflection of α -particles which occurs when they approach nuclei. When a particle hit a scintillating screen it caused a flash of light, which was observed by eye and manually recorded. The transfer of momentum to each α -particle was known by the direction of the emitted particle and its energy, which was first measured by using the fact that particles have short ranges in air. The ranges were calibrated against the energies from known sources. The subject of this book will be the deduction of nuclear properties

from the distributions of momentum transfers to one or more particles in a nuclear reaction. The work of the first twenty years provides several interesting and important examples.

The other major source of information about nuclei was the measurement by Aston (1927) of nuclear masses with an accuracy of one ten-thousandth of the proton mass or 0.1 million electron volts (MeV) in energy. Study of the radioactive decay mechanism itself provided yet another source of information. The early developments in the study of nuclear reactions were summarized and explained by Rutherford (1929) in the opening address of a Royal Society discussion on the structure of atomic nuclei which is reproduced in Part 2. At this stage the known particles were the α -particle, the proton and the electron, although the existence of the neutron had been conjectured by Rutherford to explain the difference between the nuclear charge and mass numbers.

Successes with the α -particle probe came rapidly. In the earliest work the energies of the α -particles were not resolved and it was assumed that they were elastically scattered. The nucleus was discovered by the fact that the probability of finding an α -particle scattered at a certain angle with a certain energy is given by the Rutherford law for scattering by a heavy point nucleus where the force is the Coulomb repulsion. Knowledge of the mass of the nucleus confirmed this interpretation very accurately. The Rutherford law was obtained from classical mechanics. Larger scattering angles are due to particles approaching the nucleus more closely and scattering angles for the same distance of approach are smaller for higher energies.

Rutherford's group at the Cavendish Laboratory in Cambridge were able to observe scattering at backward angles, in practice about 135° , for which in the case of high enough energies and small enough nuclear charges the Coulomb scattering law broke down. It was assumed that the particle had come close enough to the nucleus to enter a region where the law of force was not the Coulomb law, but was given by some other strong, short-range effect. Measurement of the energy at which the Rutherford law broke down, which was not very well defined because of diffraction

effects due to the quantum nature of the process, enabled an order of magnitude estimate of the nuclear radius to be obtained. This was the first use of a nuclear reaction to obtain information about nuclei.

Very soon it became possible to obtain deeper information about the energy relationships of nuclei by observing events in which an α -particle was absorbed by the target nucleus with the emission of a proton or a γ -ray or both. By measuring the energies of all the particles concerned and comparing them with Aston's mass measurements the relationship between mass and energy was confirmed. A reaction in which a particle a and target T interact to produce a final state particle b and residual nucleus R is called a $T(a, b)R$ reaction. We use the word "particle" to refer to the nucleus which is accelerated or detected.

One of the earliest reactions studied was $\text{Al}^{27}(\alpha, p)\text{Si}^{30}$. In addition to elastically scattered α -particles, two groups of protons, each characterized by a different energy, were observed. The energy of the first group corresponded to the mass difference between the initial and final particles and nuclei. The energy of the second group was lower, indicating that Si^{30} was left in an excited state which subsequently decayed to the ground state with the emission of a γ -ray of a certain energy. γ -rays of the right energy were in fact also found among the reaction products.

The use of the α -particle probe to discover excited states of nuclei developed very rapidly and led to the modern study of nuclear spectroscopy. Here interest in a reaction is limited to the fact that it will produce the required excited state and perhaps enable some of its properties to be determined. The spin, parity and decay rates of the states are examined by observing the radiations that they emit. We will be concerned with a deeper understanding of the mechanism of the reaction itself. Sometimes this understanding leads to new ways of obtaining spectroscopic information.

Preliminary information with great significance came again from the reaction $\text{Al}^{27}(\alpha, p)\text{Si}^{30}$, this time concerning the probability of the α -particle reacting with the target. Chadwick, Constable and

Pollard (1931) found that the reaction occurred preferentially at four different energies between about 4 MeV and 5.3 MeV. Resonance was said to occur in the α, Al^{27} system at these energies. The widths of the resonances were about 0.25 MeV. Clearly the variation of reaction probability with energy and angle may be expected to yield information about nuclear structure.

The first radical change in our knowledge of nuclei occurred in 1932 with the identification by Chadwick of neutrons in the reaction $\text{Be}^9(\alpha, n)\text{C}^{12}$. Not only does the discovery of the neutron give us a starting point for the understanding of nuclear structure. It also provides us with a nuclear probe which has the valuable property that it reacts only with nuclear matter and not with the Coulomb field so that nuclear reactions at very low energy may be studied.

Another development occurred in 1932 which pioneered the study of reactions. This was the artificial acceleration of charged particles by Cockcroft and Walton (1932) at the Cavendish Laboratory. At first protons were accelerated to 0.6 and 0.8 MeV. The first artificially produced nuclear reaction was $\text{Li}^7(p, \alpha)\text{He}^4$. The invention of accelerating machines promised new probes, for example protons, deuterons, and even heavier ions, higher beam intensities and, more important, higher energies.

The Cockcroft-Walton machine accelerated particles in a single step from a terminal at ground potential to a high potential terminal for which the steady potential was provided by a bank of condensers and a half-wave rectifier. The single acceleration idea was developed independently at Princeton by Van de Graaff whose machine provided the high potential by means of an electrostatic charge delivered by a belt. Early Van de Graaff accelerators could accelerate protons to 1 MeV. A 2.5 MeV machine was built at Princeton in 1936.

A machine which promised much higher energies was the cyclotron. Protons are confined to closed orbits by a magnetic field and accelerated in one or two small steps for each turn by a radio-frequency potential from which they are shielded when it is in the wrong direction for acceleration. The cyclotron was invented in

1929 by Lawrence at the University of California at Berkeley. The first model produced a beam of 80 KeV protons in 1930. Lawrence and Livingstone (1932) built a larger machine which was ready in 1932 to accelerate protons to about 1 MeV. Their first studies of nuclear reactions were published in that year, shortly after those of Cockcroft and Walton.

In this chapter we will discuss quantitative ideas which were developed to explain reactions before 1932 and we will see the effect of the development of the low-energy neutron probe.

1.1 Simple Quantitative Ideas

Given a monoenergetic collimated beam one knows the initial momentum in a reaction. It is therefore of interest to know the energy and direction of the emergent particle (or particles) and of course the probability of finding a particle emerging with a certain momentum. We will mainly be concerned with reactions involving a single emergent particle.

The object of an experiment is first to identify the final state energy so that separate quantum states may be studied. A quantum state of the system consisting of the residual nucleus plus the emergent particle is called a channel, specifically an exit channel. The quantum state of the system consisting of the incident particle and the target is the entrance channel. For a particle with only one internal quantum state there is one channel for every quantum state of the nucleus. The α -particle has only one quantum state for low energies. Its first excited state is at about 20 MeV. Both the proton and neutron have spin $\frac{1}{2}$ and therefore two states of spin projection. They may be identified separately by measuring the spin polarization of the beams. This is a refinement which will be mentioned later. The spin-dependent forces are quite small and it is a good first approximation to treat protons and neutrons as if they have only one quantum state. A particular reaction is characterized by the entrance and exit channels.

If one knows the quantum states of the initial and final systems, one can find the probability of the reaction occurring as a function

of the momentum transfer from the incident to the emergent particle. This probability is conveniently expressed in terms of the differential cross-section which is a quantity dependent only on the properties of the reacting particles and not on experimental conditions such as the intensity of the incident beam. The differential cross-section for a reaction at a given energy E is denoted $d\sigma(\theta, \phi, E)/d\Omega$ and defined by

$$dN = I [d\sigma(\theta, \phi, E)/d\Omega] d\Omega, \quad (1.1)$$

where dN particles are scattered per second into an element of solid angle $d\Omega$ making an angle (θ, ϕ) with the incident beam. The incident beam intensity is I particles per unit area per second. In many cases the scattering centre is spherically symmetric so that the problem has axial symmetry. The angle ϕ need not then be specified.

The first example of nuclear information being obtained from measurements of the differential cross-section as a function of momentum transfer is the Rutherford law for elastic scattering, in which the system remains in the entrance channel. In this case

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{1}{4} \left(\frac{ZZ'e^2}{2E} \right)^2 \text{cosec}^4 \frac{\theta}{2}, \quad (1.2)$$

where E is the incident energy in the centre of mass system, Z and Z' are the charge numbers of the target and probe respectively and e is the charge of the proton. Since E depends on the target mass M it is possible to obtain M from the experiment and verify either the Rutherford formula or an independent mass measurement.

The momentum transfer P in this experiment is given for the centre of mass system by

$$P = 2(2\mu E)^{1/2} \sin \theta/2, \quad (1.3)$$

where $\mu = mM/(m+M)$ is the reduced mass for an incident particle of mass m . The differential cross-section as a function of momentum transfer is given by

$$\frac{d\sigma(P)}{d\Omega} = 4\mu^2(ZZ'e^2)^2 P^{-4}. \quad (1.4)$$

The unit of length in nuclear physics is the fermi (fm) which is 10^{-13} cm. Differential cross-sections, however, are customarily expressed in millibarns (mb) per steradian (sr) where $1 \text{ mb} = 10^{-1} \text{ fm}^2 = 10^{-27} \text{ cm}^2$. Cross-sections integrated over angles are expressed in barns. $1 \text{ barn} = 10^{-24} \text{ cm}^2$. The unit of energy is 1 MeV.

If the Rutherford law breaks down for scattering angles greater than θ_0 we say that the radius of the nucleus is equal to the impact parameter of the trajectory whose asymptote makes an angle θ_0 with the incident direction.

$$R = \frac{ZZ'e^2}{2E} \cot \frac{\theta_0}{2}. \quad (1.5)$$

We see that for a given scattering angle particles of higher energy come closer to the nucleus. An idea of the magnitudes involved is obtained by calculating the radius of Al^{27} for an experiment in which anomalous scattering at 135° sets in at 0.9 MeV. The original experimental curve for this reaction is shown in Rutherford's address in Part 2. The value $R = 6 \text{ fm}$ is obtained very quickly from (1.5) by making use of two well-known quantities, the mass of the electron

$$m_e c^2 = 0.511 \text{ MeV}$$

and the classical radius of the electron

$$e^2/m_e c^2 = 2.82 \text{ fm}.$$

There is considerable uncertainty in the determination of the critical energy. Modern experiments with high-energy electron probes are analysed by quantum theory to yield smaller values of R .

In order to observe nuclear properties it is necessary that the probe be able to enter the nucleus. This means that it must have at least as much energy as the value of the Coulomb potential at