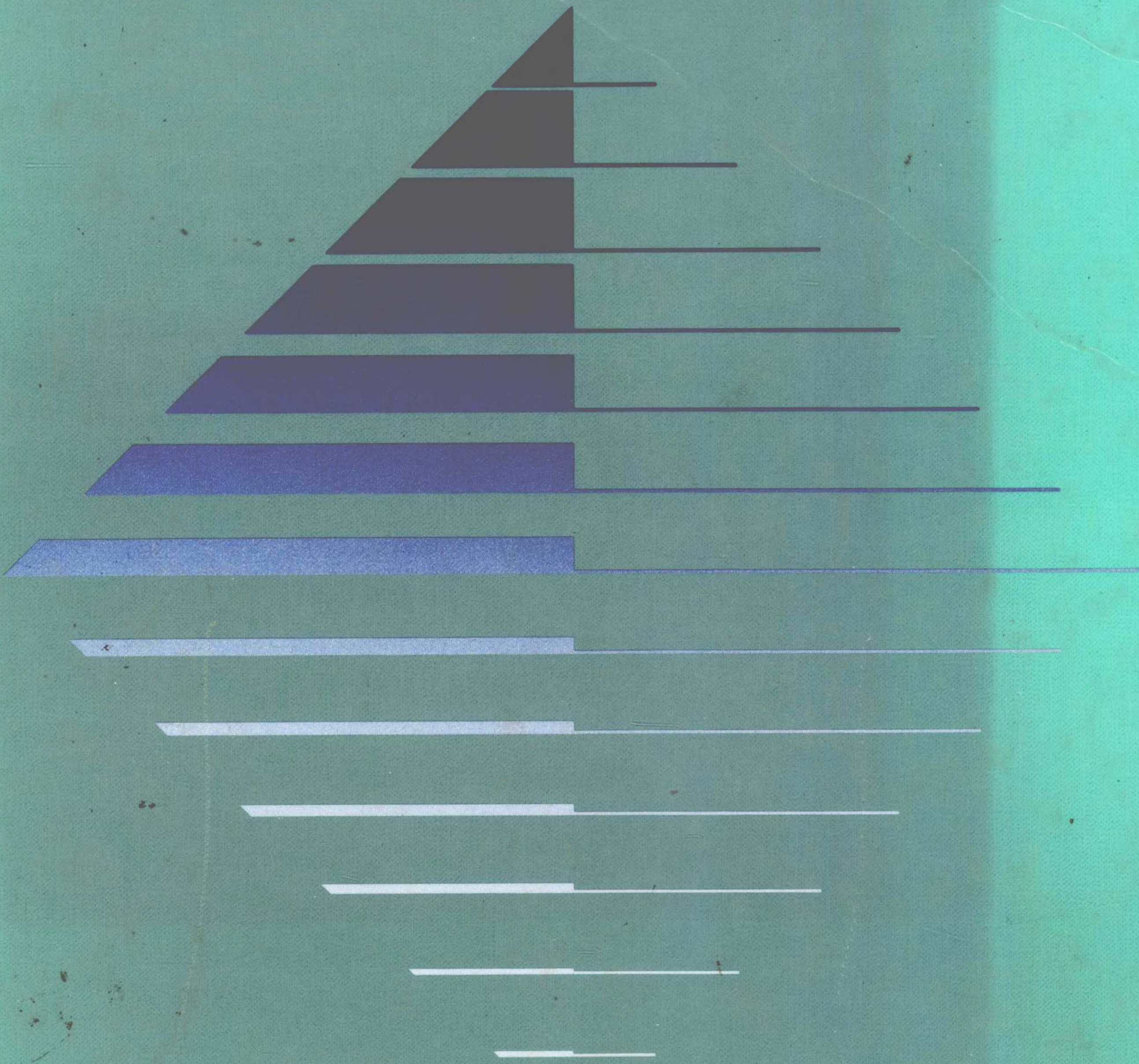
FINITE MATHEMATICS

WITH APPLICATIONS / FRANK S. BUDNICK



FINITE MATHEMATICS

With Applications

FRANK S. BUDNICK

›f Rhode Island

McGRAW-HILL BOOK COMPANY

New York St. Louis San Francisco Auckland Bogotá Hamburg Johannesburg
London Madrid Mexico Montreal New Delhi Panama Paris
São Paulo Singapore Sydney Tokyo Toronto

TO

My Parents—Mr. and Mrs. Willard L. Budnick
with Much Love and Appreciation

FINITE MATHEMATICS

With Applications

1 2 3 4 5 6 7 8 9 0 VNHVNH 8 9 8 7 6 5 4

ISBN 0-07-008861-6

This book was set in Palatino by Progressive Typographers, Inc.
The editors were Cheryl L. Mehalik and Joseph F. Murphy;
the cover was designed by Nicholas Krenitsky;
the production supervisor was Phil Galea.
Von Hoffmann Press, Inc., was printer and binder.

Library of Congress Cataloging in Publication Data

Budnick, Frank S.
Finite mathematics.

Includes bibliographical references and index.
1. Mathematics—1961- . 2. Business mathematics.
3. Social sciences—Mathematics. I. Title.
QA39.2.B8 1985 510 84-20076
ISBN 0-07-008861-6

PREFACE

INTRODUCTION

Mathematics is an integral part of the education of students in business, economics, and the social sciences. There is increasingly a desire to improve the level of quantitative sophistication possessed by graduates in these types of programs. The objective is not to make mathematicians of these students, but to make them as comfortable as possible in an environment which increasingly makes use of quantitative analysis and the computer. Students are discovering that they must integrate mathematics, statistical analysis, and the computer in both required and elective courses within their programs. Furthermore, organizations are becoming more effective users of quantitative tools and the computer. Decision makers will be better equipped to operate within this type of environment if they are familiar with the more commonly used types of quantitative analyses and the technology of the computer. Such familiarity can assist them in being better "critics" and "users" of these tools, and hopefully, better decision makers.

DESIGN OF BOOK

This is an applied finite mathematics book. The primary market for this book is a one-term course for students in business, economics, and the social sciences. The course is likely to be found in two-year and four-year schools and taken during the freshman or sophomore years.

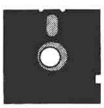
Specific features of the book include:

1 A style which carefully develops and reinforces topics.

2 An applied orientation which motivates students and provides a sense of purpose for studying mathematics.

3 Annotated summaries of actual applications of selected mathematical techniques.

4 An approach which first develops the mathematical concept and then reinforces with applications.



5 A design which incorporates and encourages the use of computers. Specific features include: (*a*) the illustration of relevant software in selected chapters, (*b*) annotated lists of software packages which are available commercially or privately, and (*c*) specially identified exercises and minicases (see symbol at left) which have computer requirements (the requirements typically involve writing a short program or utilizing existing software packages).

6 Enriched "minicases," at the end of 10 of the 11 chapters, which provide challenging applications.

7 An optional, end-of-text appendix which provides a general review of key algebra principles.

8 Notes to students which provide special insights.

9 "Points for Thought and Discussion" which allow students to pause for a moment and reconsider a concept or example from a different perspective. Their purpose is to reinforce and extend the student's understanding.

10 A multitude of other learning aids, including over 250 solved examples, over 1,200 exercises, chapter tests, chapter objectives, and summary lists of key terms and concepts as well as important formulas.

11 An instructor's manual which contains answers to all exercises and tests, suggestions for different course structures, a bank of questions for constructing quizzes and tests, and listings of some generic software which can be adapted to different computer systems and used by instructors and their students.

OVERVIEW OF CHAPTERS

Chapter 1 Linear Equations

This chapter discusses the algebraic and graphical characteristics of linear equations. **(Prerequisites: none.)**

Chapter 2 Linear Functions: Applications

This chapter introduces the concept of mathematical functions. After a discussion of general properties and characteristics of functions, the focus turns to linear functions, their characteristics, and applications. The chapter ends with a discussion of break-even analysis. **(Prerequisites: Chapter 1.)**

Chapter 3 Systems of Linear Equations

This chapter discusses systems of linear equations, graphical characteristics (where appropriate), and algebraic methods for determining solution sets. The Gaussian elimination method is the main technique presented for finding these solution sets. **(Prerequisites: Chapter 1.)**

Chapter 4 Matrix Algebra

This chapter presents the concept of a matrix and the algebra of matrices. Aside from discussing the basic matrix operations through the matrix inverse, the last section of the chapter presents a good selection of applications. **(Prerequisites: Chapter 3.)**

Chapter 5 Linear Programming: An Introduction

This is the first of three chapters which discuss linear programming and its extensions. Instructors can cover this chapter only and provide their students with a good overview of linear programming models and their application. If they desire a more in-depth treatment, Chapter 6, Chapter 7, or both can be covered. Chapter 5 provides an overview of the general structure of linear programming models, a good selection of applications, a discussion of graphical solution methods, an overview of the simplex method, and a discussion of computer solution methods. An annotated list of linear programming software is provided at the end of the chapter. **(Prerequisites: Chapter 3.)**

Chapter 6 The Simplex Method

This chapter is optional and presents details of the simplex method. Special phenomena such as alternative optimal solutions, no feasible solution, and unbounded solutions are discussed, in terms of the simplex. The last section of the chapter discusses the dual problem. **(Prerequisites: Chapter 5.)**

Chapter 7 Additional Applications and Extensions of Linear Programming

This chapter is also optional. It is a chapter not usually found in finite mathematics books. It is intended for the instructor who would like to take students a bit farther in linear programming. Included in this chapter are discussions of the *transportation model*, the *assignment model*, *integer programming*, and *goal programming*. Any or all of these models can be covered. For each model, the general form and assumptions are discussed. This is followed by sample applications and an overview of solution methods. Computer software packages are illustrated for the transportation and assignment models, and an annotated listing of available software packages for each of the four models is presented at the end of the chapter. **(Prerequisites: Chapter 5.)**

Chapter 8 Introduction to Probability Theory

This is the first of three chapters which deal with probability and its applications. This chapter begins with a brief review of sets and set operations. This is followed by a discussion of counting methods, culminating with permutations and combinations. The remainder of the chapter presents basic probability concepts and their application. **(Prerequisites: none.)**

Chapter 9 Probability Distributions

This chapter continues the discussion of probability by presenting probability distributions. The chapter begins with a discussion of random variables, frequency distributions, and probability distributions. This is followed by a discussion of measures of central tendency and measures of variability. The last two sections present the binomial distribution and its application and the normal probability distribution and its application. **(Prerequisites: Chapter 8.)**

Chapter 10 Selected Applications

This last of three chapters dealing with probability focuses upon three areas of application: *Markov processes*, *decision theory*, and *game theory*. Instructors may choose to cover any or all of these three areas of application. **(Prerequisites: Chapter 9.)**

Chapter 11 Mathematics of Finance

This chapter presents the basic methods of computation for compound interest. Both the fixed payment and annuity situations are presented. A special section presents computation methods for mortgages, and the last section of the chapter discusses cost-benefit analysis. **(Prerequisites: none.)**

Appendix A A Review of Algebra (Optional)

This appendix is an optional review of selected topics in algebra. The topics selected are those which are needed for study in this book as well as others which might be required for further courses. This appendix assumes some prior background in algebra.

ACKNOWLEDGMENTS

I wish to express my sincere appreciation to those persons who have contributed either directly or indirectly to this project. I wish to thank Jerome Bloomberg, Essex Community College; Barbara Bulmahn, Indiana University–Purdue University at Ft. Wayne; Donald Clark, Purdue University at Calumet; Richard Crownover, University of Missouri; Garrett J. Etgen, University of Houston; William Ramaley, Ft. Lewis College; and David Schedler, Virginia Commonwealth University.

I want to thank the people at McGraw-Hill with whom I worked directly. These persons include Cheryl Mehalik, Joseph Murphy, and Nick Krenitsky.

I also want to thank Jon Naughton and Rohit (Thuky) Thukral for their assistance in developing solution sets, proofreading, and computer programming. Thanks again to Ede Williams for her superb work in typing the manuscript and instructor's manual.

Finally, I would like to acknowledge my family—Jane, Chris, Scott, and Kerry. I love them all very much!

Frank S. Budnick

CONTENTS

LINEAR EQUATIONS

1

CHAPTER OBJECTIVES

- Provide a thorough understanding of the algebraic and graphical characteristics of linear equations
- Provide the tools which will allow one to determine the equation which represents a linear relationship
- Illustrate a variety of applications of linear equations

In this text, one major area of study is that of linear mathematics. This chapter is the first of seven chapters which focus upon linear mathematics and its applications. Linear mathematics is significant for a number of reasons:

1 Many of the real-world phenomena which we might be interested in representing mathematically either are linear or can be approximated reasonably well using linear relationships. As a result, linear mathematics is widely applied.

2 The analysis of linear relationships is generally easier than that of nonlinear relationships.

3 Some methods used in nonlinear mathematics are similar to, or extensions of, those used in linear mathematics. Consequently, having a good understanding of linear mathematics is prerequisite to the study of nonlinear mathematics.

1.1 CHARACTERISTICS OF LINEAR EQUATIONS

General Form

LINEAR EQUATION WITH TWO VARIABLES

A linear equation involving two variables x and y has the standard form

$$ax + by = c \tag{1.1}$$

where a, b, and c are real numbers *and* a and b cannot both equal zero.

Notice that linear equations are *first-degree* equations. Each variable in the equation is raised (implicitly) to the first power. The presence of terms having exponents other than 1 (for example, x^2) would exclude an equation from being considered linear. The presence of terms involving a product of the two variables (for example, $2xy$) would also exclude an equation from being considered linear.

The following are all examples of linear equations involving two variables:

	Eq. (1.1) Parameters		
	a	b	c
$2x + 5y = -5$	2	5	−5
$-x + \frac{1}{2}y = 0$	−1	$\frac{1}{2}$	0
$x/3 = 25$	$\frac{1}{3}$	0	25
(*Note:* $x/3 = \frac{1}{3}x$)			
$2s - 4t = -\frac{1}{2}$	2	−4	$-\frac{1}{2}$

(*Note:* The names of the variables may be different from x and y.)

The following are examples of equations which are *not* linear. Can you explain why?

$$2x + 3xy - 4y = 10$$
$$x + y^2 = 6$$
$$\sqrt{u} + \sqrt{v} = -10$$

In attempting to identify the form of an equation (linear versus nonlinear), an equation is linear if it *can be* written in the form of Eq. (1.1). A quick glance at the equation

$$2x = \frac{5x - 2y}{4} + 10$$

might lead to the false conclusion that it is not linear. However, multiplying both sides of the equation by 4 and moving all variables to the left-hand side yields $3x + 2y = 40$, which is in the form of Eq. (1.1).

LINEAR EQUATION WITH *n* VARIABLES

A linear equation involving n variables $x_1, x_2, x_3, \ldots, x_n$ has the general form

$$a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n = b \qquad \textbf{(1.2)}$$

where $a_1, a_2, a_3, \ldots, a_n$ and b are real numbers and *not all* $a_1, a_2, a_3, \ldots, a_n$ equal zero.

Each of the following is an example of a linear equation:

$$3x_1 - 2x_2 + 5x_3 = 0$$
$$-x_1 + 3x_2 - 4x_3 + 5x_4 - x_5 + 2x_6 = -80$$
$$5x_1 - x_2 + 4x_3 + x_4 - 3x_5 + x_6 - 3x_7 + 10x_8 - 12x_9 + x_{10} = 1{,}250$$

We will spend much of our time in this book discussing equations and mathematical functions that involve two variables. Aside from the fact that the arithmetic is a little easier, another important reason for concentrating on the two-variable situation is that these functions can be graphed to provide a visual frame of reference. Equation (1.2), however, generalizes the definition of a linear equation for those instances when we venture beyond two variables.

Representation Using Linear Equations

Given a linear equation having the form $ax + by = c$, the ***solution set*** for the equation is the set of all ordered pairs (x, y) which satisfy the equation. Using **set notation** the solution set S can be specified as

$$\boxed{S = \{(x, y) | ax + by = c\}} \qquad \textbf{(1.3)}$$

Verbally, this notation states that the solution set S consists of **elements** (x, y) *such that* (the vertical line) the equation $ax + by = c$ is satisfied. For any linear equation, S consists of an infinite number of elements; that is, *there are an infinite number of pairs of values* (x, y) *which satisfy any linear equation involving two variables.*

To determine any pair of values which satisfy a linear equation, assume a value for one of the variables, substitute this value into the equation, and solve for the corresponding value of the other variable. This method assumes that both variables are included in the equation (i.e., $a \neq 0$ and $b \neq 0$).

EXAMPLE 1

Given the equation

$$2x + 4y = 16$$

(*a*) Determine the pair of values which satisfies the equation when $x = -2$.
(*b*) Determine the pair of values which satisfies the equation when $y = 0$.

SOLUTION

(*a*) Substituting $x = -2$ into the equation, we have

$$2(-2) + 4y = 16$$
$$4y = 20$$

and

$$y = 5$$

When $x = -2$, the pair of values satisfying the equation is $x = -2$ and $y = 5$, or $(-2, 5)$.

(*b*) Substituting $y = 0$ into the equation gives

$$2x + 4(0) = 16$$
$$2x = 16$$

and

$$x = 8$$

When $y = 0$, the pair of values satisfying the equation is (8, 0).

EXAMPLE 2

Product Mix A company manufactures two different products. For the coming week 120 hours of labor are available for manufacturing the two products. Work-hours can be allocated for production of either product. In addition, since both products generate a good profit, management is interested in using all 120 hours during the week. Each unit produced of product A requires 3 hours of labor and each unit of product B requires 2.5 hours.

(*a*) Define an equation which states that total work-hours used for producing x units of product A and y units of product B equal 120.

(*b*) How many units of product A can be produced if 30 units of product B are produced?

(*c*) If management decides to produce one product only, what is the maximum quantity which can be produced of product A? The maximum of product B?

SOLUTION

(*a*) We can define our variables as follows:

x = number of units produced of product A
y = number of units produced of product B

The desired equation has the following structure.

$$\text{Total hours used in producing products } A \text{ and } B = 120 \quad \textbf{(1.4)}$$

What we need, then, is the expression for the left-hand side of the equation.

NOTE

You may well have a mental model for the left side of this equation—it is simply a matter of recognizing its form and stating it. Try it by asking yourself how many hours would be used if you produced 1 unit of each product? 2 units of each? 10 units of product A and 20 of product B? If you are able to answer these questions, you are using a mental model. Look back at the definitions of x and y and state the model that allows you to answer these questions.

As you reason through the structure of the left side of Eq. (1.4), the final equation might evolve as follows:

$$\begin{array}{c}\text{Total hours used}\\ \text{in producing}\\ \text{product } A\end{array} + \begin{array}{c}\text{total hours used}\\ \text{in producing}\\ \text{product } B\end{array} = 120 \quad \textbf{(1.5)}$$

Since the total hours required to produce either product equals hours required per unit produced times number of units produced, Eq. (1.5) reduces to

$$\boxed{3x + 2.5y = 120} \tag{1.6}$$

Is that the answer you reached?

(*b*) If 30 units of product *B* are produced, then $y = 30$. Therefore

$$\begin{aligned} 3x + 2.5(30) &= 120 \\ 3x &= 45 \\ x &= 15 \text{ units} \end{aligned}$$

A pair of values satisfying Eq. (1.6) is (15, 30). In other words, *one combination* of the two products which will fully utilize the 120 hours is 15 units of product *A* and 30 units of product *B*.

(*c*) If management decides to manufacture product *A* only, no units of product *B* are produced, or $y = 0$. If $y = 0$,

$$\begin{aligned} 3x + 2.5(0) &= 120 \\ 3x &= 120 \\ x &= 40 \end{aligned}$$

Therefore 40 is the maximum number of units of product *A* which can be produced using the 120 hours.

If management decides to manufacture product *B* only, $x = 0$ and

$$3(0) + 2.5y = 120$$

or

$$y = 48 \text{ units}$$

EXAMPLE 3

We stated earlier that there are an infinite number of pairs of values (x, y) which satisfy any linear equation. In Example 2, are there any members of the solution set which might not be realistic in terms of what the equation represents?

SOLUTION

In Example 2, x and y represent the number of units produced of the two products. Since *negative* production is not possible, negative values of x and y are not meaningful. There are negative values which satisfy Eq. (1.5). For instance, if $y = 60$, then

$$\begin{aligned} 3x + 2.5(60) &= 120 \\ 3x &= -30 \\ x &= -10 \end{aligned}$$

In addition to negative values, it is possible to have decimal or fractional values for x and y. For example, if $y = 40$,

$$\begin{aligned} 3x + 2.5(40) &= 120 \\ 3x &= 20 \\ x &= 6\tfrac{2}{3} \end{aligned}$$

Given all the pairs of *nonnegative* values for x and y which satisfy Eq. (1.6), noninteger values may not be meaningful given the nature of the products.