

# **Sliding Mode Control of Switching Power Converters**

---

## **Techniques and Implementation**

**Siew-Chong Tan  
Yuk-Ming Lai  
Chi Kong Tse**



**CRC Press**  
Taylor & Francis Group

# Sliding Mode Control of Switching Power Converters

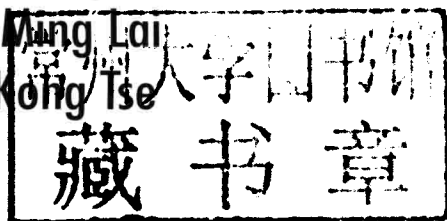
---

## Techniques and Implementation

Siew-Chong Tan

Yuk-Ming Lai

Chi Kong Tse



CRC Press

Taylor & Francis Group

Boca Raton London New York

---

CRC Press is an imprint of the  
Taylor & Francis Group, an **informa** business

CRC Press  
Taylor & Francis Group  
6000 Broken Sound Parkway NW, Suite 300  
Boca Raton, FL 33487-2742

© 2012 by Taylor & Francis Group, LLC  
CRC Press is an imprint of Taylor & Francis Group, an Informa business

No claim to original U.S. Government works

Printed in the United States of America on acid-free paper  
Version Date: 20110526

International Standard Book Number: 978-1-4398-3025-3 (Hardback)

This book contains information obtained from authentic and highly regarded sources. Reasonable efforts have been made to publish reliable data and information, but the author and publisher cannot assume responsibility for the validity of all materials or the consequences of their use. The authors and publishers have attempted to trace the copyright holders of all material reproduced in this publication and apologize to copyright holders if permission to publish in this form has not been obtained. If any copyright material has not been acknowledged please write and let us know so we may rectify in any future reprint.

Except as permitted under U.S. Copyright Law, no part of this book may be reprinted, reproduced, transmitted, or utilized in any form by any electronic, mechanical, or other means, now known or hereafter invented, including photocopying, microfilming, and recording, or in any information storage or retrieval system, without written permission from the publishers.

For permission to photocopy or use material electronically from this work, please access [www.copyright.com](http://www.copyright.com) (<http://www.copyright.com/>) or contact the Copyright Clearance Center, Inc. (CCC), 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400. CCC is a not-for-profit organization that provides licenses and registration for a variety of users. For organizations that have been granted a photocopy license by the CCC, a separate system of payment has been arranged.

**Trademark Notice:** Product or corporate names may be trademarks or registered trademarks, and are used only for identification and explanation without intent to infringe.

**Visit the Taylor & Francis Web site at**  
**<http://www.taylorandfrancis.com>**

**and the CRC Press Web site at**  
**<http://www.crcpress.com>**

# **Sliding Mode Control of Switching Power Converters**

---

**Techniques and Implementation**

---

## Preface

---

As the characteristics of power sources and electrical/electronic loads become more widely varied, nonlinear, and unpredictable, the control of the power converters that provide the necessary power processing functions will play a crucial role in optimizing performance and maintaining the needed robustness under various operating conditions. Conventional control approaches based on small-signal linear techniques are found to be incapable of achieving the necessary regulation, dynamic response, and stability requirements needed for these systems. Much research effort has been devoted to the development of advanced control methods that are able to address the complex requirements of power conversion systems. In particular, modern control theories such as sliding mode control, fuzzy control, adaptive control, etc., have been applied to the control of these systems, and their feasibility has been studied. Among them, sliding mode control has been most widely investigated and has demonstrated to be a highly promising solution for both current and future generations of power converters.

Following the trend of this development and taking advantage of the mature theoretical framework already being laid out on the subject, it is timely to present a complete exposition of the development of sliding mode controllers for power converters. This book presents an in-depth and thorough account of how such kind of controllers can be practically engineered to suit the purpose of controlling power converters. The dissemination of such knowledge is now timely and necessary, especially since the electronic industry is moving toward using renewable energy sources and widely varying loads, which can only be adequately supported by power converters using nonlinear controllers.

We developed this book with the following objectives in mind. First, we aim to offer a comprehensive overview of the principles and methods in the application of sliding mode control to power converter systems for the general readership. Second, for the more advanced readers, we aim to provide a systematic exposition of the mathematical machineries and design principles relevant to the construction of sliding mode controllers, and building upon that, to introduce and impart new practical approaches of designing such controllers. Our third objective is to demonstrate the practical implementation of sliding mode controllers based on analog circuits and to present their supporting design rules. Our final objective is to promote an appreciation of nonlinear control in general by presenting it from a practical perspective and using terminology that is familiar to the engineers.

In a nutshell, *Sliding Mode Control of Power Converters: Techniques and*

*Implementation* will be a guide to understanding the sliding mode control principle, its application to power converters, and the practical realization of sliding mode controllers. By combining theory with application and relating mathematical concepts and models to their industrial targets, this book will be equally accessible to readers with analog circuit design, power electronics, or control engineering backgrounds. We believe that this book will be of interest to students and professionals alike in the field of electrical and electronic engineering. At the same time, we believe that our approach to the modeling and implementation of sliding mode controllers for power converters will help power electronics and IC industry professionals design effective and high-performance controllers for power converters.

The book is organized as follows. We will begin in Chapter 1 with a discussion on the basic principles and theory of sliding mode control to familiarize readers with the core terminology and background of sliding mode control. This is followed by Chapter 2, in which we will give a review on the basics of power converters and their control. A short discussion on the common types of control techniques available and the current progress of the research work on the control of power converters are also provided. Next, in Chapter 3, we move on to address the important concepts, operating principles, and properties of the sliding mode controller that are relevant to the application of power converters. Here, we will also provide a detailed review of the state-of-the-art research work and some common practices in the development of sliding mode controllers for power converters.

Then, in Chapter 4, the practical design process of sliding mode controllers based on hysteresis modulation for the power converters is described. In Chapter 5, the problem of switching frequency variation in traditional sliding mode control, due to the deviation of operating conditions, and the effectiveness of applying adaptive control solutions in sliding mode control in alleviating the problem, will be thoroughly covered. Chapter 6 introduces a practical technique for implementing sliding mode control with fixed frequency for power converters operating in continuous conduction mode. Then, the idea of implementing fixed-frequency sliding mode controllers using equivalent control is further explored in Chapter 7. This includes the derivation of the system models and sliding mode control laws for the discontinuous conduction mode converter counterparts. In Chapter 8, we extend our discussion to the design and practical circuit implementation of the pulse-width-modulation-based sliding mode controllers.

Up to this stage of the book, our discussions have been focused on the design and implementation of sliding mode controllers using the output voltage of the converters as the control variable for constructing the sliding manifold. For Chapters 9 to 11, the design and implementation of the sliding mode controllers are extended to cover power converters with non-conventional sliding manifolds that are constructed nonlinearly from the current error and voltage error. In Chapter 9, the design and implementation of sliding mode controllers based on current error and voltage error will be discussed. In Chapter 10, the

discussion will be focused on how a reduced-state nonlinear sliding manifold may be used for the control of a high-order converter like the Ćuk converter. In Chapter 11, a non-conventional type of sliding mode controller based on a double-integral sliding surface for improving the steady-state regulation is discussed.

For the completion of this book, we must give our sincere gratitude to a number of people, institutions, and organizations. First, we would like to thank all our friends and colleagues in the Department of Electronic and Information Engineering at Hong Kong Polytechnic University, and the members of the Applied Nonlinear Systems Research Group, for their friendship, encouragement, and inspiration. Special thanks are due to Prof. Luis Martínez-Salamero, Universidad Rovira i Virgili, Tarragona, Spain, who is not only a great friend to us, but is also an important collaborator of some of our research work in this area. Luis is by far the most knowledgeable person in sliding mode control applications in power electronics that the authors personally know. We would also like to take this opportunity to extend our gratitude to Prof. Ashoka Bhat, Dr. Martin Chow, Prof. Adrian Ioinovici, Dr. Joe Liu, Dr. Franki Poon, Dr. Siu Chung Wong, and Prof. Xinbo Ruan, who are all experts in different areas of power electronics, and whom the authors have worked with and learned a great deal from. Next, we would also like to thank the staff of CRC Press, especially Ms. Leong Li Ming and Ms. Amy Blalock, for their professional and enthusiastic support of this project. The completion of this research work would not be possible without the financial support of the Hong Kong Research Grant Council and the Research Committee of Hong Kong Polytechnic University. Last, but not least, we must thank our families for their support and understanding throughout the course of our academic careers.

Finally, it is our pleasure to dedicate this book to all those who have been passionately involved in the work on sliding mode control and the control of power electronics.

*Siew-Chong Tan*

*Yuk-Ming Lai*

*Chi Kong Tse*

---

# Contents

---

<b>1</b>	<b>Introduction to Sliding Mode Control</b>	<b>1</b>
1.1	Introduction . . . . .	1
1.2	General Theory . . . . .	2
1.3	Properties of Sliding Motion . . . . .	4
1.3.1	An Ideal Control . . . . .	4
1.3.2	Practical Limitations and Chattering . . . . .	5
1.3.3	Constant Dynamics . . . . .	5
1.3.4	Quasi-Sliding Mode Control . . . . .	7
1.4	Mathematical Formulation . . . . .	7
1.4.1	Hitting Condition . . . . .	8
1.4.2	Existence Condition . . . . .	9
1.4.3	Stability Condition . . . . .	9
1.4.4	System with Linear Sliding Manifold . . . . .	10
1.4.5	System with Nonlinear Sliding Manifold . . . . .	11
1.4.5.1	Ideal Sliding Dynamics . . . . .	11
1.4.5.2	Equilibrium Point . . . . .	11
1.4.5.3	Linearization of Ideal Sliding Dynamics . . . . .	11
1.4.5.4	Remarks . . . . .	12
1.5	Equivalent Control . . . . .	13
1.6	Types of Implementation . . . . .	14
1.6.1	Relay and Signum Functions . . . . .	14
1.6.2	Hysteresis Function . . . . .	15
1.6.3	Equivalent Control Function . . . . .	16
<b>2</b>	<b>Overview of Power Converters and Their Control</b>	<b>19</b>
2.1	Introduction . . . . .	19
2.2	Basic DC–DC Converters . . . . .	20
2.3	Operating Modes of DC–DC Converters . . . . .	22
2.4	Overview of Control . . . . .	23
2.5	Factors Influencing Control Performances . . . . .	23
2.5.1	Switching Frequency . . . . .	23
2.5.2	Energy Storage Elements . . . . .	24
2.5.3	Control Gains . . . . .	24
2.6	Common Control Techniques . . . . .	26
2.6.1	Hysteretic Controllers . . . . .	26
2.6.2	Pulse-Width Modulation Controllers . . . . .	28



2.6.3	Design Approaches . . . . .	31
2.6.4	Problems of Small-Signal Models and Compensation . . . . .	31
2.7	Control Methodologies in Research . . . . .	32
2.7.1	Adaptive Control . . . . .	32
2.7.2	Fuzzy Logic Control . . . . .	32
2.7.3	Artificial Neural Network Control . . . . .	33
2.7.4	One-Cycle Control . . . . .	33
2.7.5	Sliding Mode Control . . . . .	33
<b>3</b>	<b>Sliding Mode Control in Power Converters</b>	<b>35</b>
3.1	Introduction . . . . .	35
3.2	Review of Literature . . . . .	36
3.2.1	Earliest Works . . . . .	36
3.2.2	Higher-Order Converters . . . . .	36
3.2.3	Parallel-Connected Converters . . . . .	37
3.2.4	Theoretical Works . . . . .	37
3.2.5	Practical Works . . . . .	38
3.2.6	Constant Frequency SM Controllers . . . . .	39
3.2.7	Remarks . . . . .	39
3.3	Characteristics of SM Control as Applied to DC–DC Converters	41
3.3.1	General Principle of SM Control Implementation . . . . .	41
3.3.2	Constant Dynamics in Power Converters . . . . .	42
3.3.3	Quasi-Sliding Mode Control in Power Converters . . . . .	44
3.3.4	Conventional Hysteresis-Modulation-Based Implementation . . . . .	44
3.4	Fixed-Frequency SM Controller in Power Converters . . . . .	45
3.4.1	Pulse-Width Modulation-Based Sliding Mode Controller . . . . .	46
3.4.2	Duty-Ratio Control . . . . .	47
3.5	Some Design Guidelines . . . . .	48
3.6	Practical Issues in Analog Implementation . . . . .	51
<b>4</b>	<b>Hysteresis-Modulation-Based Sliding Mode Controllers</b>	<b>53</b>
4.1	Introduction . . . . .	53
4.2	Theoretical Derivation . . . . .	54
4.2.1	Mathematical Model of Buck Converter . . . . .	54
4.2.2	Design of an Ideal SM Voltage Controller . . . . .	55
4.2.3	Design of a Practical SM Voltage Controller . . . . .	60
4.2.3.1	Redefinition of Sliding Line . . . . .	60
4.2.3.2	Introduction of Hysteresis Band . . . . .	61
4.2.3.3	Calculation of Switching Frequency . . . . .	62
4.3	A Standard Design Procedure . . . . .	65
4.3.1	Standard SMVC Converter Model . . . . .	65
4.3.2	Design Steps . . . . .	66
4.3.2.1	Step 1: Current Sensing Gain—H . . . . .	66

4.3.2.2	Step 2: Voltage Divider Network— $\beta$ . . . . .	66
4.3.2.3	Step 3: Gain of Differential Amplifier— $U_V$ . . . . .	66
4.3.2.4	Step 4: Calculation of Hysteresis Band— $\kappa$ . . . . .	67
4.3.2.5	Step 5: Design of Schmitt Trigger— $U_S$ . . . . .	68
4.4	Experimental Results . . . . .	69
4.4.1	Verification of Design Equation . . . . .	69
4.4.2	Steady-State Performance . . . . .	70
4.4.3	Load Variation . . . . .	72
4.4.4	Line Variation . . . . .	72
4.4.5	$\alpha$ Variation . . . . .	73
4.4.6	ESR Variation . . . . .	77
4.5	Further Discussion . . . . .	77
4.5.1	Advantages . . . . .	77
4.5.2	Disadvantages . . . . .	78
4.5.3	Possible Solutions . . . . .	79
<b>5</b>	<b>Hysteresis-Modulation-Based Sliding Mode Controllers with Adaptive Control</b> . . . . .	<b>81</b>
5.1	Introduction . . . . .	81
5.2	Examination of Conventional HM-Based SM Controlled Converters . . . . .	82
5.2.1	Mathematical Model . . . . .	82
5.2.2	Problems Identification . . . . .	83
5.2.2.1	Experimental Observation . . . . .	83
5.2.2.2	Analytical Explanation . . . . .	85
5.2.3	Possible Solutions . . . . .	86
5.3	Adaptive Feedforward Control Scheme . . . . .	87
5.3.1	Theory . . . . .	87
5.3.2	Implementation Method . . . . .	89
5.4	Adaptive Feedback Control Scheme . . . . .	90
5.4.1	Theory . . . . .	90
5.4.2	Implementation Method . . . . .	91
5.5	Experimental Results and Discussions . . . . .	94
5.5.1	Line Variation . . . . .	94
5.5.2	Load Variation . . . . .	100
<b>6</b>	<b>General Approach of Deriving PWM-Based Sliding Mode Controller for Power Converters in Continuous Conduction Mode</b> . . . . .	<b>105</b>
6.1	Introduction . . . . .	105
6.2	Background . . . . .	106
6.3	The Approach . . . . .	107
6.3.1	System Modeling . . . . .	107
6.3.2	Controller Design . . . . .	110
6.3.2.1	Derivation of Existence Conditions . . . . .	111

- 6.3.2.2 Derivation of Control Equations for PWM-Based Controller . . . . . 113
  - 6.3.3 Remarks . . . . . 115
    - 6.3.3.1 Controller Structure . . . . . 115
    - 6.3.3.2 Performance Comparison with HM-Based SM Controllers . . . . . 115
    - 6.3.3.3 Comparing the PWM-Based SM Controller Approach to the Nonlinear PWM Controller Design Approach . . . . . 117
    - 6.3.3.4 Load Resistance Dependence . . . . . 117
    - 6.3.3.5 Maximum Duty Ratio . . . . . 118
    - 6.3.3.6 Soft-Starting and Over-Current Protection Devices . . . . . 118
- 6.4 Simulation Results and Discussions . . . . . 118
  - 6.4.1 Buck Converter . . . . . 118
    - 6.4.1.1 Steady-State Performance . . . . . 119
  - 6.4.2 Boost Converter . . . . . 122
    - 6.4.2.1 Steady-State Performance . . . . . 122
  - 6.4.3 Buck-Boost Converter . . . . . 122
    - 6.4.3.1 Steady-State Performance . . . . . 122

- 7 General Approach for Deriving PWM-Based Sliding Mode Controller for Power Converters in Discontinuous Conduction Mode . . . . . 131**
  - 7.1 Introduction . . . . . 131
  - 7.2 State-Space Converter Model of the DC-DC Converters under DCM . . . . . 132
  - 7.3 The Approach . . . . . 134
    - 7.3.1 System Modeling . . . . . 135
    - 7.3.2 Controller Design . . . . . 137
      - 7.3.2.1 Derivation of Existence Conditions . . . . . 137
      - 7.3.2.2 Derivation of Control Equations for PWM-Based Controller . . . . . 140
  - 7.4 Simulation Results and Discussions . . . . . 145
    - 7.4.1 Buck Converter . . . . . 146
      - 7.4.1.1 Steady-State Performance . . . . . 146
      - 7.4.1.2 Transient Performance . . . . . 148
    - 7.4.2 Boost Converter . . . . . 148
      - 7.4.2.1 Steady-State Performance . . . . . 148
      - 7.4.2.2 Transient Performance . . . . . 150
    - 7.4.3 Buck-Boost Converter . . . . . 152
      - 7.4.3.1 Steady-State Performance . . . . . 154
      - 7.4.3.2 Transient Performance . . . . . 154
  - 7.5 Other Application of DCM SM Control: Hybrid Dual-Operating-Mode Controllers . . . . . 160

7.5.1	Background . . . . .	160
7.5.2	Architecture . . . . .	161
7.5.3	Simulation Results and Discussions . . . . .	162
7.5.3.1	Buck Converter . . . . .	162
7.5.3.2	Boost Converter . . . . .	165
<b>8</b>	<b>Design and Implementation of PWM-Based Sliding Mode Controllers for Power Converters</b>	<b>167</b>
8.1	Introduction . . . . .	168
8.2	PWM-based SM Voltage Controller for Buck Converters . .	168
8.2.1	Mathematical Model . . . . .	169
8.2.2	Existence Condition with Design Parameters Consider- ation . . . . .	169
8.2.3	Selection of Sliding Coefficients . . . . .	171
8.2.4	Implementation of Controller . . . . .	174
8.2.4.1	Design Procedure . . . . .	174
8.2.4.2	Parameters of Controllers . . . . .	176
8.2.4.3	Parameters of 10 kHz Bandwidth Controller	177
8.2.4.4	Parameters of 20 kHz Bandwidth Controller	177
8.2.4.5	Parameters of Adaptive FeedForward Ramp Generator . . . . .	178
8.2.5	Results and Discussions . . . . .	178
8.2.5.1	Steady-State Performance . . . . .	178
8.2.5.2	Load Variation Analysis . . . . .	178
8.2.5.3	Line Variation Analysis . . . . .	182
8.2.5.4	Dynamic Performance . . . . .	188
8.2.5.5	A Comparison with Classical PWM Voltage- Mode Controller . . . . .	189
8.3	PWM-Based SM Voltage Controller for Boost Converters . .	191
8.3.1	Mathematical Model . . . . .	191
8.3.2	Implementation of Controller . . . . .	192
8.3.2.1	Control Signal Computation . . . . .	192
8.3.2.2	Bandwidth of Ramp Voltage Generator . . .	192
8.3.2.3	Duty-Ratio Protection . . . . .	192
8.3.3	Experimental Prototype . . . . .	193
8.3.4	Experimental Results and Discussions . . . . .	194
8.3.4.1	Measured Signals . . . . .	194
8.3.4.2	Ensuring Duty-Ratio Protection . . . . .	194
8.3.4.3	Testing of Variable Ramp Signal Generation	195
8.3.4.4	Control Signals at Different Input Voltage . .	196
8.3.4.5	Regulation Performance . . . . .	198
8.3.4.6	Performance Comparison with Peak Current- Mode Controller . . . . .	199
8.3.4.7	Operation in Discontinuous Conduction Mode	202

<b>9</b>	<b>Sliding Mode Control with a Current Controlled Sliding Manifold</b>	<b>205</b>
9.1	Introduction . . . . .	205
9.2	The Need for Current-Mode Control in Boost-Type Converters	206
9.3	Sliding Mode Current Controller . . . . .	207
9.3.1	Generating a Suitable Reference Current Profile . . .	207
9.3.2	Sliding Surface . . . . .	208
9.3.3	Dynamical Model of Controller/Converter System and Its Equivalent Control . . . . .	208
9.3.4	Architecture of Controller . . . . .	209
9.3.5	Existence Condition . . . . .	211
9.3.6	Stability Condition . . . . .	211
9.3.6.1	Ideal Sliding Dynamics . . . . .	212
9.3.6.2	Equilibrium Point Analysis . . . . .	213
9.3.6.3	Linearization of Ideal Sliding Dynamics . . .	213
9.3.7	An Empirical Approach of Selecting the Sliding Coefficients . . . . .	214
9.3.8	Additional Remarks . . . . .	216
9.4	Results and Discussions . . . . .	216
9.4.1	Regulation Performance . . . . .	217
9.4.2	Dynamic Performance . . . . .	219
<b>10</b>	<b>Sliding Mode Control with a Reduced-State Sliding Manifold for High-Order Converters</b>	<b>223</b>
10.1	Introduction . . . . .	223
10.2	Review of Conventional Sliding Mode Controllers for Ćuk Converters . . . . .	224
10.2.1	State-Space Model of Ćuk Converter . . . . .	224
10.2.2	Full-State SM Controller . . . . .	225
10.2.3	Reduced-State SM Controller . . . . .	225
10.2.3.1	Voltage-Mode Control SM Controller . . . .	225
10.2.3.2	Current-Mode Control SM Controller . . . .	226
10.3	Constant-Frequency Reduced-State Sliding Mode Current Controller . . . . .	226
10.3.1	Sliding Surface . . . . .	227
10.3.2	Dynamical Model of Controller/Converter System and Its Equivalent Control . . . . .	227
10.3.3	Architecture of Controller . . . . .	228
10.3.4	Existence Condition . . . . .	229
10.3.5	Stability Condition . . . . .	230
10.3.5.1	Ideal Sliding Dynamics . . . . .	230
10.3.5.2	Equilibrium Point . . . . .	231
10.3.5.3	Linearization of Ideal Sliding Dynamics . . .	231
10.3.6	Selection of Sliding Coefficients . . . . .	235
10.3.7	Further Comments . . . . .	235

10.4 Results and Discussion . . . . .	235
10.4.1 Steady-State Performance . . . . .	236
10.4.2 Dynamic Performance . . . . .	238
<b>11 Indirect Sliding Mode Control with Double Integral Sliding Surface</b>	<b>241</b>
11.1 Introduction . . . . .	241
11.2 Problem Identification . . . . .	242
11.2.1 Review of Hysteresis-Modulation-Based Sliding Mode Controllers . . . . .	242
11.2.2 Review of Indirect Sliding Mode Controllers . . . . .	244
11.2.3 Analytical Explanation for the Presence of Steady-State Error in Indirect ISM-Controlled Converter . . . . .	245
11.3 A Possible Solution . . . . .	246
11.4 Application of Double-Integral Sliding Surface to PWM-Based Types of Indirect Sliding Mode Controllers . . . . .	247
11.4.1 Double-Integral Sliding Mode Controllers . . . . .	247
11.4.2 Architecture of DISM Controllers in PWM Form . . . . .	249
11.4.3 Existence Condition . . . . .	251
11.4.4 Stability Condition . . . . .	253
11.4.4.1 Ideal Sliding Dynamics . . . . .	255
11.4.4.2 Equilibrium Point . . . . .	255
11.4.4.3 Linearization of Ideal Sliding Dynamics . . . . .	256
11.5 Results and Discussions . . . . .	257
11.5.1 Simulation Result of PWM-Based DISM Buck Converter . . . . .	257
11.5.2 Experimental Result of PWM-Based DISM Boost Converter . . . . .	259
<b>Bibliography</b>	<b>267</b>
<b>Index</b>	<b>279</b>

---

# *Introduction to Sliding Mode Control*

---

## CONTENTS

1.1	Introduction .....	1
1.2	General Theory .....	2
1.3	Properties of Sliding Motion .....	4
1.3.1	An Ideal Control .....	4
1.3.2	Practical Limitations and Chattering .....	5
1.3.3	Constant Dynamics .....	5
1.3.4	Quasi-Sliding Mode Control .....	7
1.4	Mathematical Formulation .....	7
1.4.1	Hitting Condition .....	8
1.4.2	Existence Condition .....	9
1.4.3	Stability Condition .....	9
1.4.4	System with Linear Sliding Manifold .....	10
1.4.5	System with Nonlinear Sliding Manifold .....	11
1.4.5.1	Ideal Sliding Dynamics .....	11
1.4.5.2	Equilibrium Point .....	11
1.4.5.3	Linearization of Ideal Sliding Dynamics .....	11
1.4.5.4	Remarks .....	12
1.5	Equivalent Control .....	13
1.6	Types of Implementation .....	14
1.6.1	Relay and Signum Functions .....	14
1.6.2	Hysteresis Function .....	15
1.6.3	Equivalent Control Function .....	16

---

## 1.1 Introduction

The earliest published works that introduced the concept of sliding mode (SM) control can be traced back to the 1930s, and the earliest forms of SM control realization were successfully applied for ship-course control and the control of DC generators [102]. In fact, the development of the theory and applications of SM control were first initiated by Russian engineers, and in the 1950s, the theoretical framework that later facilitated the widespread applications of SM control was reported in the Russian literature [21, 101, 102]. The work was subsequently disseminated outside Russia in English written manuscripts by Itkis (1976) and Utkin (1977) [21]. Since then, the SM control theory has

aroused a lot of interests from control theoreticians and practicing engineers around the world.

In simplest terms, the SM control is a kind of nonlinear control which has been developed primarily for the control of variable structure systems [21, 68, 85, 101, 102]. Technically, it consists of a time-varying state-feedback discontinuous control law that switches at a high frequency from one continuous structure to another according to the present position of the state variables in the state space, the objective being to force the dynamics of the system under control to follow exactly what is desired and pre-determined.

The main advantage of a system with SM control characteristics is that it has guaranteed stability and robustness against parameter uncertainties [102]. Moreover, being a control method that has a high degree of flexibility in its design choices, the SM control method is relatively easy to implement as compared to other nonlinear control methods. Such properties make SM control highly suitable for applications in nonlinear systems, accounting for their wide utilization in industrial applications, e.g., electrical drivers, automotive control, furnace control, etc. [21].

In this book, we are concerned with a particular class of variable structure engineering systems, known as power electronics converters. The aim of this introductory chapter is to introduce the basic concepts and mathematical background of SM control that are necessary for understanding the discussions covered in the following chapters.

---

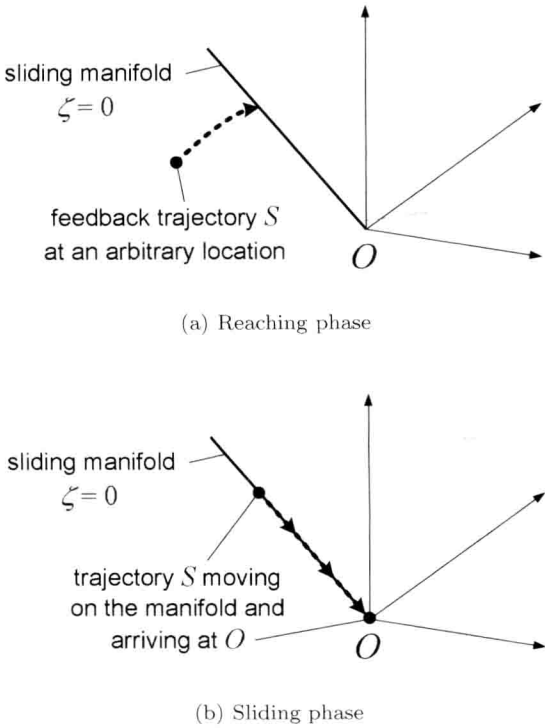
## 1.2 General Theory

Let us start by considering a system in a three-dimensional space. Imagine that there exists a plane in this space. On this plane, there is a point  $O$ , which we call the equilibrium point. This equilibrium point represents a stable attractor where any trajectory that touches it will settle upon it. It is also a point which we would like to drive the trajectory of our system to.

Next, we consider that the system's controlled trajectory is arbitrarily located in space and is far away from the plane. Without any control action, the trajectory will move according to the natural characteristics of the system. However, when a control action is given, the trajectory can be altered in a "preferred way." The direction in which the trajectory moves is dependent on the type of control action given. A series of different control actions may be given to the system such that regardless of its initial condition, the controlled trajectory will first move toward the plane, and upon reaching the plane, will slide along the plane toward and eventually settle upon  $O$ .

A control such as this is known as SM control. The plane which guides the trajectory is called the sliding plane or sliding surface, or more generally, the sliding manifold. The control actions required for performing the SM control





**FIGURE 1.1**

Graphical representations of SM control process: (a) Reaching phase—illustrating trajectory  $S$  moving toward the sliding manifold irrespective of its initial condition; and (b) Sliding phase—illustrating trajectory  $S$  moving on the sliding manifold and stopping at the origin  $O$ .

will involve very fast switching between different control functions. The sectors of the space in which the trajectory can be made to perform SM control is called the sliding regimes.

With all these terminologies explained, let us put the definition of SM control in a more formal sense. For any given system, if a sliding regime exists and the sliding manifold  $\zeta = 0$  possesses a stable equilibrium point  $O$ , when operated in sliding mode, the feedback tracking trajectory  $S$ , regardless of its location, will be driven toward the sliding manifold, and upon hitting the manifold, it will induce the control of the system to switch alternately between two or more discrete control functions  $U_1, U_2, \dots$ , etc., at an infinite frequency, such that the system's trajectory will be trapped precisely on the sliding manifold such that  $S = \zeta = 0$ , and eventually the trajectory will be directed toward the desired equilibrium point  $O$ .