Stability of Structures

Principles and Applications

Chai H. Yoo
Sung C. Lee

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CHAI H. YOO
Auburn University

SUNG C. LEE
Dongguk University

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PREFACE

The subject of this book is the stability of structures subjected to external loading that induces compressive stresses in the body of the structures. The structural elements examined are beams, columns, beam-columns, frames, rectangular plates, circular plates, cylindrical shells, and general shells. Emphasis is on understanding the behavior of structures in terms of load-displacement characteristics; on formulation of the governing equations; and on calculation of the critical load.

Buckling is essentially flexural behavior. Therefore, it is imperative to examine the condition of equilibrium in a flexurally deformed configuration (adjacent equilibrium position). The governing stability equations are derived by both the equilibrium method and the energy method based on the calculus of variations invoking the Trefftz criterion.

Stability analysis is a topic that fundamentally belongs to nonlinear analysis. The fact that the eigenvalue procedure in modern matrix and/or finite element analysis is a fortuitous by-product of incremental nonlinear analysis is a reaffirming testimony. The modern emphasis on fast-track education designed to limit the number of required credit hours for core courses in curriculums left many budding practicing structural analysts with gaping gaps in their understanding of the theory of elastic stability. Many advanced works on structural stability describe clearly the fundamental aspects of general nonlinear structural analysis. We believe there is a need for an introductory textbook such as this, which will present the fundamentals of structural stability analysis within the context of elementary nonlinear flexural analysis. It is believed that a firm grasp of these fundamentals and principles is essential to performing the important interpretation required of analysts when computer solutions are adopted.

The book has been planned for a two-semester course. The first chapter introduces the buckling of columns. It begins with the linear elastic theory and proceeds to include the effects of large deformations and inelastic behavior. In Chapter 2 various approximate methods are illustrated along with the fundamentals of energy methods. The chapter concludes by introducing several special topics, some of them advanced, that are useful in understanding the physical resistance mechanisms and consistent and rigorous mathematical analysis. Chapters 3 and 4 cover buckling of beam-columns. Chapter 5 presents torsion in structures in some detail, which is one of the least-well-understood subjects in the entire spectrum of structural mechanics. Strictly speaking, torsion itself does not belong to a work on structural stability, but it needs to be covered to some extent if one is to have a better understanding of buckling accompanied with torsional behavior. Chapters 6 and 7 consider stability of framed structures in conjunction with torsional behavior of structures. Chapters 8 to 10 consider buckling of plate elements, cylindrical shells, and general shells. Although the book is devoted primarily to analysis, rudimentary design aspects are also discussed.

The reader is assumed to have a good foundation in elementary mechanics of deformable bodies, college-level calculus, and analytic geometry, and some exposure to differential equations. The book is designed to be a textbook for advanced seniors and/or first-year graduate students in aerospace, civil, mechanical, engineering mechanics, and possibly naval architects and shipbuilding fields and as a reference book for practicing structural engineers.

Needless to say, we have relied heavily on previously published work. Consequently, we have tried to be meticulous in citing the works and hope that we have not erred on the side of omission.

AUTHORS BIOGRAPHY

Chai H. Yoo, PhD., PE., F.ASCE, is professor emeritus at Auburn University. He has over 40 years of teaching, research, and consulting experience. He received his BS from Seoul National University in Korea and his MS and PhD degrees from the University of Maryland. He is the recipient of the 2008 ASCE Shortridge Hardesty Award for his outstanding research and practical engineering efforts dealing with the strength and stability of thin-walled sections, especially as it applies to bridge girders. Author of over 75 refereed journal papers and a book on curved steel girder bridges, Dr. Yoo has been active on numerous ASCE and SSRC technical committee activities. He was the principal investigator/project director of NCHRP Project 12-38 that produced AASHTO Guide Specifications for Horizontally Curved Steel Girder Highway Bridges with Design Examples for I-Girder and Box-Girder Bridges.

Sung C. Lee, PhD., M.ASCE, is professor and head of the Department of Civil and Environmental Engineering at Dongguk University in Seoul, Korea. He received his BS from Seoul National University in Korea, his MS from Oregon State University, and his PhD from Auburn University. He is the author of over 100 refereed journal articles, technical papers, and proceedings. Dr. Lee's breakthrough research on the strength and stability of web panels in plate- and box-girders renders a correct understanding on a topic that has been controversial over the last five decades.

CONTENTS

CHAPTER 1

Buckling of Columns

Contents

1.1. INTRODUCTION

A physical phenomenon of a reasonably straight, slender member (or body) bending laterally (usually abruptly) from its longitudinal position due to compression is referred to as buckling. The term *buckling* is used by engineers as well as laypeople without thinking too deeply. A careful examination reveals that there are two kinds of buckling: (1) bifurcation-type buckling; and (2) deflection-amplification-type buckling. In fact, most, if not all, buckling phenomena in the real-life situation are the deflection-amplification type. A bifurcation-type buckling is a purely conceptual one that occurs in a perfectly straight (geometry) homogeneous (material) member subjected to a compressive loading of which the resultant must pass

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though the centroidal axis of the member (concentric loading). It is highly unlikely that any ordinary column will meet these three conditions perfectly. Hence, it is highly unlikely that anyone has ever witnessed a bifurcation-type buckling phenomenon. Although, in a laboratory setting, one could demonstrate setting a deflection-amplification-type buckling action that is extremely close to the bifurcation-type buckling. Simulating those three conditions perfectly even in a laboratory environment is not probable.

Structural members resisting tension, shear, torsion, or even short stocky columns fail when the stress in the member reaches a certain limiting strength of the material. Therefore, once the limiting strength of material is known, it is a relatively simple matter to determine the load-carrying capacity of the member. Buckling, both the bifurcation and the deflection-amplification type, does not take place as a result of the resisting stress reaching a limiting strength of the material. The stress at which buckling occurs depends on a variety of factors ranging from the dimensions of the member to the boundary conditions to the properties of the material of the member. Determining the buckling stress is a fairly complex undertaking.

If buckling does not take place because certain strength of the material is exceeded, then, why, one may ask, does a compression member buckle? Chajes (1974) gives credit to Salvadori and Heller (1963) for clearly elucidating the phenomenon of buckling, a question not so easily and directly explainable, by quoting the following from *Structure in Architecture*:

> *A slender column shortens when compressed by a weight applied to its top, and, in so doing, lowers the weight's position. The tendency of all weights to lower their position is a basic law of nature. It is another basic law of nature that, whenever there is a choice between different paths, a physical phenomenon will follow the easiest path. Confronted with the choice of bending out or shortening, the column finds it easier to shorten for relatively small loads and to bend out for relatively large loads. In other words, when the load reaches its buckling value the column finds it easier to lower the load by bending than by shortening.*

Although these remarks will seem excellent to most laypeople, they do contain nontechnical terms such as choice, easier, and easiest, flavoring the subjective nature. It will be proved later that buckling is a phenomenon that can be explained with fundamental natural principles.

If bifurcation-type buckling does not take place because the aforementioned three conditions are not likely to be simulated, then why, one may ask, has so much research effort been devoted to study of this phenomenon? The bifurcation-type buckling load, the critical load, gives the upper-bound solution for practical columns that hardly satisfies any one of the three conditions. This will be shown later by examining the behavior of an eccentrically loaded cantilever column.

1.2. NEUTRAL EQUILIBRIUM

The concept of the stability of various forms of equilibrium of a compressed bar is frequently explained by considering the equilibrium of a ball (rigid-body) in various positions, as shown in Fig. 1-1 (Timoshenko and Gere 1961; Hoff 1956).

Although the ball is in equilibrium in each position shown, a close examination reveals that there are important differences among the three cases. If the ball in part (a) is displaced slightly from its original position of equilibrium, it will return to that position upon the removal of the disturbing force. A body that behaves in this manner is said to be in a state of stable equilibrium. In part (a), any slight displacement of the ball from its position of equilibrium will raise the center of gravity. A certain amount of work is required to produce such a displacement. The ball in part (b), if it is disturbed slightly from its position of equilibrium, does not return but continues to move down from the original equilibrium position. The equilibrium of the ball in part (b) is called unstable equilibrium. In part (b), any slight displacement from the position of equilibrium will lower the center of gravity of the ball and consequently will decrease the potential energy of the ball. Thus in the case of stable equilibrium, the energy of the system is a minimum (local), and in the case of unstable equilibrium it is a maximum (local). The ball in part (c), after being displaced slightly, neither returns to its original equilibrium position nor continues to move away upon removal of the disturbing force. This type of equilibrium is called neutral equilibrium. If the equilibrium is neutral, there is no change in energy during a displacement in the conservative force system. The response of the column is very similar to that of the ball in Fig. 1-1. The straight configuration of the column is stable at small loads, but it is unstable at large loads. It is assumed that a state of neutral equilibrium exists at the

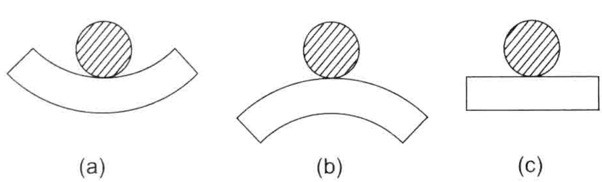

Figure 1-1 Stability of equilibrium

transition from stable to unstable equilibrium in the column. Then the load at which the straight configuration of the column ceases to be stable is the load at which neutral equilibrium is possible. This load is usually referred to as the critical load.

To determine the critical load, eigenvalue, of a column, one must find the load under which the member can be in equilibrium, both in the straight and in a slightly bent configuration. How slightly? The magnitude of the slightly bent configuration is indeterminate. It is conceptual. This is why the free body of a column must be drawn in a slightly bent configuration. The method that bases this slightly bent configuration for evaluating the critical loads is called the method of neutral equilibrium (neighboring equilibrium, or adjacent equilibrium).

At critical loads, the primary equilibrium path (stable equilibrium, vertical) reaches a bifurcation point and branches into neutral equilibrium paths (horizontal). This type of behavior is called the buckling of bifurcation type.

1.3. EULER LOAD

It is informative to begin the formulation of the column equation with a much idealized model, the Euler[1] column. The axially loaded member shown in Fig. 1-2 is assumed to be prismatic (constant cross-sectional area) and to be made of homogeneous material. In addition, the following further assumptions are made:

1. The member's ends are pinned. The lower end is attached to an immovable hinge, and the upper end is supported in such a way that it can rotate freely and move vertically, but not horizontally.
2. The member is perfectly straight, and the load P, considered positive when it causes compression, is concentric.
3. The material obeys Hooke's law.
4. The deformations of the member are small so that the term $(y')^2$ is negligible compared to unity in the expression for the curvature, $y''/[1+(y')^2]^{3/2}$. Therefore, the curvature can be approximated by y''.[2]

[1] The Euler (1707–1783) column is due to the man who, in 1744, presented the first accurate column analysis. A brief biography of this remarkable man is given by Timoshenko (1953). Although it is customary today to refer to a simply supported column as an Euler column, Euler in fact analyzed a flag-pole-type cantilever column in his famous treatise according to Chajes (1974).

[2] y' and y'' denote the first and second derivatives of y with respect to x. Note: $\lfloor y''\rfloor < \lfloor y'\rfloor$ but $\lfloor y'\rfloor \approx$ thousandths of a radian in elastic columns.

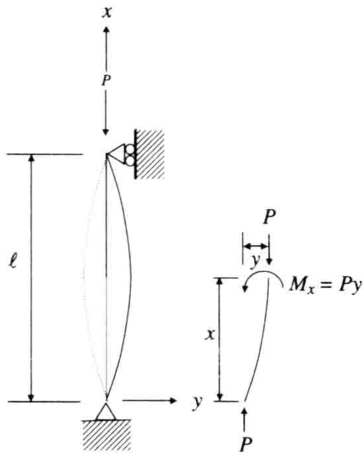

Figure 1-2 Pin-ended simple column

$$-\frac{M}{EI} = \frac{y''}{[1 + (y')^2]^{3/2}} \simeq y'' \tag{1.3.1}$$

From the free body, part (b) in Fig. 1-2, the following becomes immediately obvious:

$$EIy'' = -M(x) = -Py \quad \text{or} \quad EIy'' + Py = 0 \tag{1.3.2}$$

Equation (1.3.2) is a second-order linear differential equation with constant coefficients. Its boundary conditions are

$$y = 0 \quad \text{at } x = 0 \quad \text{and} \quad x = \ell \tag{1.3.3}$$

Equations (1.3.2) and (1.3.3) define a linear eigenvalue problem. The solution of Eq. (1.3.2) will now be obtained. Let $k^2 = P/EI$, then $y'' + k^2y = 0$. Assume the solution to be of a form $y = \alpha e^{mx}$ for which $y' = \alpha m e^{mx}$ *and* $y'' = \alpha m^2 e^{mx}$. Substituting these into Eq. (1.3.2) yields $(m^2 + k^2)\alpha e^{mx} = 0$.

Since αe^{mx} cannot be equal to zero for a nontrivial solution, $m^2 + k^2 = 0$, $m = \pm ki$. Substituting gives

$$y = C_1 \alpha e^{kix} + C_2 \alpha e^{-kix} = A \cos kx + B \sin kx$$

A and B are integral constants, and they can be determined by boundary conditions.

$$y = 0 \quad at\ x = 0 \Rightarrow A = 0$$

$$y = 0 \quad \text{at } x = \ell \Rightarrow B \sin k\ell = 0$$

As $B \neq 0$ (if $B = 0$, then it is called a trivial solution; $0 = 0$), $\sin k\ell = 0 \Rightarrow k\ell = n\pi$
where $n = 1, 2, 3, \ldots$ but $n \neq 0$. Hence, $k^2 = P/EI = n^2\pi^2/\ell^2$, from which it follows immediately

$$P_{cr} = \frac{n^2\pi^2 EI}{\ell^2} \quad (n = 1, 2, 3, ..) \tag{1.3.4}$$

The eigenvalues P_{cr}, called critical loads, denote the values of load P for which a nonzero deflection of the perfect column is possible. The deflection shapes at critical loads, representing the eigenmodes or eigenvectors, are given by

$$y = B \sin \frac{n\pi x}{\ell} \tag{1.3.5}$$

Note that B is undetermined, including its sign; that is, the column may buckle in any direction. Hence, the magnitude of the buckling mode shape cannot be determined, which is said to be immaterial.

The smallest buckling load for a pinned prismatic column corresponding to $n = 1$ is

$$P_E = \frac{\pi^2 EI}{\ell^2} \tag{1.3.6}$$

If a pinned prismatic column of length ℓ is going to buckle, it will buckle at $n = 1$ unless external bracings are provided in between the two ends.

A curve of the applied load versus the deflection at a point in a structure such as that shown in part (a) of Fig. 1-3 is called the equilibrium path. Points along the primary (initial) path (vertical) represent configurations of the column in the compressed but straight shape; those along the secondary path (horizontal) represent bent configurations. Equation (1.3.4) determines a periodic bifurcation point, and Eq. (1.3.5) represents a secondary (adjacent or neighboring) equilibrium path for each value of n. On the basis of Eq. (1.3.5), the secondary path extends indefinitely in the horizontal direction. In reality, however, the deflection cannot be so large and yet satisfies the assumption of rotations to be negligibly small. As P in Eq. (1.3.4) is not a function of y, the secondary path is horizontal. A finite displacement formulation to be discussed later shows that the secondary equilibrium path for the column curves upward and has a horizontal tangent at the critical load.