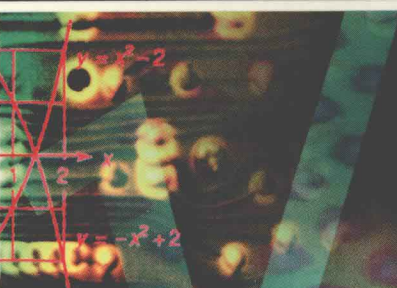
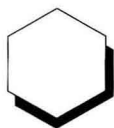


MASTERING MATHEMATICS FOR ELECTRICAL AND ELECTRONIC ENGINEERING

NOEL M. MORRIS



Mastering



Mathematics for Electrical and Electronic Engineering

Noel M. Morris

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MACMILLAN

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First published 1994 by
THE MACMILLAN PRESS LTD
Houndmills, Basingstoke, Hampshire RG21 2XS
and London
Companies and representatives
throughout the world

ISBN 0-333-59359-6

A catalogue record for this book is available
from the British Library.

Copy-edited and typeset by Povey-Edmondson
Okehampton and Rochdale, England

Printed in China

10	9	8	7	6	5	4	3	2	1
03	02	01	00	99	98	97	96	95	94

Mastering

Mathematics for Electrical and
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To Laura and Alex



Preface

Mathematics is an essential tool in the armoury of electrical and electrical technicians and engineers and, in particular, there are certain branches of mathematics which have special relevance to them. This book covers the general background of mathematics, and places special emphasis on the needs of electrical and electronic engineering.

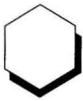
The book attends to the requirements not only of students studying on their own, but also those following BTEC ONC, OND, HNC and HND courses, and will be specially appropriate to A-level students intending to transfer to degree and other courses.

As well as dealing with the essential mathematics of electrical and electronic courses, the book contains a large number of worked examples, which help the reader to explore the subject to greater depth. At the end of each chapter you will find not only many self-test questions (with answers at a later stage), which will help you test your grasp of the subject matter, but also a summary of important facts included in the chapter.

A 'plus' feature of the book is the chapter on SPICE software (Simulation Program with Integrated Circuit Emphasis), which is of special relevance to the solution of all types of circuits. This software is readily available, and a wide range of circuit solutions are described in chapter 16. I am especially indebted to Mr P. Goss, Technical Manager of ARS Microsystems, for his advice and assistance in the matter.

I would like to thank my wife for her support, without which the writing of this book would not have been possible.

NOEL M. MORRIS



How to use this book

Mathematics forms the backbone of Electrical and Electronic Engineering courses, and this book will take the reader through a significant part of his or her education.

The chapters of the book are listed below, together with the type of course for which they are best suited. The symbol **A** designates those suitable for science-based A-level courses, **N** designates those of particular value to BTEC ONC/D courses, and **H** signifies those of interest to BTEC HNC/D courses.

Chapter

- | | |
|----|---|
| 1 | Fractions, roots and powers – A, N |
| 2 | Numbers and numbering systems – A, N, H |
| 3 | Logarithms, the decibel and the Neper – A, N, H |
| 4 | Algebra – A, N, H |
| 5 | Simultaneous equations – A, N, H |
| 6 | Trigonometry – A, N, H |
| 7 | Further trigonometric skills – A, N, H |
| 8 | Mensuration – A, N |
| 9 | Graphs – A, N, H |
| 10 | Vectors and phasors – A, N, H |
| 11 | Complex numbers – A, N, H |
| 12 | Differentiation – A, N, H |
| 13 | Integration – A, N, H |
| 14 | Transients in electrical circuits – A, H |
| 15 | Boolean algebra and logic circuits – A, N, H |
| 16 | Computer solution of electric circuits – A, N, H |

Chapter 16 includes details of one of the most important computer packages for use with electronic and electrical circuit analysis, namely **SPICE** (Simulation Program with Integrated Circuit Emphasis). There is, however, a wide range of computer packages available for the solution of what otherwise can be very complex problems. For example, **DERIVE** and **MATHCAD** are very suitable for the solution of mathematical problems. This type of software is likely to revolutionise the teaching of mathematics.

Most software is available through software houses which advertise in computer magazines. A wide range of very low cost mathematical and scientific software is also available through **SHAREWARE** suppliers, which are also advertised in computer magazines.



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