RATIONAL CHOICE THEORY

CRITICAL CONCEPTS IN THE SOCIAL SCIENCES

Edited by
MICHAEL ALLINGHAM

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Critical Concepts in the Social Sciences

Edited by
Michael Allingham

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THE GENERAL IRRELEVANCE OF THE GENERAL IMPOSSIBILITY THEOREM

Gordon Tullock

Source: *Quarterly Journal of Economics* 81 (1967): 256–70.

A phantom has stalked the classrooms and seminars of economics and political science for nearly fifteen years. This phantom, Arrow's General Impossibility Theorem, has been generally interpreted as proving that no sensible method of aggregating preferences exists.[1] The purpose of this essay is to exorcise the phantom, not by disproving the theorem in its strict mathematical form, but by showing that it is insubstantial. I shall show that when a rather simple and probable type of interdependence is assumed among the preference functions of the choosing individuals, the problem becomes trivial if the number of voters is large.[2] Since most cases which require aggregation of preferences involve large numbers of people, "Arrow problems" will seldom be of much importance.

In *Social Choice and Individual Values*[3] Arrow included a chapter on "Similarity as the Basis of Social Welfare Judgments"[4] in which he discussed possible lines of research which might lead to a method of avoiding the implications of his proof. In this chapter, he pointed to Black's single-peaked preference curves as particularly promising.[5] The generalization of Black's single-peaked curves to more than one dimension will give the fundamental model upon which this article is based. It may, therefore, fairly be said that the present work follows the path indicated by Arrow. The development of single-peaked preferences for two dimensions was first undertaken by Newing and Black in *Committee Decisions with Complementary Valuation*,[6] which was published at about the same time as *Social Choice and Individual Values* and presumably was not known to Arrow. Newing and Black, however, did not give much consideration to cases in which there were large numbers of voters. The model to be used here will involve many voters and will be used to examine the general impossibility theorem.

The proof of Arrow's theorem requires, as one of its steps, the cyclical majority or paradox of voting.[7] In addition to the mathematical reasons, the emphasis on the paradox is appropriate since the method of "aggregating preferences" which immediately occurs to the average citizen of a democracy is majority voting. This article is intended to demonstrate that majority voting will, indeed, always be subject to the paradox of voting, but that this is of very little importance. Majority voting will not produce a "perfect" answer, but the answer it does produce will not be significantly "worse" than if the paradox of voting did not exist. Any choice process involving large numbers of people will surely be subject to innumerable minor defects with the result that the outcome, if considered in sufficient detail, will always deviate from Arrow's conditions. The deviation may, however, be so small that it makes no practical difference.

Most majority voting procedures have arrangements which bring the voting to an end before every tiny detail of the proposal has been subject to a vote. These rules (frequently informal rather than part of the rules of order) mean that when the voting is brought to a stop there almost certainly remain minor changes in the result which a majority would approve if it were possible to bring them to a vote. Thus the outcome will be, in Arrow's terms, imposed, but it will be very close to a perfect result. As an example, suppose a body of men is voting on the amount of money to be spent on something, with the range under consideration running from zero to $10,000,000. The preferences of these men are single-peaked. Majority voting will eventually lead to the selection of the optimum of the median voter as the outcome. If, however, the procedure is such that proposals to change the amount of money by $100 or less cannot be entertained,[8] then the outcome will normally not be at the optimum, but it will be within $100 of it. This result does not meet Arrow's conditions, but there is no reason to be disturbed by this fact.

In order to demonstrate that the cyclical majority is equally unimportant in real world "preference aggregation," let us consider a group of voters deciding two matters, say appropriations for the Army and the Navy by majority voting. In Figure I the vertical dimension is the appropriation for the Army and the horizontal for the Navy. The individual voters each have an optimum combination and a preference mountain which has the usual characteristics. For simplicity let us further assume that the voter's optima are evenly distributed over the space, and that their indifference curves are all perfect circles centering on their optima. The last two assumptions do not correspond with reality and will be eliminated at a later stage. Let us further assume that the number of voters is great enough so that the space can serve as a proxy for the voters. Putting it differently, of two areas in the issue space of Figure I, the larger will contain the optima of more voters than will the smaller. This makes it possible to use simple Euclidian geometry as an analytical tool.

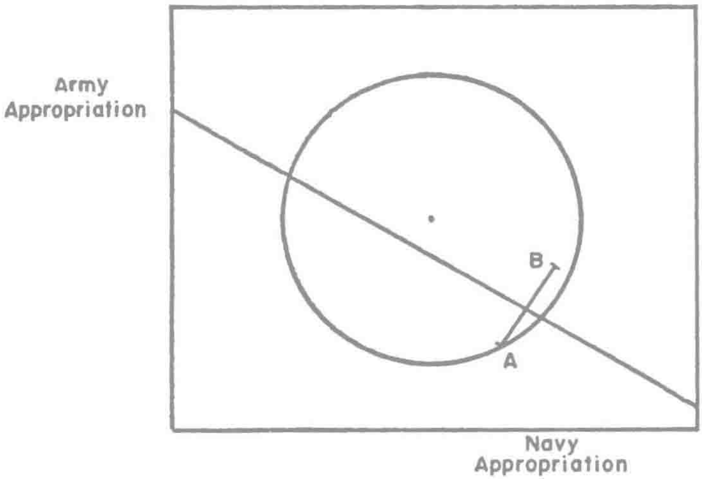

Figure I

Suppose we wish to determine whether motion *B* on Figure I can defeat the status quo, represented by *A*, by a simple majority vote. Since we are assuming that all indifference curves are perfect circles around the individuals optimum, each voter will simply vote for the alternative which is closest to his optimum. If we connect *A* and *B* with a straight line and erect a perpendicular bisector on this line, then *B* will be closer to the optima of all individuals whose optima lie on the same side of the bisector as *B* and *A* will similarly be closest to all optima which lie on *A*'s side of the bisector. We can compare the votes for each alternative by simply noting the area of the rectangle on each side of the bisector. As a shorthand method, if the perpendicular bisector runs through the center of the rectangle there will be an equal number of votes for *A* and *B*. If it does not, then the alternative on the same side of the perpendicular bisector as the center will win. The locus of all points which will tie with *A* is a circle around the center running through *A*, and *A* can beat any point outside the circle but will be beaten by any point inside. Clearly no cycles are possible. The process will lead into the center eventually, since, of any pair of alternatives, the one closer to the center will always win.

This might be called the perfect geometrical model, in which the number of voters whose optima fall in a given area is exactly proportional to its area. Given that the voters are finite in number, small discontinuities would appear. Two areas that differ little in size might have the same number of voters; indeed, the smaller might even have more. Cycles are, therefore, possible, but they would become less and less important as the number of choosing individuals increases. In Figure II we have a point, *A*, in our standard type of issue space, and I have drawn a circle around it. For

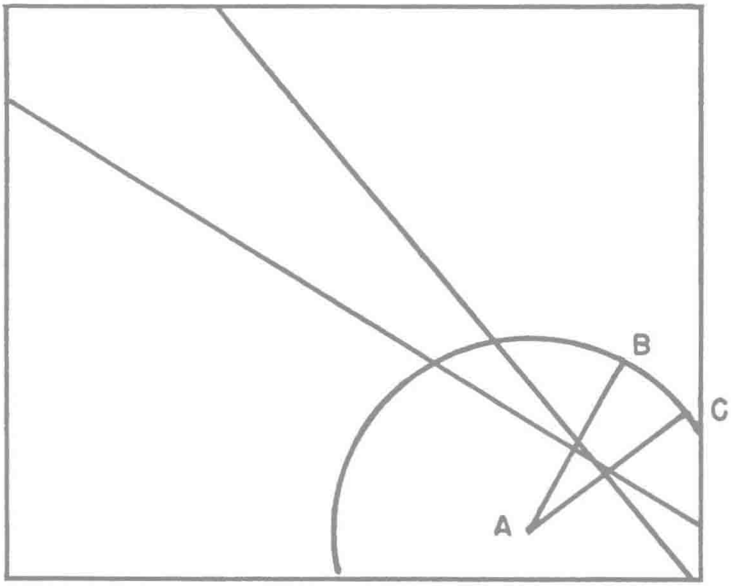

Figure II

convenience I shall assume 999,999 voters. Whether any given point on this circle can beat *A* depends upon how the perpendicular bisector of the line connecting it with *A* partitions the voters' optima. We can conceive ourselves as moving around the circle, trying each point on it against *A*. *B* would beat *A*, but *C* would not. With a finite number of voters, the changes between motions would be discontinuous. *B*, for example, might get 602,371 votes to *A*'s 397,628. As we moved around the circle towards *C* there would be a small space in which this vote would stay unchanged, then it would suddenly shift to 602,370 against 397,629, which would also persist for a short segment of the circle. Needless to say, the segment in which the vote did not change would be extremely small, but it would exist.

If we consider a point which gets only a bare majority over *A*, 500,000 votes to 499,999, and move along the circle towards *B*, there will be a finite gap during which the vote does not change, and then it will shift to 500,000 for *A* and 499,999 for the alternative.[9] Given this finite distance, however, it is sure that at least occasionally a point can be beaten by another point which is more distant from the center than it is. In other words, it will be possible for majority voting to move away from the center as well as to move towards it. This phenomenon makes cycles possible.[10]

Granting these discontinuities, however, we could still draw a line separating those points which could get a majority over any given point from those that could not. With our 999,999 voters this line would no doubt appear to be the circle of Figure I to the naked eye. Examining it through a microscope, however, we should find that it was not exactly circular and that there would be small areas which could get a majority over the original point, but which lay farther from the center than that point. Note, however, that these areas would be very small. If our original point is far from the center (as is *A* in either Figure I or Figure II), then the area which could get a majority over *A* but which lies farther from the center would be tiny compared with the area which could get a majority and which lay closer to the center.

Under these circumstances, unless proposals for changes are introduced in a very carefully controlled and planned manner, the voting process would in all probability lead to rapid movement toward the center.[11] Unfortunately the convergence need not continue until the absolute center is reached.[12] For close to the center, the area which is preferred to *A* and is closer to the center, is much smaller than initially. It is therefore more probable than at first that the preferred alternative to *A* would be farther from the center than *A*. Cycling becomes more probable. When we get very close to the center a point randomly selected from among those which could get a majority over the given point would have a good chance of being farther from the center than it is. At this point, however, most voters will feel that new proposals are splitting hairs, and the motion to adjourn will carry.

Discussion of the point is simplified by the use of a particular type of line which we will call a "median line." A median line is a line passing through two individuals' optima and dividing the remaining optima either into two equal "halves" or, if the number of optima is odd, into two groups one of which has one more optima than the other. Figure III shows one such line and a point, *A*, which is not on the line. If, from point *A*, we drop a perpendicular to the median line, then the point at the base of the perpendicular, *A*', will be closer to all the points on the other side of the line and the two points on the line than is *A*. It can, therefore, get a majority over *A*. Actually there would be a small lozenge, as in Figure III, outlining points which could get a majority over *A*. The geometry of this lozenge, however, will vary somewhat depending upon the exact location of the individual optima, so we will confine ourselves to the simple perpendicular relationship.

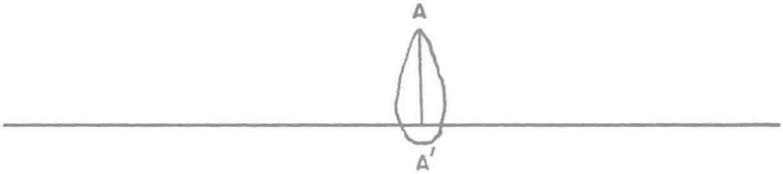

Figure III

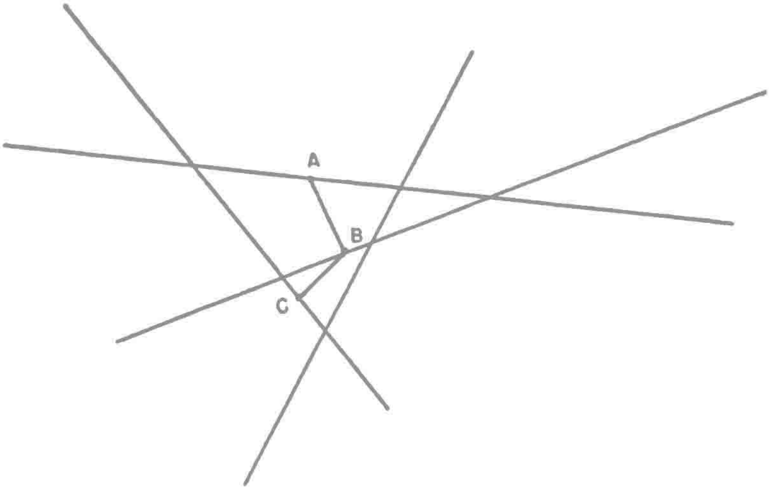

Figure IV

Most of these median lines would intersect in a tiny area in the center of the issue space. If we greatly magnified this area and drew in only a few of the median lines, we would get something which looked like Figure IV. If we start with point *A*, then our theorem indicates that *B* can get a majority over it. *C*, on the other hand, can get a majority over *B*. Similarly it is obvious that there would be other points which can get majorities over *C*. Starting with any point in this general area, it will be possible to select points which will obtain a majority over it. Thus, there is no point which can get a majority over all other points.

The area in which the bulk of the bisectors intersect is, of course, very small, but in some cases the point of intersection might be far away from the center of the issue space. Suppose that there are an odd number of points and we select one which is near the extreme outer edge of the issue space. It may be possible to draw through this point two lines each of which pass through another point, and each of which divide the optima so that there is only one more on one side of the line than on the other. The angle between these two lines would be extremely small, but by exaggerating this angle we get the situation shown in Figure V. Above this pair of lines there would be 499,999 optima and below the same number. The three points lying on the lines make up our total of 999,999. If we start at any point on either of these lines, such as *A*, we can drop a perpendicular to the other and thus obtain a point which can get a majority over the first point. From this second point, we can then drop a perpendicular to the first line and obtain a point which can get a majority over it. By continuing this process we can eventually approach the intersection point which, by assumption, lies at the outer edge of our space. Thus it is possible, by simple majority voting, to reach points

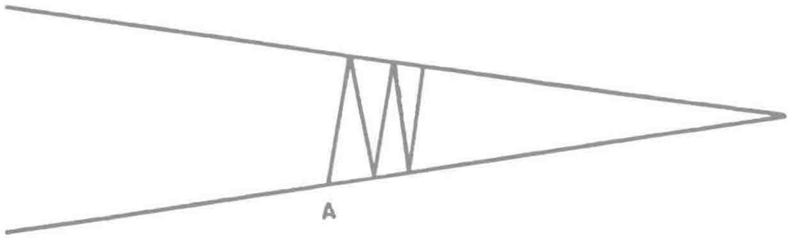

Figure V

at almost any portion of the issue space. Needless to say, this sort of series of votes is highly unlikely. It can be easily recognized because it would involve a long series of votes in each of which there was only a one-vote difference between the majority and the minority. Since we never see this in the real world, we can feel reasonably confident that this type of movement away from the center does not occur.

Since standard voting procedures do not permit infinitely fine adjustment, the fact that majority voting would not lead to a unique solution seems of very little importance. Black defines a "majority motion" as a proposition "which is able to obtain a simple majority over all of the other motions concerned."[13] The rules of procedure make it unlikely that such a motion will be selected by majority voting. The outcome should be a motion which could not get a simple majority over *all* other motions, but only over those other motions which differ enough so that they can be put against it under the procedural rules. The result is an approximation, but a reasonably satisfactory one. Thus if there is no true majority motion, if endless cycling were the predicted outcome of efforts to obtain perfect adjustment, this would not change the outcome at all if the cycles would only involve motions proposing such small changes that they could be ruled out of order. Even if the cycles slightly enlarged the area in which the voting system was indeterminate, this would be a trivial defect. Only if the cycles would involve "moves" substantially larger than the minimum permitted by the procedural system would they be a significant problem.

The investigation of the likely size of cycles in the real world can proceed by making assumptions about the distribution of voters and the rules of order and then calculating the likelihood of cycles among motions which differ enough so that they could be voted on, or by observing the real world.[14] There would seem to be two possible explanations for this paucity of examples of the phenomenon. Either it does not occur very commonly, which would be in accord with the theoretical considerations given above, or it is hard to detect the presence of cycles even when they are present.

In order to examine the possibility that the shortage of real world examples of cycles is explained not by their rarity, but by the difficulty of detecting them, let us consider the actual methods of voting used in most representative bodies. Under Robert's Rules, or the innumerable variants which exist, the procedure is quite complicated. We need not examine these rules in detail; a simplified generalization of them will suffice. Let us, therefore, examine the following system. A motion is made to move from the status quo. An amendment to this motion may then be proposed, and various subamendments to the amendment. All of the amendments and subamendments can be regarded as separate proposals. The distinguishing characteristic of this system is that a whole set of proposals is made before any of them are voted upon, and then that they are voted upon in a fixed order which is known in advance.

Suppose that the status quo is *A*. *B* is offered as a motion. *C* would be offered only if its sponsor thought it could beat both *A* and *B*. (Or, in special circumstances, if it might lead to a blocking cycle. This will be discussed below.) But people do make mistakes. Let us suppose, then, that someone in error offers amendment *C*. This is followed by subamendment *D*, which has been correctly calculated and can beat *A*, *B*, and *C*. For our purposes, we may use a simple set of rules providing that motions, amendments and subamendments are voted upon in reverse order from that in which they are proposed. Thus *D* would be put against *C*, would win; would be put against *B*, and would win; and then would be put against the status quo, *A*, and win again. Note that *C*, the miscalculation, has no effect on the voting except to delay it slightly. Such mistakes will certainly be made; hope springs eternal in the human breast, but they have no effect on the outcome. We can, therefore, ignore them.

Let us now consider the possibility of cycles. In Figure VI, we start with the status quo at *A*. Suppose that a motion, *B*, were introduced which could beat *A*. Either by accident, or by calculation, another motion *C*, might be introduced which could beat *B*, but would be beaten by *A*. Deliberate contrivance of such a cycle by people who prefer *A* to *B*, but realize that *B* would win in a direct confrontation would be rational. The existence of a cycle, of course, does not prevent other amendments from being offered. *D*, for example, could beat any member of the cycle.

If, however, *D* were not offered, the voting on *C*, *B*, and *A* would not immediately lead to an apparent cycle because *A* would be chosen if they were voted on in the order specified. This type of concealed cycle, however, should not lead to a stable result. Once the voting has led to a return to the status quo, further proposals for change would be strongly urged. If there were strict rules forbidding the reintroduction of a measure which had been voted down,[15] then some other proposal, say *B*', would be offered. We could expect to see essentially the same series of proposals and amendments retraced again and again. The absence of this kind of repetition in actual legislative practice is evidence that there are few concealed cycles in real world legislative activity.

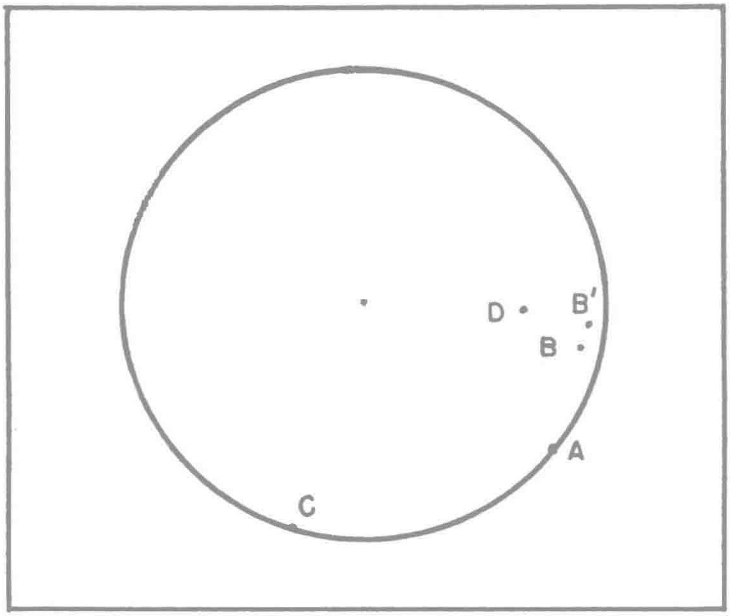

Figure VI

So far we have been mostly concerned with the situation when there are only two variables and they are continuous. The generalization of the conclusions to many-dimensional issue space is obvious, but the effect of shifting to a noncontinuous variable may not be. In Figure VII we show a situation where the two variables are both discontinuous, and hence only certain points in the issue space are possible. The principal difference which this restriction makes is that there are now far fewer points which can get a majority over some given point. Point *A*, for example, is dominated only by the six points marked with *x*'s. The likelihood that one of the points that dominate *A* is equally or farther distant from the center than *A*, is reduced when the total number of points is small, and the likelihood that there will be a set of points which are in a cycle is small. The tiny central area where every point is dominated by some other is apt to be nonexistent simply because there are too few points in this region. On the other hand, cycles are only unlikely, not impossible. If a cycle does occur, it is likely to be of more than trivial significance if the distances between the points are sizable and the cycle must, therefore, involve sizable differences of policy. In sum, with discontinuous issue space, cycles must be rare but are apt to be important when they occur.

Our discussion so far has been based upon a special type of interdependence of the preference structures of individuals. It is assumed that social states, products, and laws differ in a number of characteristics.[16] Each