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Anton Deitmar

A First Course in Harmonic Analysis

Second Edition

调和分析基础教程 第2版

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Preface to the second edition

This book is intended as a primer in harmonic analysis at the upper undergraduate or early graduate level. All central concepts of harmonic analysis are introduced without too much technical overload. For example, the book is based entirely on the Riemann integral instead of the more demanding Lebesgue integral. Furthermore, all topological questions are dealt with purely in the context of metric spaces. It is quite surprising that this works. Indeed, it turns out that the central concepts of this beautiful and useful theory can be explained using very little technical background.

The first aim of this book is to give a lean introduction to Fourier analysis, leading up to the Poisson summation formula. The second aim is to make the reader aware of the fact that both principal incarnations of Fourier theory, the Fourier series and the Fourier transform, are special cases of a more general theory arising in the context of locally compact abelian groups. The third goal of this book is to introduce the reader to the techniques used in harmonic analysis of noncommutative groups. These techniques are explained in the context of matrix groups as a principal example.

The first part of the book deals with Fourier analysis. Chapter 1 features a basic treatment of the theory of Fourier series, culminating in L^2-completeness. In the second chapter this result is reformulated in terms of Hilbert spaces, the basic theory of which is presented there. Chapter 3 deals with the Fourier transform, centering on the inversion theorem and the Plancherel theorem, and combines the theory of the Fourier series and the Fourier transform in the most useful Poisson summation formula. Finally, distributions are introduced in chapter 4. Modern analysis is unthinkable without this concept that generalizes classical function spaces.

The second part of the book is devoted to the generalization of the concepts of Fourier analysis in the context of locally compact abelian groups, or LCA groups for short. In the introductory Chapter 5 the entire theory is developed in the elementary model case of a finite abelian group. The general setting is fixed in Chapter 6 by introducing the notion of LCA groups; a modest amount of topology enters at this stage. Chapter 7 deals with Pontryagin duality; the dual is shown to be an LCA group again, and the duality theorem is given.

The second part of the book concludes with Plancherel's theorem in Chapter 8. This theorem is a generalization of the completeness of the Fourier series, as well as of Plancherel's theorem for the real line.

The third part of the book is intended to provide the reader with a first impression of the world of non-commutative harmonic analysis. Chapter 9 introduces methods that are used in the analysis of matrix groups, such as the theory of the exponential series and Lie algebras. These methods are then applied in Chapter 10 to arrive at a classification of the representations of the group SU(2). In Chapter 11 we give the Peter-Weyl theorem, which generalizes the completeness of the Fourier series in the context of compact non-commutative groups and gives a decomposition of the regular representation as a direct sum of irreducibles. The theory of non-compact non-commutative groups is represented by the example of the Heisenberg group in Chapter 12. The regular representation in general decomposes as a direct integral rather than a direct sum. For the Heisenberg group this decomposition is given explicitly.

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Exeter, June 2004 Anton Deitmar

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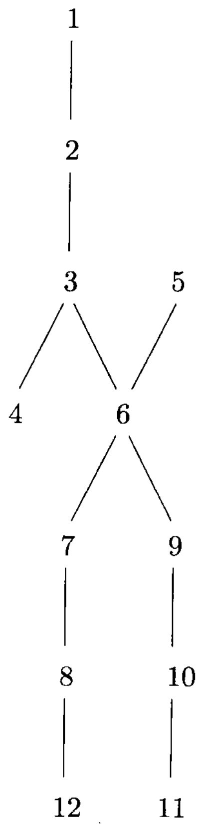

Notation We write $\mathbb{N} = \{1, 2, 3, \dots\}$ for the set of natural numbers and $\mathbb{N}_0 = \{0, 1, 2, \dots\}$ for the set of natural numbers extended by zero. The set of integers is denoted by $\mathbb{Z}$, set of rational numbers by $\mathbb{Q}$, and the sets of real and complex numbers by $\mathbb{R}$ and $\mathbb{C}$, respectively.

Contents

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Part I

Fourier Analysis

Chapter 1

Fourier Series

The theory of Fourier series is concerned with the question of whether a given periodic function, such as the plot of a heartbeat or the signal of a radio pulsar, can be written as a sum of simple waves. A *simple wave* is described in mathematical terms as a function of the form $c\sin(2\pi kx)$ or $c\cos(2\pi kx)$ for an integer k and a real or complex number c.

The formula

$$e^{2\pi ix} = \cos 2\pi x + i\sin 2\pi x$$

shows that if a function f can be written as a sum of exponentials

$$f(x) = \sum_{k\in\mathbb{Z}} c_k e^{2\pi ikx},$$

for some constants c_k, then it also can be written as a sum of simple waves. This point of view has the advantage that it gives simpler formulas and is more suitable for generalization. Since the exponentials $e^{2\pi ikx}$ are complex-valued, it is therefore natural to consider complex-valued periodic functions.

1.1 Periodic Functions

A function $f : \mathbb{R} \to \mathbb{C}$ is called *periodic of period* $L > 0$ if for every $x \in \mathbb{R}$,

$$f(x + L) = f(x).$$

If f is periodic of period L, then the function

$$F(x) = f(Lx)$$

is periodic of period 1. Moreover, since $f(x) = F(x/L)$, it suffices to consider periodic functions of period 1 only. For simplicity we will call such functions just *periodic*.

Examples. The functions $f(x) = \sin 2\pi x$, $f(x) = \cos 2\pi x$, and $f(x) = e^{2\pi i x}$ are periodic. Further, every given function on the half-open interval $[0, 1)$ can be extended to a periodic function in a unique way.

Recall the definition of an *inner product* $\langle .,. \rangle$ on a complex vector space V. This is a map from $V \times V$ to $\mathbb{C}$ satisfying

- for every $w \in V$ the map $v \mapsto \langle v, w \rangle$ is $\mathbb{C}$-linear,
- $\langle v, w \rangle = \overline{\langle w, v \rangle}$,
- $\langle .,. \rangle$ is positive definite, i.e., $\langle v, v \rangle \geq 0$; and $\langle v, v \rangle = 0$ implies $v = 0$.

If f and g are periodic, then so is $af + bg$ for $a, b \in \mathbb{C}$, so that the set of periodic functions forms a complex vector space. We will denote by $C(\mathbb{R}/\mathbb{Z})$ the linear subspace of all *continuous* periodic functions $f : \mathbb{R} \to \mathbb{C}$. For later use we also define $C^\infty(\mathbb{R}/\mathbb{Z})$ to be the space of all infinitely differentiable periodic functions $f : \mathbb{R} \to \mathbb{C}$. For f and g in $C(\mathbb{R}/\mathbb{Z})$ let

$$\langle f, g \rangle = \int_0^1 f(x)\overline{g(x)}dx,$$

where the bar means complex conjugation, and the integral of a complex-valued function $h(x) = u(x) + iv(x)$ is defined by linearity, i.e.,

$$\int_0^1 h(x)dx = \int_0^1 u(x)dx + i\int_0^1 v(x)dx.$$

The reader who has up to now only seen integrals of functions from $\mathbb{R}$ to $\mathbb{R}$ should take a minute to verify that integrals of complex-valued functions satisfy the usual rules of calculus. These can be deduced from the real-valued case by splitting the function into real and imaginary part. For instance, if $f\colon [0, 1] \to \mathbb{C}$ is continuously differentiable, then $\int_0^1 f'(x)\, dx = f(1) - f(0)$.