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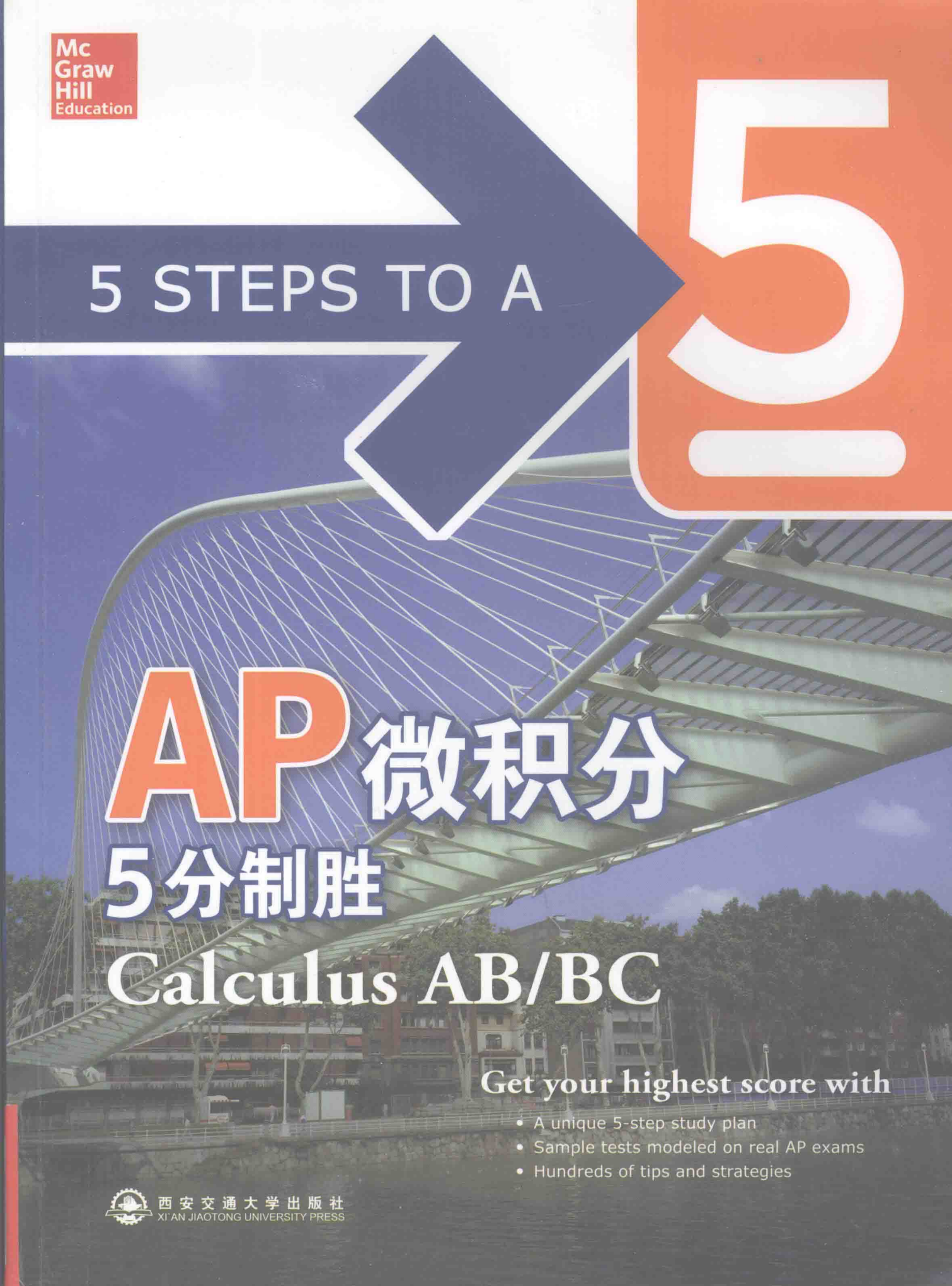
AP微积分 5分制胜

Calculus AB/BC



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AP微积分 5分制胜 Calculus AB/BC

William Ma
Carolyn Wheater 编著

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前　言

AP项目（Advanced Placement Program）始于1955年，由美国大学理事会（the College Board）主持，是在高中阶段开设的具有大学水平的课程，即大学预修课程。AP项目目前设有34门课程和考试，它可以使有余力、有能力、成绩优秀的高中生有机会先修部分美国大学基础课程以获得大学学分，因此吸引了很多成绩优秀的学生选修。目前，已有60多个国家的几千所大学把AP学分作为其入学参考标准，其中包括哈佛大学、耶鲁大学、牛津大学、剑桥大学等世界知名大学。

美国每年约有200万高中毕业生，他们都要参加美国高考SAT和AP课程的考试。美国高中生会在11年级时完成SAT考试，在12年级（高中最后一年）完成两件大事：第一，根据SAT的考试成绩申请大学和奖学金；第二，选修AP课程，并进行备考。在高中选修AP课程和通过AP考试不仅是对学生能力和学业水平的证明，还可以使学生：1. 在申请大学时具有很大的优势。美国大学把学生在AP考试中的表现作为衡量其是否能够胜任大学学习的依据。从美国大学录取顾问委员会公布的影响大学录取因素的比较分析可以看出，AP成绩以80.3%的影响力位居第一，因为它向学校充分展示了学生的才智、专长及学习能力。2. 进入大学后，可以获得大学学分，免修同类课程，提早选修更高级的课程或跳级。3. 提前毕业。4. 节省大学学费。在美国，初等教育是免费的，但高等教育是收费的。选修的AP课程越多，免修的大学课程也就越多，节省的学费也就越多。另外，对中国学生而言，除了可以获得美国大学学分、省时省钱外，还可以在国内提前适应美国大学课程。

AP考试成绩的评定为5分制，满分5分表示极为优秀，4分为优秀，3分相当于合格，即可为大多数学校所接受，2分为可能有资格，1分则不予推荐。 AP考试在每年5月份举行一次，为期两周。每门课程的考试时间约为2～3个小时，考试费用为每科1000元人民币或1400元港币左右。

更多信息可查询以下网站：

AP考试官网：http://www.collegeboard.com

AP国内报名网站：http://apchina.net.cn

香港考务局报名网址：https://www2.hkeaa.edu.hk

为满足国内考生对AP考试资料日益增长的需求，我们从美国知名教育出版公司McGraw-Hill Education引进了本系列AP考试丛书，共包括7本，分别为《AP微观/宏观经济学5分制胜》、《AP统

计学5分制胜》、《AP微积分5分制胜》、《AP美国历史5分制胜》、《AP物理5分制胜》、《AP生物5分制胜》和《AP化学5分制胜》。AP各学科分册由AP考试相关领域专家编写，精准把握考试命题特点，设计“五步”高效学习方案，总结与考试相关的学科内容和要点，精选针对性练习以及全真模拟试题，并配以答案和准确详尽的解析。本系列丛书适用于备考AP的所有考生，便于考生巩固所学，紧抓重点，取得高分。

本书为其中的《AP微积分5分制胜》。

怎样才能在AP微积分考试中取得优异成绩呢？怎样才能得到满分5分呢？其实你已经开始第一步了，因为你正在阅读这本书。你需要做的下一步就是确保读懂书中内容并做完全部习题。近年来，AP微积分考试发生了许多改变。例如，现在的试题不再强调冗长枯燥的代数运算，而是期望考生具备解决各种问题的能力，包括以图表或文字的形式出现的题目。对于很多题目，你都可以使用计算器来得出答案。

我们拥有多年AP微积分的教学经验，并与考生和其他教师有过很多交流，十分了解考生在AP微积分考试中会遇到的一些困难。例如，有些考生抱怨不能读懂题目的含义，也有考生说即使给出了解题方法，他们还是无法理解解题步骤。遇到这样的情况，谁能不感觉到沮丧呢？本书将为考生解决这些问题。书中的题目能附图表的都附有图表，解题方法会逐一呈现。本书从关于极限与连续的章节开始。这部分内容通常在初级微积分课程中教授。如果你对这些概念已经十分熟悉，那就可以略过这些内容，从第六章开始学习。

那么，怎样才能在AP微积分考试中取得满分5分呢？

第一步：从本书第二章中给出的三个学习计划中挑选一个适合你自己的。

第二步：利用第三章的诊断测试（Diagnostic Exam）来检测你的备考程度。

第三步：通过学习第四章提供的应试技巧，研究考试成功的策略。

第四步：通过学习第五章至第十四章的全部内容来巩固获取高分所必需的知识点。

第五步：利用本书提供的模拟测试（Practice Exams）来树立考试信心。

正如大部分AP考试专家所说的那样，“首先必须理解，然后练习。”

祝你好运！

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INTRODUCTION: THE FIVE-STEP PROGRAM

How is this Book Organized?

This book begins with an introduction to the Five-Step Program followed by 14 chapters reflecting the 5 steps.

- Step 1 provides an overview of the AP Calculus AB and BC Exams, and offers three study plans for preparing for the two AP Calculus Exams.
- Step 2 contains a diagnostic test with answers and explanations.
- Step 3 offers test-taking strategies for answering both multiple-choice and free-response questions, and for using a graphing calculator.
- Step 4 consists of 10 chapters providing a comprehensive review of all topics covered on the AP Calculus Exams, with Chapter 14 containing materials tested only on the BC exam. At the end of each chapter (beginning with Chapter 5), you will find a set of practice problems with solutions, a set of cumulative review problems with solutions, and a Rapid Review section giving you the highlights of the chapter.
- Step 5 provides four full practice AP Calculus Exams (2 AB exams and 2 BC exams) with answers, explanations, and worksheets to compute your score.

The book concludes with a summary of math formulas and theorems related to the AP Calculus Exams. *(Please note that the exercises in this book are done with the TI-89 Graphing Calculator.)*

Introducing the Five-Step Preparation Program

This book is organized as a five-step program to prepare you to succeed in the AP Calculus AB/BC Exams. These steps are designed to provide you with vital skills, strategies, and the practice that can lead you to that perfect 5. Here are the 5 steps.

Step 1: Set Up Your Study Program

In this step you will read an overview of the AP Calculus AB and BC Exams, including a summary of topics covered in both exams and a description of the format of both exams. You will also follow a process to help determine which of the following preparation programs is right for you:

- Full school year: September through May.
- One semester: January through May.
- Six weeks: Basic training for the exam.

Step 2: Determine Your Test Readiness

In this step you will take a diagnostic multiple-choice exam in calculus. This pre-test should give you an idea of how prepared you are to take the real exam before beginning to study for the actual AP Calculus AB and BC Exams.

Step 3: Develop Strategies for Success

In this step you will learn strategies that will help you do your best on the exam. These strategies cover both the multiple-choice and free-response sections of the exam.

- Learn to read multiple-choice questions.
- Lean how to answer multiple-choice questions.
- Learn how to plan and write answers to the free-response questions.

Step 4: Review the Knowledge You Need to Score High

In this step you will learn or review the material you need to know for the test. This review section takes up the bulk of this book. It contains:

- A comprehensive review of AP Calculus AB and BC.
- A set of practice problems.
- A set of cumulative review problems beginning with Chapter 5.
- A rapid review summarizing the highlights of the chapter.

Step 5: Build Your Test-taking Confidence

In this step you will complete your preparation by testing yourself on practice exams. We have provided you with two complete exams in AP Calculus AB and two complete exams in AP Calculus BC, and scoring guides for both of them. Although these practice exams are not reproduced questions from the actual AP calculus exams, they mirror both the material tested by AP and the way in which it is tested.

Finally, at the back of this book you will find additional resources to aid your preparation. These include:

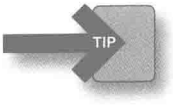
- A brief bibliography.
- A list of websites related to the AP Calculus exam.
- A summary of formulas and theorems related to the AP Calculus exams.

Introduction to the Graphics Used in this Book

To emphasize particular skills and strategies, we use several icons throughout this book. An icon in the margin will alert you that you should pay particular attention to the accompanying text. We use these icons:

This icon points out a very important concept or fact that you should not pass over.

This icon calls your attention to a strategy that you may want to try.



This icon indicates a tip that you might find useful.

This icon identifies material covered only in the BC exam.

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CONTENTS

STEP 1 **Set Up Your Study Plan 1**

1 **What You Need to Know About the AP Calculus AB/BC Exams 3**

1.1 What Is Covered on the AP Calculus Exams? 4

1.2 What Is the Format of the AP Calculus AP/BC Exams? 4

1.3 What Are the Advanced Placement Exam Grades? 5

How Is the AP Calculus Exam Grade Calculated? 5

1.4 Which Graphing Calculators Are Allowed for the Exam? 6

Calculators and Other Devices Not Allowed for the AP Calculus Exam 7

Other Restrictions on Calculators 7

2 **How to Plan Your Time 8**

2.1 Three Approaches to Preparing for the AP Calculus Exam 8

Overview of the Three Plans 8

2.2 Calendar for Each Plan 10

Summary of the Three Study Plans 13

STEP 2 **Determine Your Test Readiness 15**

3 **Take a Diagnostic Exam 17**

3.1 Getting Started! 20

3.2 Diagnostic Test 20

3.3 Answers to Diagnostic Test 27

3.4 Solutions to Diagnostic Test 28

3.5 Calculate Your Score 39

Short-Answer Questions 39

AP Calculus AB/BC Diagnostic Exam 39

STEP 3 **Develop Strategies for Success 41**

4 **How to Approach Each Question Type 43**

4.1 The Multiple-Choice Questions 44

4.2 The Free-Response Questions 44

4.3 Using a Graphing Calculator 45

4.4 Taking the Exam 46

What Do I Need to Bring to the Exam? 46

Tips for Taking the Exam 47

STEP 4 **Review the Knowledge You Need to Score High 49**

5 **Limits and Continuity** 51

5.1 The Limit of a Function 52
Definition and Properties of Limits 52
Evaluating Limits 52
One-Sided Limits 54
Squeeze Theorem 57

5.2 Limits Involving Infinities 59
Infinite Limits (as $x \to a$) 59
Limits at Infinity (as $x \to \pm\infty$) 61
Horizontal and Vertical Asymptotes 63

5.3 Continuity of a Function 66
Continuity of a Function at a Number 66
Continuity of a Function over an Interval 66
Theorems on Continuity 66

5.4 Rapid Review 69

5.5 Practice Problems 71

5.6 Cumulative Review Problems 72

5.7 Solutions to Practice Problems 72

5.8 Solutions to Cumulative Review Problems 75

6 **Differentiation** 77

6.1 Derivatives of Algebraic Functions 78
Definition of the Derivative of a Function 78
Power Rule 81
The Sum, Difference, Product, and Quotient Rules 82
The Chain Rule 83

6.2 Derivatives of Trigonometric, Inverse Trigonometric, Exponential, and Logarithmic Functions 84
Derivatives of Trigonometric Functions 84
Derivatives of Inverse Trigonometric Functions 86
Derivatives of Exponential and Logarithmic Functions 87

6.3 Implicit Differentiation 89
Procedure for Implicit Differentiation 89

6.4 Approximating a Derivative 92

6.5 Derivatives of Inverse Functions 94

6.6 Higher Order Derivatives 96

BC 6.7 Indeterminate Forms 97
L'Hôpital's Rule for Indeterminate Forms 97

6.8 Rapid Review 97

6.9 Practice Problems 99

6.10 Cumulative Review Problems 100

此为试读,需要完整PDF请访问: www.ertongbook.com

6.11 Solutions to Practice Problems 100
6.12 Solutions to Cumulative Review Problems 103

7 Graphs of Functions and Derivatives 105

7.1 Rolle's Theorem, Mean Value Theorem, and Extreme Value Theorem 105
Rolle's Theorem 106
Mean Value Theorem 106
Extreme Value Theorem 109
7.2 Determining the Behavior of Functions 110
Test for Increasing and Decreasing Functions 110
First Derivative Test and Second Derivative Test for Relative Extrema 113
Test for Concavity and Points of Inflection 116
7.3 Sketching the Graphs of Functions 122
Graphing without Calculators 122
Graphing with Calculators 123
7.4 Graphs of Derivatives 125
BC
7.5 Parametric, Polar, and Vector Representations 130
Parametric Curves 130
Polar Equations 131
Types of Polar Graphs 131
Symmetry of Polar Graphs 132
Vectors 133
Vector Arithmetic 134
7.6 Rapid Review 135
7.7 Practice Problems 139
7.8 Cumulative Review Problems 141
7.9 Solutions to Practice Problems 142
7.10 Solutions to Cumulative Review Problems 149

8 Applications of Derivatives 151

8.1 Related Rate 151
General Procedure for Solving Related Rate Problems 151
Common Related Rate Problems 152
Inverted Cone (Water Tank) Problem 153
Shadow Problem 154
Angle of Elevation Problem 155
8.2 Applied Maximum and Minimum Problems 157
General Procedure for Solving Applied Maximum and Minimum Problems 157
Distance Problem 157
Area and Volume Problem 158
Business Problems 161
8.3 Rapid Review 162

8.4 Practice Problems 163
8.5 Cumulative Review Problems 165
8.6 Solutions to Practice Problems 166
8.7 Solutions to Cumulative Review Problems 173

9 More Applications of Derivatives 176

9.1 Tangent and Normal Lines 176
Tangent Lines 176
Normal Lines 182
9.2 Linear Approximations 185
Tangent Line Approximation (or Linear Approximation) 185
Estimating the nth Root of a Number 187
Estimating the Value of a Trigonometric Function of an Angle 187
9.3 Motion Along a Line 188
Instantaneous Velocity and Acceleration 188
Vertical Motion 190
Horizontal Motion 190
BC 9.4 Parametric, Polar, and Vector Derivatives 192
Derivatives of Parametric Equations 192
Position, Speed, and Acceleration 193
Derivatives of Polar Equations 193
Velocity and Acceleration of Vector Functions 194
9.5 Rapid Review 197
9.6 Practice Problems 198
9.7 Cumulative Review Problems 200
9.8 Solutions to Practice Problems 201
9.9 Solutions to Cumulative Review Problems 206

10 Integration 208

10.1 Evaluating Basic Integrals 209
Antiderivatives and Integration Formulas 209
Evaluating Integrals 211
10.2 Integration by U-Substitution 214
The U-Substitution Method 214
U-Substitution and Algebraic Functions 214
U-Substitution and Trigonometric Functions 216
U-Substitution and Inverse Trigonometric Functions 217
U-Substitution and Logarithmic and Exponential Functions 219
BC 10.3 Techniques of Integration 222
Integration by Parts 222
Integration by Partial Fractions 223
10.4 Rapid Review 224
10.5 Practice Problems 225

10.6 Cumulative Review Problems 226
10.7 Solutions to Practice Problems 227
10.8 Solutions to Cumulative Review Problems 230

11 Definite Integrals 232

11.1 Riemann Sums and Definite Integrals 233
Sigma Notation or Summation Notation 233
Definition of a Riemann Sum 234
Definition of a Definite Integral 235
Properties of Definite Integrals 236
11.2 Fundamental Theorems of Calculus 238
First Fundamental Theorem of Calculus 238
Second Fundamental Theorem of Calculus 239
11.3 Evaluating Definite Integrals 242
Definite Integrals Involving Algebraic Functions 242
Definite Integrals Involving Absolute Value 243
Definite Integrals Involving Trigonometric, Logarithmic, and Exponential Functions 244
Definite Integrals Involving Odd and Even Functions 246
BC 11.4 Improper Integrals 247
Infinite Intervals of Integration 247
Infinite Discontinuities 248
11.5 Rapid Review 249
11.6 Practice Problems 250
11.7 Cumulative Review Problems 251
11.8 Solutions to Practice Problems 252
11.9 Solutions to Cumulative Review Problems 255

12 Areas and Volumes 258

12.1 The Function $F(x) = \int_a^x f(t)dt$ 259
12.2 Approximating the Area Under a Curve 263
Rectangular Approximations 263
Trapezoidal Approximations 267
12.3 Area and Definite Integrals 268
Area Under a Curve 268
Area Between Two Curves 273
12.4 Volumes and Definite Integrals 277
Solids with Known Cross Sections 277
The Disc Method 281
The Washer Method 286
BC 12.5 Integration of Parametric, Polar, and Vector Curves 290
Area, Arc Length, and Surface Area for Parametric Curves 290
Area and Arc Length for Polar Curves 291
Integration of a Vector-Valued Function 292

12.6 Rapid Review 293
12.7 Practice Problems 296
12.8 Cumulative Review Problems 297
12.9 Solutions to Practice Problems 298
12.10 Solutions to Cumulative Review Problems 306

13 More Applications of Definite Integrals 309

13.1 Average Value of a Function 310
Mean Value Theorem for Integrals 310
Average Value of a Function on $[a, b]$ 311
13.2 Distance Traveled Problems 313
13.3 Definite Integral as Accumulated Change 316
Business Problems 316
Temperature Problem 317
Leakage Problems 318
Growth Problem 318
13.4 Differential Equations 319
Exponential Growth/Decay Problems 319
Separable Differential Equations 321
BC 13.5 Slope Fields 324
BC 13.6 Logistic Differential Equations 328
BC 13.7 Euler's Method 330
Approximating Solutions of Differential Equations by Euler's Method 330
13.8 Rapid Review 332
13.9 Practice Problems 334
13.10 Cumulative Review Problems 336
13.11 Solutions to Practice Problems 337
13.12 Solutions to Cumulative Review Problems 343

14 Series (For Calculus BC Students Only) 346

BC 14.1 Sequences and Series 347
Convergence 347
14.2 Types of Series 348
p-Series 348
Harmonic Series 348
Geometric Series 348
Decimal Expansion 349
14.3 Convergence Tests 350
Integral Test 350
Ratio Test 351
Comparison Test 351
Limit Comparison Test 352

14.4 Alternating Series 353
Error Bound 354
Absolute Convergence 354
14.5 Power Series 354
Radius and Interval of Convergence 355
14.6 Taylor Series 355
Taylor Series and MacLaurin Series 355
Common MacLaurin Series 357
14.7 Operations on Series 357
Substitution 357
Differentiation and Integration 358
Error Bounds 359
14.8 Rapid Review 360
14.9 Practice Problems 362
14.10 Cumulative Review Problems 363
14.11 Solutions to Practice Problems 363
14.12 Solutions to Cumulative Review Problems 366

STEP 5 **Build Your Test-Taking Confidence 369**

AP Calculus AB Practice Exam 1 371
AP Calculus AB Practice Exam 2 397
AP Calculus BC Practice Exam 1 424
AP Calculus BC Practice Exam 2 451

Appendixes 479
Formulas and Theorems 481
Bibliography and Web Sites 489

Set Up Your Study Plan

CHAPTER 1 What You Need to Know About the AP Calculus AB/BC Exams

CHAPTER 2 How to Plan Your Time

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