INTRODUCTION TO SUPERSYMMETRY AND SUPERGRAVITY

Extended Second Edition

超对称和超引力导论 第2版

Peter West

$$\{Q_A{}^i, \bar{Q}_{\dot{B}j}\} = -2i(\sigma^a)_{A\dot{B}}\delta^i_j P_a\,; \quad \{Q_A{}^i, Q_B{}^j\} = \epsilon_{AB} Z^{ij}$$

$$[Q_A{}^i, P_a] = 0; \qquad [T_r, T_s] = f_{rst} T_t$$

$$[Q^i_A, T_r] = (U_r)^i{}_j Q^j_A\,, \quad [Q^i_A, J_{ab}] = \frac{1}{2}(\sigma_{ab})_A{}^B Q^i{}_B$$

$$[Z^{ij}, \text{any generator}] = 0\,.$$

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INTRODUCTION TO SUPERSYMMETRY AND SUPERGRAVITY

Preface to the Second Edition

"When swept out of its normal channel, life scatters into innumerable streams. It is difficult to foresee which it will take in its treacherous and winding course. Where today it flows in shallows, like a rivulet over sand banks, so shallow that the shoals are visible, tomorrow it will flow quickly and full."

Mikhail Sholokhov

"Men willingly believe what they wish"

Caesar

The first edition of this book was completed in 1986, however, much of the material was written long before. It focused on the development of four-dimensional supersymmetric models including supergravity with emphasis on their ultraviolet properties. Already in 1983, our understanding of the finiteness of rigid supersymmetric theories had led to the realization that supersymmetry was most unlikely to solve the celebrated inconsistency of quantum mechanics and gravity. This, and the fact that many aspects of supersymmetric theories had been worked out, lead to a search for new ideas. It was inevitable that string theory, which had been extensively developed in the late 1960's and early 1970's would be revived from its dormant state. We recall that supersymmetry was discovered independently in two ways, one of which was within the superstring which contained it as a symmetry. Also during the dormant stage, theoreticians had developed BRST symmetry, conformal models, the vertex operator representation of Lie algebras, the use of the gauge group E_8 for grand unified models, even within the context of ten-dimensional supersymmetric theories and gained further understanding of anomalies. All these enabled the solution of some of the problems which the original pioneers of string theory had encountered.

The superstring possesses two-dimensional supersymmetry on its world sheet. Consequently, it requires a knowledge of two-dimensional supersymmetry algebras and the models which realize them. The more interesting string theories also possess space-time supersymmetry and it is thought that any contact they have with reality is via a four-dimensional supersymmetric theory. Hence, the study of string theory involves a knowledge of supersymmetry in general.

Although there are a number of arguments to suggest that supersymmetry may have a place in Nature, there is at present no experimental evidence for supersymmetry. Hopefully, the next generation of accelerators will find such evidence. However, even if this is not the case it would seem that supersymmetry has entered the culture of both mathematics and theoretical physics and will be the subject of study for the foreseeable future both by itself and within the context of superstring theories. Supersymmetry has introduced into theoretical physics, almost at a subconscious level, a number of ideas which are likely to be important. This process has become further apparent from the development of string theory; our understanding of which has been revealed to be far from complete and probably requiring the introduction of new physical concepts. One likely casualty is our notion of space-time, our perception of which has been subtlety changing as supersymmetry and string theory have evolved.

Taking into account the above developments, I have taken this opportunity to add to the second edition five and a half new chapters on two-dimensional supersymmetry, extending the short discussion of this subject which already existed in the first edition. After an introduction, the topics discussed are two-dimensional supersymmetry algebras, their irreducible representations, rigid and local (i.e., supergravity) theories of supersymmetry both in x-space and superspace. These theories include the actions of the superstring and the heterotic string. We also give a discussion of the superconformal algebras in two dimensions and the final chapter contains an account of Green's functions and operator product expansions for $N = 2$ superconformal models.

These chapters should equip the reader with the supersymmetry necessary for the study of superstring theory and superconformal models. However, another virtue is that two-dimensional theories, having fewer component fields, are in general much simpler than their four-dimensional analogues and so illustrate more clearly many of the principles of supersymmetric theories given in the earlier chapters. I have also made a number of minor changes in the original text and supplied a number of problems in Appendix C. I have, however, resisted any temptation to include any string theory other than that contained in the first edition. The reader may find much preliminary material for a book, by this author, on string theory in the reviews of Refs. 245 and 258.

I would like to thank Paul Howe and George Papadopoulos for many discussions on two-dimensional supersymmetry and Myrna Guarisco for typing the manuscript.

Geneva
June, 1989

Preface

This book has evolved out of a number of courses that I have given on supersymmetry and supergravity. While giving these lectures I became convinced of the need for a book which contained a pedagogical introduction to most aspects of supersymmetric theories. Although the content of this book has been to some extent constrained by those areas that were my research activities at the time I wrote my lecture notes, most major areas relevant for an introduction are covered, as well as some more advanced topics. Some of the latter are concerned with the quantum properties of supersymmetric theories and the construction of supergravity theories. The final chapter contains a discussion of free gauge covariant string field theory.

Supersymmetric theories have had an important influence on the theoretical physics community. It has encouraged the quest for a single unified theory of physics and has lead to a wider understanding of what can constitute the space-time we live in. On a more general level, it has made more acceptable the study of ideas which are at first sight rather distantly related to experimental data. Some effort has been made to present a step-by-step and necessarily technical derivation of the results. However, by studying the subject itself, it is hoped that the reader will also come to appreciate more fully the concepts that may be abstracted from supersymmetric theories.

I would like to express my gratitude to King's College, the California Institute of Technology and CERN where this manuscript was written and typed. I also wish to thank my collaborators for the insights they have shared with me.

London
June, 1986

Peter West

Contents

Chapter 1

Introduction

"Thus fortune on our first endeavour smiles"

Virgil

This book has grown out of a number of lectures given at various summer schools and is intended to be an introduction to Supersymmetry and Supergravity. It is pedagogical in the sense that complete proofs are almost always given. After more than ten years of intensive work on supersymmetric theories, it is impossible to cover the vast literature in a single volume. Rather than discuss many topics in an incomplete way, we have decided to restrict our attention to those areas which are essential for a beginner to learn, are important for the development of the subject as a whole and can be covered in a reasonably short space in a pedagogical manner. This included almost all of rigid supersymmetry and $N = 1$ supergravity. Excluded were the extended supergravity theories, superstring theories, Kaluza-Klein dimensional reduction and an extensive discussion of the phenomenological implications of supersymmetry. The reader is referred to Appendix B for some reviews on these important subjects.

In view of the introductory nature of this review, we have not attempted to give the most systematic treatment, or give the complete mathematical background, for fear that the reader should become lost in the details. Often, only those features which are necessary for a step-by-step derivation of the desired result are given. In this sense, this review is rather low-brow, but hopefully easily understandable. It is to be expected that the reader should feel that he is on the tip of the iceberg of some deeper framework. This is particularly true, for example, in Chapter 14 on superspace.

A substantial part of the review has been written at the same time as the book of Ref. 6. This is a more scholarly work in which the conceptual, as well as mathematical theory behind supersymmetry is given. From this base the entire theory of supersymmetry is derived in detail in a complete manner. The resulting book is somewhat lengthy, but is systematic and complete in those topics that it covers. Care has been taken to avoid too much overlap, and it is hoped that these two works will complement each other. In the later chapters, however, there is some similarity and I would like to thank Peter van Nieuwenhuizen for reading these chapters and making many helpful suggestions.

* * *

Supersymmetry was discovered by Golfand and Liktman.[1] A theory invariant under a non-linear realization of supersymmetry was given by Akulov and Volkov.[2] In a separate development,[7] supersymmetry was introduced as a two-dimensional symmetry of the world sheet within the context of string theories. However, supersymmetry only became widely known when this two-dimensional symmetry was generalized to four dimensions and used to construct the Wess-Zumino model.[3]

To this day, there is no firm evidence that supersymmetry is realized in Nature. Neither is there any completely compelling reason to believe that supersymmetry is required to resolve any of the paradoxes of our present theories of physics. However, it is possible that supersymmetry may be required to explain the new phenomena found already, or in the near future, in particle accelerators. On the theoretical side, there are also some reasons to hope that supersymmetry is required. In Nature, there are at least two vastly different energy scales: the weak scale (100 GeV) and the Planck scale (10^{19} GeV). There are also some reasons to believe that there should be one or more intermediate scales. Although the origin of these vastly different scales is unknown, it is considered to be natural to have a theory in which phenomena at the lowest scale are not polluted by much larger effects arising from the higher scales. Some supersymmetric theories are natural in this sense, and it is a consequence of this argument that the superpartners of the observed particles ought to have masses around the weak scale and hence should be seen in the near future (see Chapter 19). This particular property of supersymmetric theories is a consequence of the fact that the spin-zero states are related by supersymmetry to states of spin $\frac{1}{2}$.

The most uncertain aspect of the standard model of weak and electromagnetic interactions is the spin-zero sector. In fact, many of the 19 free parameters of this model arise due to the undetermined interactions of the spin-zero fields with themselves and the spin-$\frac{1}{2}$ fields. It is natural to hope that some of these free parameters are fixed in a supersymmetric theory. In fact, this has not been achieved within the context of supersymmetric models with only one supersymmetry, but it is likely to be the case should one succeed in constructing a realistic model with more than one supersymmetry.

On a different mass scale, it has been hoped that supersymmetric theories would provide a consistent theory of gravity and quantum mechanics and, at the same time, unify gravity with all the other forces of nature. The most promising candidates that could achieve this long-awaited result are superstring theories. In this context it is, however, worth remembering that gravity may not be a fundamental force, but may arise due to a dynamical mechanism.

It is striking that supersymmetry seems as if it might provide answers to so many of the outstanding unanswered questions. What is remarkable is that, were supersymmetry to answer these questions, then it would relate phenomena at the Planck scale to those at the weak scale.

Although supersymmetry was not universally embraced at its conception, in more recent years it has become the major research interest of a substantial number of the theoretical physics community. Such a devotion of resources to a subject which has no firm contact with the observed world is not an uncommon feature of human behaviour, but it is a new phenomenon for theoretical physics. This is partly a consequence of the hopes for supersymmetry outlined above, as well as the very rich structure of supersymmetric theories that have led to a succession of developments which in turn fuelled further interest. Some of these developments include the construction of rigid extended supersymmetry theories and supergravity theories; superconformal invariance, that is, the finiteness of a class of the former theories; the further development of superstring theories; the construction of realistic supersymmetric models and the use of supersymmetry to simplify proofs of mathematical theorems.

This book begins, of course, with the supersymmetry algebra. It is shown that this algebra is a natural consequence, within the framework of quantum field theory, of demanding either a Fermi-Bose symmetry or that physics should realize the Poincaré group and an internal symmetry group in a non-trivial manner (see Chapters 2, 3, 4 and 5).

The irreducible representations of this supersymmetry algebra (Chapter 8) describe the possible on-shell states of supersymmetric theories. It is explained how one may systematically construct all supersymmetric theories from a knowledge of only their on-shell states. One simply finds the supersymmetry transformations that rotate the fields subject to their field equations (i.e., the on-shell states) into each other, and then finds an invariant action constructed from unconstrained fields. This technique is illustrated for $N = 1$ rigid supersymmetric theories in Chapters 5, 6 and 7, and the theories of rigid supersymmetry are constructed in Chapter 12. The word "rigid" means that the parameter of supersymmetry transformations is independent of space-time.

The $N = 1$ supergravity theory is constructed in Chapter 9, by first finding the linearized theory which is invariant under only rigid supersymmetry. Then, using the Noether method, we find the complete locally supersymmetric theory. The invariance of this result is established in Chapter 11.

Having obtained these supersymmetric theories, we can then couple them together. This is most easily achieved using the supersymmetric tensor calculus. As the name suggests, by analogy with general relativity, one takes multiplets of supersymmetry, that is, sets of x-space fields that transform into themselves under supersymmetry, and finds rules for combining them to form new supermultiplets. This, with a knowledge of how to construct invariants, allows one to find the most general coupling. In Chapters 12 and 13 this is performed for rigid and local supersymmetry respectively.

This supersymmetry tensor calculus is given in x-space and allows one to keep supersymmetry manifest at every step of the construction. It cannot, however, be used to keep supersymmetry manifest during quantum calculations. This is achieved at the classical and quantum level by using superspace (Chapter 14). Here, the supersymmetry arises as transformations on an 8-dimensional coset space called superspace. Four of the co-ordinates of superspace commute, while the remaining four anticommute. This construction is a generalization of the realization of the Poincaré group on Minkowski space. Supermultiplets are then given by fields, called superfields, on superspace. The formulation of the theories of rigid supersymmetry and $N = 1$ supergravity are given in Chapters 15 and 16.

In Chapter 17 we discuss how to calculate quantum effects in superspace, and in Chapter 18 the ultraviolet properties of supersymmetric theories are derived. These include the existence of a large class of finite quantum field theories. In Chapter 19, a brief discussion of the theoretical aspects of the construction of realistic models is given. Chapter 20 discusses the supermultiplet structure to which the currents in supersymmetric theories belong. This enables one to find formulations of supergravity, and also has implications for the quantum effects in supersymmetric theories.

Finally, with the new interest in string theories in mind we discuss in Chapter 21, the two-dimensional supersymmetric theories that underlie superstring theories, while, in Chapter 22, we give an introduction to the gauge covariant formulation of string theories.

Chapter 2

The Supersymmetry Algebra

In the 1960's, with the growing awareness of the significance of internal symmetries such as SU(2) and larger groups, physicists attempted to find a symmetry which would combine in a non-trivial way the space-time Poincaré group with an internal symmetry group. After much effort it was shown that such an attempt was impossible within the context of a Lie group. Coleman and Mandula[4] showed on very general assumptions that any Lie group which contained the Poincaré group P, whose generators P_a and J_{ab} satisfy the relations

$$
\begin{aligned}
&[P_a, P_b] = 0 \\
&[P_a, J_{bc}] = (\eta_{ab} P_c - \eta_{ac} P_b) \\
&[J_{ab}, J_{cd}] = -(\eta_{ac} J_{bd} + \eta_{bd} J_{ac} - \eta_{ad} J_{bc} - \eta_{bc} J_{ad})
\end{aligned}
\tag{2.1}
$$

and an internal symmetry group G with generators T_s such that

$$[T_r, T_s] = f_{rst} T_t \tag{2.2}$$

must be a direct product of P and G; or in other words

$$[P_a, T_s] = 0 = [J_{ab}, T_s] \tag{2.3}$$

They also showed that G must be of the form of a semisimple group with additional $U(1)$ groups.

It is worthwhile to make some remarks concerning the status of this no-go theorem. Clearly there are Lie groups that contain the Poincaré group and internal symmetry groups in a non-trivial manner; however the theorem states that these groups lead to trivial physics. Consider, for example, two-body scattering; once we have imposed conservation of angular momentum and momentum the scattering angle is the only unknown quantity. If there were a Lie group that had a non-trivial mixing with the Poincaré group then there would be further space-time associated generators. The resulting conservation laws will further constrain, for example, two-body scattering, and so the scattering angle can only take on discrete values. However, the scattering process is expected to be analytic in the scattering angle, θ, and hence we must conclude that the process does not depend on θ at all.

Essentially the theorem shows that if one used a Lie group that contained an internal group which mixed in a non-trivial manner with the Poincaré group then the S-matrix for all processes would be zero. The theorem assumes among other things, that the S-matrix exists and is non-trivial, the vacuum is nondegenerate and that there are no massless particles. It is important to realize that the theorem only applies to symmetries that act on S-matrix elements and not on all the other many symmetries that occur in quantum field theory. Indeed it is not uncommon to find examples of the latter symmetries. Of course, no-go theorems are only as strong as the assumptions required to prove them.

In a remarkable paper Gelfand and Likhtman[1] showed that provided one generalized the concept of a Lie group one could indeed find a symmetry that included the Poincaré group and an internal symmetry group in a non-trivial way. In this section we will discuss this approach to the supersymmetry group; having adopted a more general notion of a group, we will show that one is led, with the aid of the Coleman-Mandula theorem, and a few assumptions, to the known supersymmetry group.

Since the structure of a Lie group, at least in some local region of the identity, is determined entirely by its Lie algebra it is necessary to adopt a more general notion than a Lie algebra. The vital step in discovering the supersymmetry algebra is to introduce generators, $Q_\alpha{}^i$, which satisfy anti-commutation relations, i.e.

$$\begin{aligned} \{Q_\alpha{}^i, Q_\beta{}^j\} &= Q_\alpha{}^i Q_\beta{}^j + Q_\beta{}^j Q_\alpha{}^i \\ &= \text{some other generator} \end{aligned} \tag{2.4}$$

The significance of the i and α indices will become apparent shortly. Let us therefore assume that the supersymmetry group involves generators P_a, J_{ab}, T_s and possibly some other generators which satisfy commutation relations, as well as the generators $Q_\alpha{}^i$ $(i = 1, 2, \ldots, N)$. We will call the former generators which satisfy Eqs. (2.1), (2.2) and (2.3) to be even and those satisfying Eq. (2.4) to be odd generators.

Having let the genie out of the bottle we promptly replace the stopper and demand that the supersymmetry algebra have a Z_2 graded structure. This simply means that the even and odd generators must satisfy the rules:

$$\begin{aligned} [\text{even}, \text{even}] &= \text{even} \\ \{\text{odd}, \text{odd}\} &= \text{even} \\ [\text{even}, \text{odd}] &= \text{odd} \end{aligned} \tag{2.5}$$

We must still have the relations

$$[P_a, T_s] = 0 = [J_{ab}, T_s] \tag{2.6}$$

since the even (bosonic) subgroup must obey the Coleman-Mandula theorem.

Let us now investigate the commutator between J_{ab} and $Q_\alpha{}^i$. As a result of Eq. (2.5) it must be of the form

$$[Q_\alpha{}^i, J_{ab}] = (b_{ab})_\alpha{}^\beta Q_\beta{}^i \tag{2.7}$$

since by definition the $Q_\alpha{}^i$ are the only odd generators. We take the α indices to be those rotated by J_{ab}. As in a Lie algebra we have some generalized Jacobi identities. If we denote an even generator by B and an odd generator by F we find that

$$\begin{aligned} [[B_1, B_2], B_3] + [[B_3, B_1], B_2] + [[B_2, B_3], B_1] &= 0 \\ [[B_1, B_2], F_3] + [[F_3, B_1], B_2] + [[B_2, F_3], B_1] &= 0 \\ \{[B_1, F_2], F_3\} + \{[B_1, F_3], F_2\} + [\{F_2, F_3\}, B_1] &= 0 \\ [\{F_1, F_2\}, F_3] + [\{F_1, F_3\}, F_2] + [\{F_2, F_3\}, F_1] &= 0 \end{aligned} \tag{2.8}$$

The reader may verify, by expanding each bracket that these relations are indeed identically true.

The identity

$$[[J_{ab}, J_{cd}], Q_\alpha{}^i] + [[Q_\alpha{}^i, J_{ab}], J_{cd}] + [[J_{cd}, Q_\alpha{}^i], J_{ab}] = 0 \tag{2.9}$$

upon use of Eq. (2.7) implies the result

$$\begin{aligned} [b_{ab}, b_{cd}]_\alpha{}^\beta = &-\eta_{ac}(b_{bd})_\alpha{}^\beta - \eta_{bd}(b_{ac})_\alpha{}^\beta \\ &+ \eta_{ad}(b_{bc})_\alpha{}^\beta + \eta_{bc}(b_{ad})_\alpha{}^\beta \end{aligned} \tag{2.10}$$

This means that the $(b_{cd})_\alpha{}^\beta$ form a representation of the Lorentz algebra or in otherwords the $Q_\alpha{}^i$ carry a representation of the Lorentz group. We will select $Q_\alpha{}^i$ to be in the $(0, \frac{1}{2}) \oplus (\frac{1}{2}, 0)$ representation of the Lorentz group, i.e.

$$[Q_\alpha{}^i, J_{ab}] = \tfrac{1}{2}(\sigma_{ab})_\alpha{}^\beta Q_\beta{}^i \tag{2.11}$$

We can choose $Q_\alpha{}^i$ to be a Majorana spinor, i.e.

$$Q_\alpha{}^i = C_{\alpha\beta}\bar{Q}^{\beta i} \tag{2.12}$$

where $C_{\alpha\beta} = -C_{\beta\alpha}$ is the charge conjugation matrix (see Appendix A). This does not represent a loss of generality since, if the algebra admits complex conjugation as an involution we can always redefine the supercharges so as to satisfy (2.12) (see Note 1 at the end of this chapter).

The above calculation reflects the more general result that the $Q_\alpha{}^i$ must belong to a realization of the even (bosonic) subalgebra of the supersymmetry