Textbooks in Mathematical Sciences

Understanding Nonlinear Dynamics

非线性动力学入门

Daniel Kaplan
Leon Glass

Springer-Verlag
世界图书出版公司

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北京·广州·上海·西安

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Library of Congress Cataloging-in-Publication Data
Kaplan, Daniel, 1959–
Understanding nonlinear dynamics / Daniel Kaplan and Leon Glass.
p. cm. – (Textbooks in mathematical sciences)
Includes bibliographical references and index.
ISBN 0-387-94440-0
1. Dynamics. 2. Nonlinear theories. I. Glass, Leon, 1943–. II. Title. III. Series.
QA845.K36 1995 94-43113
515′.352–dc20

Printed on acid-free paper.

This reprint has been authorized by Springer-Verlag (Berlin/Heidelberg/New York) for sale in the People's Republic of China only and not for export therefrom
Reprinted in China by Beijing World Publishing Corporation, 1997.

ISBN 0-387-94440-0 Springer-Verlag New York Berlin Heidelberg

To Maya and Tamar. — DTK

To Kathy, Hannah, and Paul
and in memory of Daniel. — LG

Preface

This book is about *dynamics*—the mathematics of how things change in time. The universe around us presents a kaleidoscope of quantities that vary with time, ranging from the extragalactic pulsation of quasars to the fluctuations in sunspot activity on our sun; from the changing outdoor temperature associated with the four seasons to the daily temperature fluctuations in our bodies; from the incidence of infectious diseases such as measles to the tumultuous trend of stock prices.

Since 1984, some of the vocabulary of dynamics—such as *chaos, fractals,* and *nonlinear*—has evolved from abstruse terminology to a part of common language. In addition to a large technical scientific literature, the subjects these terms cover are the focus of many popular articles, books, and even novels. These popularizations have presented "chaos theory" as a scientific revolution. While this may be journalistic hyperbole, there is little question that many of the important concepts involved in modern dynamics—global multistability, local stability, sensitive dependence on initial conditions, attractors—are highly relevant to many areas of study including biology, engineering, medicine, ecology, economics, and astronomy.

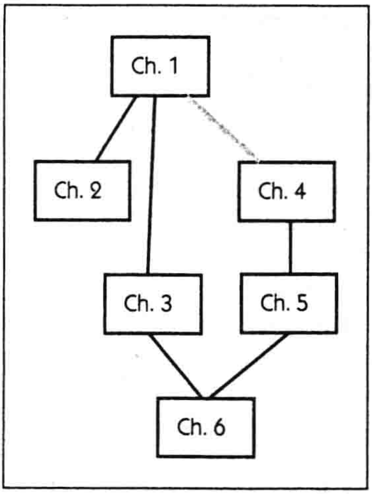
This book presents the main concepts and applications of nonlinear dynamics at an elementary level. The text is based on a one-semester undergraduate course that has been offered since 1975 at McGill University and that has been constantly updated to keep up with current developments. Most of the students enrolled in the course are studying biological sciences and have completed a year of calculus with no intention to study further mathematics. Since the main concepts of nonlinear dynamics are largely accessible using only elementary arguments, students are able to understand the mathematics and successfully carry out computations. The exciting nature and modernity of the concepts and the graphics are further stimuli that motivate students.

Mathematical developments since the mid 1970's have shown that many interesting phenomena can arise in simple finite-difference equations. These are introduced in Chapter 1, where the student is initiated into three important mathematical themes of the course: local stability analysis, global multistability, and problem solving using both an algebraic and a geometric approach. The graphical iteration of one-dimensional, finite-difference equations, combined with the analysis of the local stability of steady states, provides two complementary views of the same problem. The concept of chaos is introduced as soon as possible, after the student is able graphically to iterate a one-dimensional, finite-difference equation, and understands the concept of stability. For most students, this is the first exposure to mathematics from the twentieth century!

From the instructor's point of view, this topic offers the opportunity to refresh students' memory and skills in differential calculus. Since some students take this course several years after studying geometry and calculus, some skills have become rusty. Appendix A reviews important functions such as the Hill function, the Gaussian distribution, and the conic sections. Many exercises that can help in solidifying geometry and calculus skills are included in Appendix A.

Chapters 2 and 3 continue the study of discrete-time systems. Networks and cellular automata (Chapter 2) are important both from a conceptual and technical perspective, and because of their relevance to computers. The recent interest in neural and gene networks makes this an important area for applications and current research.

Many students are familiar with fractal images from the myriad popularizations of that topic. While the images provide a compelling motivation for studying nonlinear dynamics, the concepts of self-similarity and fractional dimension are important from a mathematical perspective. Chapter 3 discusses self-similarity and fractals in a way that is closely linked to the dynamics discussed in Chapter 1. Fractals arise from dynamics in many unexpected ways. The concept of a fractional dimension is unfamiliar initially but can be appreciated by those without advanced technical abilities. Recognizing the importance of computers in studying fractals, we use a computer-based notation in presenting some of the material.



Dependencies among the chapters.

The study of continuous-time systems forms much of the second half of the book. Chapter 4 deals with one-dimensional differential equations. Because of the importance of exponential growth and decay in applications, we believe that every science student should be exposed to the linear one-dimensional differential equation, learning what it means and how to solve it. In addition, it is essential that those interested in science appreciate the limitations that nonlinearities impose on exponential ("Malthusian") growth. In Chapter 4, algebraic analysis of the linear stability of steady states of nonlinear equations is combined with the graphical analysis of the asymptotic dynamics of nonlinear equations to provide another exposure to the complementary use of algebraic and geometric methods of analysis.

Chapter 5 deals with differential equations with two variables. Such equations often appear in the context of compartmental models, which have been proposed in diverse fields including ion channel kinetics, pharmacokinetics, and ecological systems. The analysis of the stability of steady states in two-dimensional nonlinear equations and the geometric sketching of the trajectories in the phase plane provide the most challenging aspect of the course. However, the same basic conceptual approach is used here as is used in the linear stability analyses in Chapter 1 and Chapter 4, and the material can be presented using elementary methods only.

In most students' mathematical education, a chasm exists between the concepts they learn and the applications in which they are interested. To help bridge this gap, Chapter 6 discusses methods of data analysis including classical methods (mean, standard deviation, the autocorrelation function) and modern methods derived from nonlinear dynamics (time-lag embeddings, dimension and related topics). This chapter may be of particular interest to researchers interested in applying some of the concepts from nonlinear dynamics to their work.

In order to illustrate the practical use of concepts from dynamics in applications, we have punctuated the text with short essays called "Dynamics in Action." These cover a wide diversity of subjects, ranging from the random drift of molecules to the deterministic patterns underlying global climate changes.

Following each chapter is supplementary material. The notes and references provide a guide to additional references that may be fun to read and are accessible to beginning students. A set of exercises reviewing concepts and mathematical skills is also provided for each chapter. Solutions to selected exercises are provided at the end of the book. For each chapter, we also give a set of computer exercises. The computer exercises introduce students to some of the ways computers can be used in nonlinear dynamics. The computer exercises can provide many opportunities for a term project for students.

The appropriate use of this book in a course depends on the student clientele and the orientation of the instructors. In our instruction of biological science students at McGill, emphasis has been on developing analytical and geometrical skills to carry out stability analysis and analysis of asymptotic dynamics in one-dimensional finite-difference equations and in one- and two-dimensional differential equations. We also include several lectures on neural and gene networks, cellular automata, and fractals.

Although this text is written at a level appropriate to first- and second-year undergraduates, most of the material dealing with nonlinear finite-difference and differential equations and time-series analysis is not presented in standard undergraduate or graduate curricula in the physical sciences or mathematics. This book might well be used as a source for supplementary material for traditional courses in advanced calculus, differential equations, and mathematical methods in physical sciences. The link between dynamics and time series analysis can make this book useful to statisticians or signal processing engineers interested in a new perspective on their subject and in an introduction to the research literature.

Over the years, a number of teaching assistants have contributed to the development of this material and the education of the students. Particular thanks go to Carl Graves, David Larocque, Wanzhen Zeng, Marc Courtemanche, Hiroyuki Ito, and Gil Bub. We also thank Michael Broide, Scott Greenwald, Frank Witkowski, Bob Devaney, Michael Shlesinger, Jim Crutchfield, Melanie Mitchell, Michael Frame, Jerry Marsden, and the students of McGill University Biology 309 for their many corrections and suggestions. We thank André Duchastel for his careful redrawing of many of the figures reproduced from other sources. Finally, we thank Jerry Lyons, Liesl Gibson, Karen Kosztolnyik, and Kristen Cassereau for their excellent editorial assistance and help in the final stages of preparation of this book.

McGill University has provided an ideal environment to carry out research and to teach. Our colleagues and chairmen have provided encouragement in many ways. We would like to thank in particular, J. Milic-Emili, K. Krjnevic, D. Goltzman, A. Shrier, M. R. Guevara, and M. C. Mackey. The financial support of the Natural Sciences Engineering and Research Council (Canada), the Medical Research Council (Canada), the Canadian Heart and Stroke Association has enabled us to carry out research that is reflected in the text. Finally, Leon Glass thanks the John Simon Guggenheim Memorial Foundation for Fellowship support during the final stages of the preparation of this text.

We are making available various electronic extensions to this book, including additional exercises, solutions, and computer materials. For information, please contact `understanding@cnd.mcgill.ca`.

February 1995

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About the Authors

Daniel Kaplan specializes in the analysis of data using techniques motivated by nonlinear dynamics. His primary interest is in the interpretation of irregular physiological rhythms, but the methods he has developed have been used in geophysics, economics, marine ecology, and other fields. He joined McGill in 1991, after receiving his Ph.D from Harvard University and working at MIT. His undergraduate studies were completed at Swarthmore College. He has worked with several instrumentation companies to develop novel types of medical monitors.

Leon Glass is one of the pioneers of what has come to be called chaos theory, specializing in applications to medicine and biology. He has worked in areas as diverse as physical chemistry, visual perception, and cardiology, and is one of the originators of the concept of "dynamical disease." He has been a professor at McGill University in Montreal since 1975, and has worked at the University of Rochester, the University of California in San Diego, and Harvard University. He earned his Ph.D. at the University of Chicago and did postdoctoral work at the University of Edinburgh and the University of Chicago.

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