



Nonparametric **Statistics**

A Step-by-Step Approach

Second Edition

Gregory W. Corder • Dale I. Foreman

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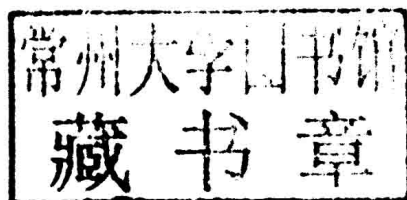
SECOND EDITION

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A Step-by-Step Approach

GREGORY W. CORDER

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NONPARAMETRIC STATISTICS

PREFACE

The social, behavioral, and health sciences have a need for the ability to use non-parametric statistics in research. Many studies in these areas involve data that are classified in the nominal or ordinal scale. At times, interval data from these fields lack parameters for classification as normal. Nonparametric statistical tests are useful tools for analyzing such data.

Purpose of This Book

This book is intended to provide a conceptual and procedural approach for nonparametric statistics. It is written so that someone who does not have an extensive mathematical background may work through the process necessary to conduct the given statistical tests presented. In addition, the outcome includes a discussion of the final decision for each statistical test. Each chapter takes the reader through an example from the beginning hypotheses, through the statistical calculations, to the final decision as compared with the hypothesis. The examples are then followed by a detailed, step-by-step analysis using the computer program SPSS®. Finally, research literature is identified which uses the respective nonparametric statistical tests.

Intended Audience

While not limited to such, this book is written for graduate and undergraduate students in social science programs. As stated earlier, it is targeted toward the student who does not have an especially strong mathematical background, but can be used effectively with a mixed group of students that includes students who have both strong and weak mathematical background.

Special Features of This Book

There are currently few books available that provide a practical and applied approach to teaching nonparametric statistics. Many books take a more theoretical approach to the instructional process that can leave students disconnected and frustrated, in need of supplementary material to give them the ability to apply the statistics taught.

It is our hope and expectation that this book provides students with a concrete approach to performing the nonparametric statistical procedures, along with their application and interpretation. We chose these particular nonparametric procedures since they represent a breadth of the typical types of analyses found in social science research. It is our hope that students will confidently learn the content presented with the promise of future successful applications.

In addition, each statistical test includes a section that explains how to use the computer program SPSS. However, the organization of the book provides effective instruction of the nonparametric statistical procedures for those individuals with or without the software. Therefore, instructors (and students) can focus on learning the tests with a calculator, SPSS, or both.

A Note to the Student

We have written this book with you in mind. Each of us has had a great deal of experience working with students just like you. Over the course of that time, it has been our experience that most people outside of the fields of mathematics or hard sciences struggle with and are intimidated by statistics. Moreover, we have found that when statistical procedures are explicitly communicated in a step-by-step manner, almost anyone can use them.

This book begins with a brief introduction (Chapter 1) and is followed with an explanation of how to perform the crucial step of checking your data for normality (Chapter 2). The chapters that follow (Chapters 3–9) highlight several nonparametric statistical procedures. Each of those chapters focuses on a particular type of variable and/or sample condition.

Chapters 3–9 each have a similar organization. They each explain the statistical methods included in their respective chapters. At least one sample problem is included for each test using a step-by-step approach. (In some cases, we provide additional sample problems when procedures differ between large and small samples.) Then, those same sample problems are demonstrated using the statistical software package SPSS. Whether or not your instructor incorporates SPSS, this section will give you the opportunity to learn how to use the program. Toward the end of each chapter, we identify examples of the tests in published research. Finally, we present sample problems with solutions.

As you seek to learn nonparametric statistics, we strongly encourage you to work through the sample problems. Then, using the sample problems as a reference, work through the problems at the end of the chapters and additional data sets provided.

New to the Second Edition

Given an opportunity to write a second edition of this book, we revised and expanded several portions. Our changes are based on feedback from users and reviewers.

We asked several undergraduate and graduate students for feedback on Chapters 1 and 2. Based on their suggestions, we made several minor changes to Chapter 1 with a goal to improve understanding. In Chapter 2, we expanded the section that describes and demonstrates the Kolmogorov–Smirnov (K-S) one-sample test.

After examining current statistics textbooks and emerging research paper, we decided to include two additional tests. We added the sign test to Chapter 3 and the Kolmogorov–Smirnov (K-S) two-sample test to Chapter 4. We also added a discussion on statistical power to Chapter 3 as requested by instructors who had adopted our book for their courses.

Since our book's first edition, SPSS has undergone several version updates. Our new edition of the book also has updated directions and screen captures for images of SPSS. Specifically, these changes reflect SPSS version 21.

We have included web-based tools to support our book's new edition. If you visit the publisher's book support website, you will find a link to a Youtube channel that includes narrated screen casts. The screen casts demonstrate how to use SPSS to perform the tests included in this book. The publisher's book support website also includes a link to a decision tree that helps the user determine an appropriate type of statistical test. The decision tree is organized using Prezi. The branches terminate with links to the screen casts on YouTube.

Gregory W. Corder
Dale I. Foreman

LIST OF VARIABLES

English Symbols

C	number of columns in a contingency table; number of categories
C_F	tie correction for the Friedman test
C_H	tie correction for the Kruskal–Wallis H -test
D, \tilde{D}	divergence between values from cumulative frequency distributions
D_i	difference between a ranked pair
df	degrees of freedom
f_e	expected frequency
f_o	observed frequency
\hat{f}_r	empirical frequency value
F_r	Friedman test statistic
g	number of tied groups in a variable
h	correction for continuity
H	Kruskal–Wallis test statistic
H_A	alternate hypothesis
H_O	null hypothesis
k	number of groups
K	kurtosis
M	midpoint of a sample
n	sample size
N	total number of values in a contingency table
p	probability
P_i	a category's proportion with respect to all categories
r_b	biserial correlation coefficient for a sample
r_{pb}	point-biserial correlation coefficient for a sample
r_s	Spearman rank-order correlation coefficient for a sample
R	number of runs; number of rows in a contingency table
R_i	sum of the ranks from a particular sample
s	standard deviation of a sample
S_k	skewness
SE	standard error
t	t statistic
t_i	number of tied values in a tie group
T	Wilcoxon signed rank test statistic
U	Mann–Whitney test statistic
\bar{x}	sample mean
y	height of the unit normal curve ordinate at the point dividing two proportions

z	the number of standard deviations away from the mean
Z	Kolmogorov–Smirnov test statistic

Greek Symbols

α	alpha, probability of making a type I error
α_B	adjusted level of risk using the Bonferroni procedure
β	beta, probability of making a type II error
θ	theta, median of a population
μ	mu, mean value for a population
ρ	rho, correlation coefficient for a population
σ	sigma, standard deviation of a population
Σ	sigma, summation
χ^2	chi-square test statistic

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NONPARAMETRIC STATISTICS: AN INTRODUCTION

1.1 OBJECTIVES

In this chapter, you will learn the following items:

- The difference between parametric and nonparametric statistics.
- How to rank data.
- How to determine counts of observations.

1.2 INTRODUCTION

If you are using this book, it is possible that you have taken some type of introductory statistics class in the past. Most likely, your class began with a discussion about probability and later focused on particular methods of dealing with populations and samples. Correlations, z -scores, and t -tests were just some of the tools you might have used to describe populations and/or make inferences about a population using a simple random sample.

Many of the tests in a traditional, introductory statistics text are based on samples that follow certain assumptions called parameters. Such tests are called *parametric tests*. Specifically, parametric assumptions include samples that

- are randomly drawn from a normally distributed population,
- consist of independent observations, except for paired values,
- consist of values on an interval or ratio measurement scale,
- have respective populations of approximately equal variances,
- are adequately large,* and
- approximately resemble a normal distribution.

*The minimum sample size for using a parametric statistical test varies among texts. For example, Pett (1997) and Salkind (2004) noted that most researchers suggest $n > 30$. Warner (2008) encouraged considering $n > 20$ as a minimum and $n > 10$ per group as an absolute minimum.

If any of your samples breaks one of these rules, you violate the assumptions of a parametric test. You do have some options, however.

You might change the nature of your study so that your data meet the needed parameters. For instance, if you are using an ordinal or nominal measurement scale, you might redesign your study to use an interval or ratio scale. (See Box 1.1 for a description of measurement scales.) Also, you might seek additional participants to enlarge your sample sizes. Unfortunately, there are times when one or neither of these changes is appropriate or even possible.

BOX 1.1

MEASUREMENT SCALES.

We can measure and convey variables in several ways. *Nominal* data, also called categorical data, are represented by counting the number of times a particular event or condition occurs. For example, you might categorize the political alignment of a group of voters. Group members could either be labeled democratic, republican, independent, undecided, or other. No single person should fall into more than one category.

A *dichotomous* variable is a special classification of nominal data; it is simply a measure of two conditions. A dichotomous variable is either discrete or continuous. A *discrete dichotomous* variable has no particular order and might include such examples as gender (male vs. female) or a coin toss (heads vs. tails). A *continuous dichotomous* variable has some type of order to the two conditions and might include measurements such as pass/fail or young/old.

Ordinal scale data describe values that occur in some order of rank. However, distance between any two ordinal values holds no particular meaning. For example, imagine lining up a group of people according to height. It would be very unlikely that the individual heights would increase evenly. Another example of an ordinal scale is a Likert-type scale. This scale asks the respondent to make a judgment using a scale of three, five, or seven items. The range of such a scale might use a 1 to represent *strongly disagree* while a 5 might represent *strongly agree*. This type of scale can be considered an ordinal measurement since any two respondents will vary in their interpretation of scale values.

An *interval* scale is a measure in which the relative distances between any two sequential values are the same. To borrow an example from the physical sciences, we consider the Celsius scale for measuring temperature. An increase from -8 to -7°C degrees is identical to an increase from 55 to 56°C .

A *ratio* scale is slightly different from an interval scale. Unlike an interval scale, a ratio scale has an absolute zero value. In such a case, the zero value indicates a measurement limit or a complete absence of a particular condition. To borrow another example from the physical sciences, it would be appropriate to measure light intensity with a ratio scale. Total darkness is a complete absence of light and would receive a value of zero.

On a general note, we have presented a classification of measurement scales similar to those used in many introductory statistics texts. To the best of our knowledge, this hierarchy of scales was first made popular by Stevens (1946). While Stevens has received agreement (Stake, 1960; Townsend & Ashby, 1984) and criticism (Anderson, 1961; Gaito, 1980; Velleman & Wilkinson, 1993), we believe the scale classification we present suits the nature and organization of this book. We direct anyone seeking additional information on this subject to the preceding citations.